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Buoyancy Driven Viscoelastic Mhd Flow Past a Porous Plate with Diffusion Thermo Effects

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***Abstract*—Diffusion thermo effect and the effect of buoyancy are studied when visco-elastic fluid past a porous plate. The fluid flow is considered as free convective incompressible two-dimensional, in presence of heat and mass transfer. Also, the magnetic field is applied in the transverse direction of the flow field. To solve the problem a method has been used called similarity technique which is incorporated to transform the non-linear partial differential equations into ordinary differential equation. MATLAB bvp4c solver is used to solve the differential equations and the effects of various flow parameters are explained and visualized graphically.**

***Index Terms*—Diffusion thermo, Buoyancy, MHD, similarity technique, MATLAB bvp4c solver**

I. INTRODUCTION

The fluid which shows both viscous and elastic nature (visco-elastic) are widely used in science, engineering and real-life technologies. viscoelastic MHD free convection are important in practical and theoretical experiment. It has significant applications in many fields specially for making soft robotics, used in renewable energy, in defence industries, crude oil extrusion, for nuclear reactor cooling, fibre spinning, in food processing, extrusion of polymer, in defence industries etc. A few areas of interest in which heat and mass transfer fluid flow combined along with the Dufour effect and buoyancy play a significant role in boundary layer control, in aerospace and aerodynamics, in blood flow modelling, for making artificial tissues and organs. It has also wide applications in geophysical studies like modelling of ice and glacier sheet, prediction of landslides and also analysis of lava and magma flows.

Cussler (1988) has studied the diffusion mass transfer in fluid systems. Chambre and Young (1958) have investigated on the diffusion of chemically reactive species in a laminar boundary layer. Gupta *et al*. (1979) have worked on free convection flow past a linearly accelerated vertical plate in the presence of energy dissipation. Kafousias and Raptis (1989) have carried the above work to study the mass transfer effects subjected to variable suction or injection. Later, the mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources has been studied by Jha and Prasad (1990).

Eckert and Drake (1972) realized the importance of thermal-diffusion and diffusion-thermo effects on MHD fluid flows. Saxena and Dubey (2011_1) have investigated the flow behavior of unsteady MHD heat and mass transfer free convection flow of a polar fluid past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion. Later, Saxena and Dubey (2011_2) have generalized their previous work in presence of radiation and variable permeability in a slip flow regime. Saravana *et al*. (2011) have investigated mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux. Kumar *et al*. (2012) have studied thermal diffusion and radiation effects on unsteady MHD flow through porous medium with variable temperature and mass diffusion in the presence of heat source or sink. Diffusion-thermo and radiation effects on MHD free convective heat and mass transfer flow past an infinite vertical plate in the presence of a chemical reaction of first order have been investigated by Raveendra Babu *et al.* (2012). Choudhury *et al*. (2012) also generalized their works. Thermal and solutal buoyancy effect on MHD boundary layer flow of a visco-elastic fluid past a porous plate with varying suction and heat source in the presence of thermal diffusion has been studied by Chandra *et al.* (2015). Reddy *et al.* (2020) have analyzed the analytical study of buoyancy effects on MHD visco-elastic fluid past an inclined plate. The extended work is carried out by Mayur *et al.* (2024) along with temperature dependent viscosity on unstable condition. Mayur *et al.* (2025) also carried out stability analysis of buoyant visco-elastic fluid flow in a vertical porous layer with horizontal through flow.

Therefore, the present study aims to analyze specifically the buoyancy and Dufour effect (diffusion-thermo) along with heat and mass transfer on visco elastic fluid flow past a porous plate when magnetic field is imposed in lateral direction of the plate. The solutions for velocity, temperature and shearing stress are obtained and compare it with the previous work done by various authors.

II. MATHEMATICAL FORMULATION

The steady two-dimensional MHD free convection with heat and mass transfer flow of an incompressible electrically conducting visco-elastic fluid (Walter's liquid model B') past a vertical porous plat in the presence of buoyancy force and diffusion thermal effect is considered. A magnetic field of strength B_0 is applied in transverse direction of the plate. Let x/- axis be taken along the plate in the vertically upward direction and y/-axis be taken along the normal to the plate. Let us assume that $R_e \ll 1$ (magnetic Reynolds number) so that the induced magnetic field is neglected here compare to applied magnetic field B_0 . Hence the basic governing flow is totally depended upon buoyancy force driven by temperature difference between the wall and the medium. Under the above assumption and using Boussinesq's approximation, the governing equations of motion are as follows:

EQUATION OF CONTINUITY:

$$\frac{\partial v'}{\partial y'} = 0 \qquad (1)$$

EQUATION OF MOMENTUM:

$$\rho v' \frac{\partial u'}{\partial y'} = \eta_0 \frac{\partial^2 u'}{\partial y'^2} + \rho g \beta (T' - T'_\infty) + \rho g \beta^* (C' - C'_\infty) - k_0 \left[v' \frac{\partial^3 u'}{\partial y'^3} \right] - \sigma B_0^2 u' \qquad (2)$$

ENERGY EQUATION:

$$\rho C_p \left(v' \frac{\partial T'}{\partial y'} \right) = k_T \left(\frac{\partial^2 T'}{\partial y'^2} \right) + S_l (T' - T'_\infty) + \left[\rho \frac{D_m K_T}{C_s} \frac{\partial^2 C'}{\partial y'^2} \right] \qquad (3)$$

CONCENTRATION EQUATION

$$v' \frac{\partial C'}{\partial y'} = D \left(\frac{\partial^2 C'}{\partial y'^2} \right) + D_l \left(\frac{\partial^2 T'}{\partial y'^2} \right) \qquad (4)$$

SUBJECT TO THE BOUNDARY CONDITIONS:

$$y' = 0 : u' = U,\ v' = -v_0 = constant,\ T' = T'_w \ , C' = C'_\infty \qquad (5)$$
$$y' \to \infty : u' \to 0, \quad T' \to T'_\infty , \quad C' \to C'_\infty$$

WE INTRODUCE THE FOLLOWING NON-DIMENSIONAL QUANTITIES:

$$u = \frac{u'}{U} , y = \frac{y'U}{\nu}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} , \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \lambda = \frac{v_0}{U} , M = \frac{\sigma B_0^2 \nu}{\rho U^2}$$

$$Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{U^3}, Gc = \frac{\nu g \beta^* (C'_w - C'_\infty)}{U^3}, Pr = \frac{\eta_0 C_p}{K_T}$$

$$Sc = \frac{\nu}{D}, k = \frac{k_0 U^2}{\rho \nu^2}, Du = \frac{D_m K_T (C'_w - C'_\infty)}{C_s C_p \nu (T'_w - T'_\infty)}, S = \frac{\nu S'}{{v_0}^2}, S_0 = \frac{D_l (T'_w - T'_\infty)}{\nu\big((C'_w - C'_\infty)\big)}$$

where Gr is the thermal Grashof number, Gc is the solutal Grashof number, Pr is the Prandtl number, M is the Hartmann number, k is the dimensionless visco-elastic parameter, Sc is the Schmidt number and Du is the coefficient of mass diffusivity, S is the heat source parameter, S_0 is the Soret number.
The non-dimensional forms of the equations (2) to (4) are

$$k\lambda \frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} - \lambda \frac{du}{dy} + Mu = -(Gr\theta + Gc\phi) \quad (6)$$

$$\frac{1}{Pr}\frac{d^2\theta}{dy^2} - \lambda \frac{d\theta}{dy} + S\theta = -Du \frac{d^2\phi}{dy^2} \quad (7)$$

$$\frac{1}{Sc}\frac{d^2\phi}{dy^2} + S_0 \frac{d^2\theta}{dy^2} - \lambda \frac{d\phi}{dy} = 0 \quad (8)$$

RELEVANT TO THE BOUNDARY CONDITIONS:
$y = 0: \quad u = 1, \theta = 1, \phi = 1$
$y \to \infty: \ u \to 0, \quad \theta \to 0, \quad \phi \to 0$ (9)

III. METHOD OF SOLUTION

To solve the equation (6), (7) and (8) we have used Matlab bv4pc solver which is a core matlab function for solving systems of ordinary differential equation governing the fluid motion with the help of relevant boundary conditions given above.
The solutions of the equations (7) and (8) which represents temperature and concentration profiles respectively are given by

$$\theta = C_4 e^{-\omega_4 y} + (1 - C_4) e^{-\omega_2 y}$$
$$\phi = e^{-\omega_2 y}$$

The Velocity Profile Is Given By
$$u = C_8 e^{-\omega_6 y} + C_9 e^{-\omega_4 y} + C_{20} e^{-\omega_2 y} + k(C_{18} e^{-\omega_4 y} + C_{19} e^{-\omega_2 y} - C_{21} e^{-\omega_6 y})$$

The non-dimensional shearing stress at the plate in the direction of free stream is given by
$$\frac{\tau'}{\rho U^2} = \tau = \left(\frac{du}{dy} + k\lambda \frac{d^2u}{dy^2}\right)_{at\ y=0}$$
$$= -\omega_6 C_8 - \omega_4 C_9 - \omega_2 C_{20} + k(-\omega_4 C_{18} - \omega_2 C_{19} + \omega_6 C_{21}) + k\lambda(\omega_6^2 C_8 + \omega_4^2 C_9 + \omega_2^2 C_{20})$$

The heat flux from the plate to the fluid in terms of Nusselt number is given by
$$Nu = \left(\frac{d\theta}{dy}\right)_{y=0} = -(\omega_4 C_4 + \omega_2 - \omega_2 C_4)$$

The mass flux at the wall in terms of Sherwood number is given by
$$Sh = \left(\frac{d\phi}{dy}\right)_{y=0} = -\omega_2$$

IV. RESULTS AND DISCUSSIONS

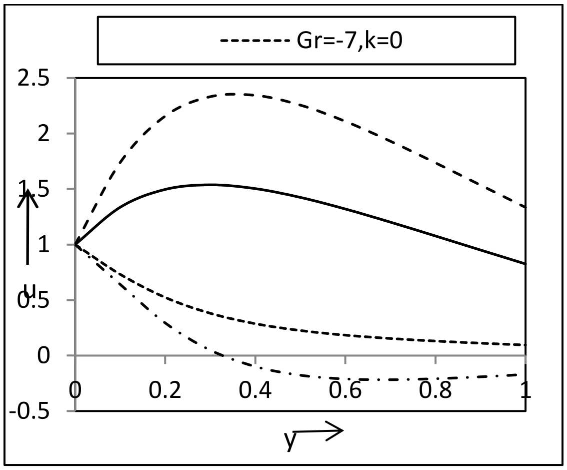

FIGURE 1.1: Velocity u against y for M=1, Gm=10, Sc=5,
S_0 =1, Du=0.5, S=5, Pr=5

For studying the behavior of velocity profile under various flow parameters we have considered positive as well as negative values of thermal Grashof number. Gr > 0 signifies that the fluid flow is through an externally cooled plate and Gr <0 indicates that the flow is through an externally heated plate. The behaviour of velocity profile for positive and negative values of Grashof number are seen in the figure 1.1. A parabolic nature is noticed in velocity profile for positive Grashof number but for negative Grashof number, a back flow is noticed for k ≤ 0.1 as the fluid moves away from the plate.

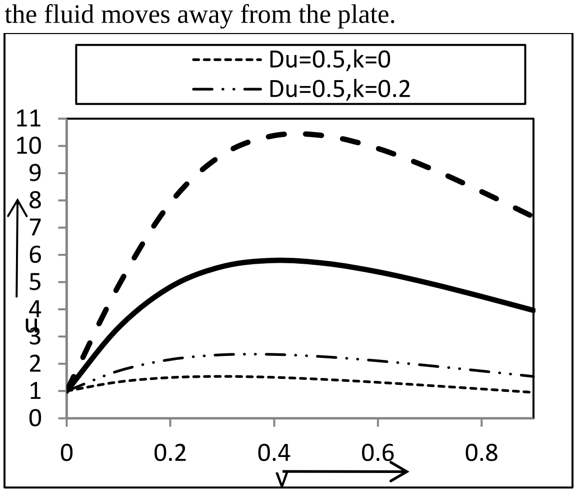

FIGURE 1.2 Velocity u against y for Sc=5, Gm=10, Gr=7
M=1, S_0=1, S=5, Pr=5.

The Dufour effect is energy flux due to mass concentration gradient. It is represented through the parameter Du. The increasing values of Dufour parameter raises the speed of fluid flows with the enhancement of visco-elastic parameter (Figure 1.2).

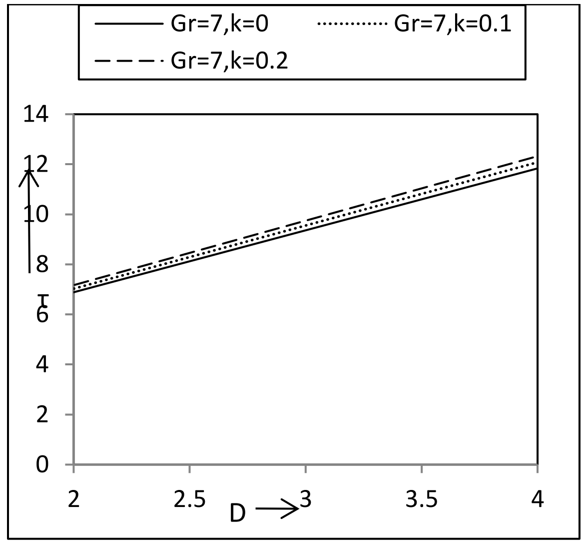

FIGURE 1.3: Shearing stress against Du for M=1, Gm=10, Sc=5
S=5, S_0=1, Pr=5

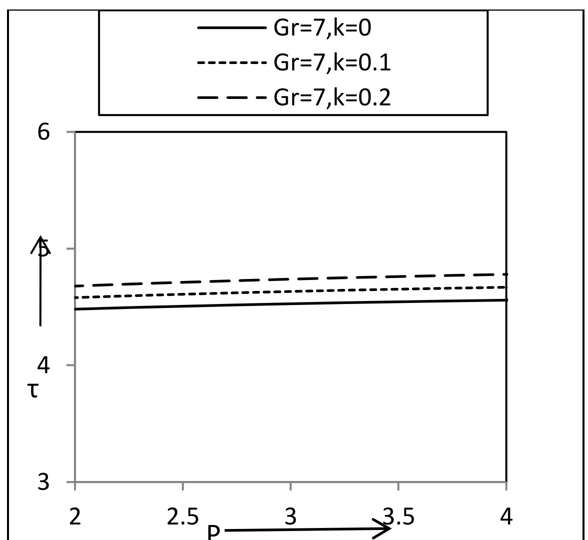

FIGURE 1.4: Shearing stress against Pr for M=1, Gm=10, Sc=5, S=5
S_0=1, Du=0.5

when visco-elastic parameter of non-Newtonian fluids increases, the fluid becomes thicker and as a consequence of this, the viscous or drag force will be enhanced compare to Newtonian fluid. Figure 1.3 and 1.4 denote the nature of shearing stress experienced by the fluid flow against Prandtl number and Dufour number respectively. In both the cases, when Prandtl number and Dufour number rises, the magnitude of shearing stress also increases along with the increasing values of visco-elastic parameter.

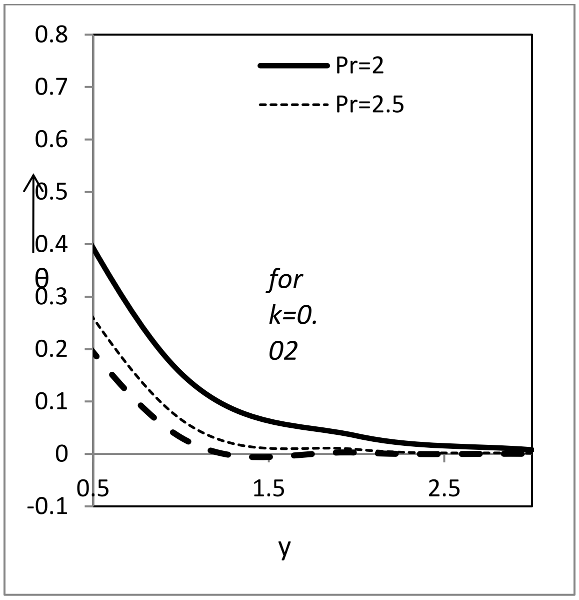

FIGURE 1.5: Temperature θ against y for M=3, Gr=5, Gm=5,
S_0=1, S=5, Du=0.5

Temperature profile θ for various values of Prandtl number (Pr) are plotted against y in the figures 1.5. Effects of Prandtl number (Pr) on temperature fields for visco-elastic fluid are illustrated in figure 1.5. It is likely to noticed that temperature profile θ decreases with y for amplified values of Pr. From this we can say that the thermal boundary layer is thinner for higher Prandtl number.

V. CONCLUSION

The numerical study of the present paper gives a theoretical idea about the effects of visco-elastic parameter on the two-dimensional steady free convective MHD flow past a vertical porous plate with buoyancy and Dufour effect taking into account the heat and mass transfer. This above study leads to the following conclusions:

- Visco-elastic parameter affects the entire velocity fields along with other physical flow parameters.
- At the vicinity of the plate the effect of visco-elasticity is maximum.
- An increase in the visco-elastic parameter amplifies the magnitude of shearing stress.
- The velocity profile accelerates for externally cooled plate but decelerates for externally heated plate in both Newtonian and visco-elastic fluids.
- The velocity profile increases with the increasing values of Dufour number in both Newtonian and visco-elastic fluids.
- The rising values of Prandtl number and Dufour number enhanced the magnitude of shearing stress.
- An increase in Prandtl number reduced the temperature profile.
- There are no significant buoyancy effects in the flow field.

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