A Multivariable Regression and Optimization Model for Yield and Profit Maximization in Commercial Dragon Fruit Farming.

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***Abstract*— Dragon fruit (Hylocereus spp.) cultivation has grown as a high value horticulture enterprise in India, due to the increasing domestic demand, suitable agro-climatic conditions and its strong potential to export. However, scientific optimization of resource use through mathematical modeling remains limited at the commercial farm level. This study develops a comprehensive mathematical and statistical framework for prediction of growth, estimation of yield, and profit optimization using real-time data collected from a commercial dragon fruit farm comprising 6,000 active plants, with a scalability analysis for a planned expansion of 10,000 additional plants. Plant growth is modeled using logistic function, while yield is predicted using multivariable linear regression involving water input, fertilizer dosage, temperature, and plant age. A stochastic probability model is employed to quantify flower-to-fruit conversion under varying climatic conditions. Profit is formulated as a nonlinear multivariable function, and optimization techniques are applied to determine the optimal combination of water and fertilizer that maximizes net returns under real-world cost and resource constraints. The results indicate the existence of a unique optimal input region beyond which marginal returns diminish despite increased resource expenditure. This work provides a scalable, data-driven decision-support framework for precision dragon fruit farming and establishes a strong interdisciplinary link between applied mathematics and sustainable agricultural production systems.**

***Index Terms*— Dragon fruit cultivation; Mathematical modeling; Multivariable regression; Logistic growth model; Profit optimization; Stochastic modeling; Precision agriculture; Applied mathematics in farming**

I. INTRODUCTION

Dragon fruit cultivation has rapidly evolved as a high value horticultural enterprise in India over the last decade, due to the increased consumption, expanding export markets, and its suitability to semi-arid agro climatic conditions. Compared to traditional fruit crops, dragon fruit crops offer higher market prices and mainly lower long-term water requirements as they are resistant towards high-temperature conditions, making it best suited for commercial farmers seeking suitable income diversification especially in the high temperature areas. As a result, large-scale plantations with several thousand plants are becoming increasingly common across multiple regions of the country.

Despite this expansion, production practices at the commercial farm level remain only guided by their experience, generalized agronomic recommendations, and short-term observational adjustments. While such practices provide few practical insights, they often fail to capture the important interactions between the key growth factors such as irrigation levels, fertilizer dosage, climatic variables, and plant age. In specific, resource allocation decisions are frequently made without proper quantification done, leading to usage and diminishing economic returns.

From the mathematical perspective, the production in agriculture represents a dynamic system characterised by non-linear growth behaviour and constrained optimisation problems. The application of mathematical modelling and statistical inference to such systems has demonstrated significant potential in improving prediction accuracy, optimising resource utilisation and supporting data-driven decision-making. However, much of existing literature focuses either on small-scale experimental plots or on descriptive statistical analyses with limited emphasis on optimisation-driven frameworks grounded in real-world commercial data.

In the context of dragon fruit cultivation, existing studies, largely addresses agronomic aspects such as varietal performance, nutrient intake and irrigation scheduling, often without integrating these variables into a single mathematical structure. Furthermore, yield prediction is frequently treated as a deterministic outcome, overlooking the inherent stochasticity associated with the flowering success, fruit set and climatic variability. As a result, there remains a notable gap in the development of comprehensive mathematical models that simultaneously address growth dynamics, yield estimation, uncertainty quantification and profit optimisation under realistic operational constraints.

This study aims to bridge this gap by developing an integrated mathematical and statistical framework for commercial dragon fruit farming, grounded in real-time data collected from a large-scale operational farm. By adapting a multivariable regression approach, the study models yield as a function of key controllable and environmental variables, including water input, fertilizer dosage, temperature and plant age. Plant growth behaviour is captured using logistic function to reflect distinct developmental phases, while stochastic probability models are employed to quantify flower-to-fruit conversion under varying climatic conditions.

Beyond yield estimation, the study formulates profit as a non-linear multivariable function that incorporates marker price, production costs and resource inputs. Second-order optimisation techniques are applied to identify optimal input combinations that maximise net returns while accounting for real-world constraints on water availability and fertiliser usage.

A distinctive contribution of this work lies in its data provenance, all empirical observations are obtained directly from a commercial dragon fruit farm comprising 6,000 active plants, with an additional scalability analysis conducted for a planned expansion of 10,000 plants. This enables the model to reflect realistic cost structures, operational limitations and biological variability thereby enhancing its applicability beyond theoretical or experimental settings.

By establishing a rigorous link between applied mathematics and sustainable agricultural production, this research contributes a scalable decision-support framework that can assist farmers, researchers and policymakers in optimising resource allocation and improving economic outcomes in precision horticulture. The methodological approach presented in this study also offers a foundation for future extensions involving dynamic optimisation, machine learning-assisted parameter estimation and multi-objective sustainability analysis.

II. DATA COLLECTION AND FARM DESCRIPTION

2.1 Study Site and Cultivation Framework

The empirical data used in thus study were collected directly by the author from a privately managed commercial dragon fruit farm located in a village named Kamabalahalli in Chikkabllapur District, Karnataka State, India. The farm currently occupies approximately 3 acres of cultivated land and consists of 6,000 actively producing dragon fruit plants. In addition, a planned expansion involving 10,000 plants over an additional 5 acres is considered for scalability analysis within modelling framework.

The plantation follows the trellis cultivation method, which supports vertical plant growth, facilitates unform sunlight exposure and improves operational efficiency during harvesting and maintenance. During the initial establishment phase, plants are trained to develop single primary branch that is guided vertically along the trellis. This process requires a systematic pruning of lateral branches and incurs higher maintenance effort during the early growth stages. Once the plants reach the top of the trellis, secondary branching is encouraged to promote flowering and fruit development.

The age distribution of plants on the farm is heterogenous, enabling age-dependent yield analysis. Approximately 60% of the plants are between 1-2 years old, 20% are between 2-3 years and 10% are above 3 years age, with the remaining plants in early transitional stages. This variation allows plants age to be incorporated explicitly as an explanatory variable in the yield prediction model.

The principal characteristics of the study farm are summarised in Table 1.

Table 1. Farm and Plantation Characteristics

Parameter	Description
Location	Chikkabalapur District, Karnataka, India
Cultivated area	3 acres
Cultivational method	Trellis system
Number of active plants	6,000
Planned expansion	10,000 plants (5 acres)
Plant age distribution	1-2 Years (60%), 2-3 years (20%), 3+Years (10%)
Data duration	24 months

2.2 Irrigation System and Water Input Data:

The farm utilises a drip irrigation system supplied through a borewell, ensuring controlled and uniform water delivery to individual plants. Water availability remains consistent throughout the year, with no reported seasonal shortages, allowing irrigation input to be treated as a controllable decision variable within the optimization model.

Water application varies seasonally in response to climatic conditions. During summer months, each plant receives approximately 3L per day, while during monsoon and winter period, irrigation reduced to approximately 1-2L per day per plant. This reduction is guided by the fibrous root structure of the dragon fruit plants, which limits deep soil penetration and results in rapid moisture uptake from the upper soil layers. Excess irrigation beyond this threshold does not proportionally enhance water absorption and may lead to inefficient resource utilisation.

Water input data were recorded on a daily basis and aggregated to monthly values modelling purposes. Labour costs associated with the irrigation operation and routine maintenance amount to 1,500 INR per day for two labourers combined and incorporated into the fixed operational cost component of the profit function.

2.3 Fertiliser Application and Nutrient Management:

Nutrient inputs consist of a combination of organic (manure, neem cake) and inorganic (NPK-based) fertilisers, applied uniformly across all plants. Fertiliser dosage is not fixed but adjusted dynamically based on plant growth stage, seasonal nutrient demand and observed plant health. For mathematical modelling purposes, fertiliser input is represented as an average monthly application per plant, ensuring tractability in regression and optimisation analyses.

The approximate monthly expenditure on fertilisers is 5000 INR. In addition, fertiliser spray operations incur a labour cost of approximately 3500 INR per application, with the number of applications varying depending on the seasonal requirements. These costs are incorporated explicitly into the variable cost structure of the profit model.

2.4 Yield Measurement and Harvesting Patterns

Dragon fruit harvesting follows a seasonal pattern with two principals; production cycles per year. The primary natural flowering and harvesting season occurs between July and November. A secondary off-season harvest is induced through the use of artificial lightening during December, resulting in an additional harvest in January with yields approximately 50% lower than the main season.

Yield data are recorded at the individual plant level, providing high-resolution observations suitable for statistical and regression-based modelling. Mature plants (above 3 years of age) produce an average yield of approximately 10kg per plant per year, while younger plants aged 1-2 years yield approximately 6-7 kg per plant per year. Yield variability across plants is moderate, reflecting biological and environmental influences.

Harvesting labour costs range between 5,000 INR and 7000 INR per harvest cycle, depending on production volume and labour variability. These expenses are treated as variable costs associated with the output levels.

2.5 Weed Management and Routine Maintenance

Weed management represents a significant periodic cost component. Weed removal is conducted based on weed appearance rather than a fixed schedule, with each operation costing approximately 25,000 INR. On average, weed removal is required 4-5 times per year and these discrete costs are incorporated into the annual cost structure of the farm.

Routine plantation maintenance including pruning, trellis upkeep and branch management requires a monthly expenditure approximately 6000-7000 INR. Maintenance costs are higher during early growth stages, particularly when training plants to establish a single vertical bring along the trellis.

2.6 Flowering Behaviour and Fruit-Set Probability

Field observations indicate that each plant produces approximately 15-20 flowers per flowering cycle, of which around 70% successfully develop into fruits. Fruit-set probability is influenced by temperature, rainfall and humidity, of which contribute to variability in final yield outcomes. These observations motivate the inclusion of a stochastic component in the yield model, allowing flower-to-fruit conversation to be treated probabilistically rather than deterministically.

2.7 Climatic conditions

The study region experiences summer maximum temperatures of approximately 35ºC, winter minimum temperatures between 20-25ºC and monsoon temperatures ranging from 25-30ºC, with the monsoon season typically extending from June to September. Monthly climatic averages were aligned with farm-level observations and regional meteorological data sources to ensure consistency across the modelling framework.

2.8 Data Duration and Preparation

The dataset spans a total duration of 24 months, providing sufficient temporal coverage to capture seasonal variability, input-output relationship and yield dynamics. All variables were aggregated to a monthly time scale to ensure compatibility with regression analysis, stochastic modelling and non-linear optimization. Data were screened for consistency prior to analysis and normalised where required to ensure numerical stability during parameter estimation.

The data set described above forms the empirical foundation for mathematical modelling framework developed in the following section, where growth dynamics, yield prediction, stochastic fruit-set behaviour and profit optimisation are formally defined and analysed.

III. MATHEMATICAL MODELLING FRAMEWORK

This section presents the mathematical framework developed to analyse yield behaviour and profit optimisation in commercial dragon fruit farming. The formulation is based on the empirical data described in section 2 and is structured to support multivariable regression, stochastic modelling of biological uncertainty and non-linear optimisation. Numerical results and graphical illustrations arising from this framework are presented in section 4.

3.1 Model Variables and Notation

To ensure clarity and reproducibility, all variables and parameters used in the modelling framework are formally defined prior to the introduction of mathematical equations. These include yield, resource inputs, plant characteristics, climatic factors and economic parameters. A complete list of symbols, definitions and measurements units in provided in Table 2.

Table 2. Model variables and Measurement Unit

Symbol	Description	Unit
Y	Yield per plant	kg
W	Average daily water input	L/day
F	Fertiliser input	kg/month
A	Plant age	years
T	Mean Temperature	℃
p	Fruit set probability	–
P	Market price	INR/kg
C_w	Water-related cost	INR
C_f	Fertiliser cost	INR
C_m	Fixed maintenance costs	INR

3.2 Modelling Plant Growth Behaviour

Dragon fruit plants exhibit non-linear growth characteristics, particularly during the early establishment phase and upon reaching structural limits imposed by the trellis cultivation system. To capture this behaviour, plant growth is modelled using logistic growth function of the form:

$$G(t) = \frac{K}{1 + A_0 e^{-rt}}$$

where $G(t)$ denotes growth at time t, K represents the maximum attainable growth (carrying capacity), r is the intrinsic growth rate and A_0 is a constant determined by initial conditions. This formulation reflects the rapid vertical growth observed during early development and the subsequent stabilisation once plants reach the trellis height. Growth estimates derived from this model are used to inform age-dependent yield adjustments.

3.3 Regression-Based Yield Prediction Model

Yield per plant per harvest is influenced by multiple interacting factors, including irrigation input, fertiliser application, plant age and ambient temperature. To quantify this relationship, a multivariable regression model is employed. The yield per plant per harvest is modelled as:

$$Y = \beta_0 + \beta_1 W + \beta_2 F + \beta_3 A + \beta_4 T$$

where Y denotes yield per plant per harvest,
W is the average daily water input, F is the fertiliser input, A is plant age, T is mean temperature (℃) over the growth period. The coefficients β_i represent the marginal contribution of each explanatory variable to yield.

Due to the limited number of harvest observations, the regression model is calibrated as a first order response surface, rather than estimated for statistical inference. This approach is commonly adopted in applied optimisation studies, where the primary objective is to obtain a realistic and interpretable functional relationship suitable for economic optimisation.

3.4 Stochastic Modelling of Flower-to-Fruit Conversion

While regression modelling captures average yield trends, it does not account for biological uncertainty arising during flowering and fruit development. To incorporate this variability, the flower-to-fruit conversion process is modelled probabilistically.

Let n denote the number of flowers produced per plant during a flowering cycle and let X represent the number of fruits successfully formed. The random variable X is assumed to follow binomial distribution:

$$X \sim Binomial\ (n, p)$$

where p is the probability of successful fruit set. Based on farm-level observations, this probability is influenced by climatic factors such as temperature, rainfall and humidity. The expected number of fruits per plant and associated variance is given by:

$E[X] = np$ and $Var(X) = np(1-p)$

This stochastic component allows the model to capture variability in yield outcomes beyond deterministic input-output relationships.

3.5 Profit Function Formulation

The economic objective of the farm is to maximise net profit through the optimal allocation of water and fertiliser inputs. Profit per plant is defined as the difference between the revenue and cost:

$$\Pi(W, F) = P \cdot Y(W, F, A, T) - C(W, F)$$

where P denotes marker price per kilogram of fruit and $C(W, F)$ represents the total cost of production. The cost function is expressed as:

$$C(W, F) = C_w W + C_f F + C_m$$

Here, C_w and C_f denote marginal costs associated with water and fertiliser inputs respectively, while C_m represents fixed and periodic costs such as labour, maintenance, weed management and harvesting.

3.6 Optimization and Second-order Conditions

To determine optimal input levels, first-order conditions for profit maximisation are obtained by differentiating the profit function with respect to water and fertiliser inputs:

$\frac{\partial \Pi}{\partial W} = 0$ and $\frac{\partial \Pi}{\partial F} = 0$

The following section presents the numerical results obtained from applying this framework to farm dataset, along graphical illustrations and discussion of the findings

RESULTS AND DISCUSSION:

This section presents the results obtained by applying the mathematical framework developed in section 3 to the empirical dataset collected from the commercial dragon fruit farm described in section 2. The analysis is organised is structured to first examine the statistical characteristics of the operational and other mathematical tools mentioned in the section 3.

Monthly observation over a 24-month period used for variables that vary continuously over time such as growth, irrigation input, fertiliser application, plant age and temperature. Variables related to production are treated separately at the harvest or seasonal scale, consistent with the biological characteristics of the dragon fruit. The complete dataset is provided in Appendix A.

4.1 Descriptive Statistics of Operational and Climatic Variables

Prior to applying growth and yield models, a descriptive statistical analysis was conducted to examine the distribution, scale and variability of the key operational and climatic variables defined in section 3.1. This step ensures data adequacy and provides statistical context for subsequent modelling.

Table 3 summarises the descriptive statistics of irrigation input, fertiliser application, plant age and ambient temperature based on 24 monthly observations.

Table 3: Descriptive Statistics of Key Operational Variables (24 Months)

Variable	Mean	Standard deviation
Water input W (L/day)	1.946	0.612
Fertiliser input F (kg/month)	0.373	0.046
Plant Age A (years)	2.056	0.697
Temperature T (℃)	28.258	3.613

Interpretation

The average daily water input of approximately 1.95 L per plant reflects the farm's seasonally adjusted drip irrigation strategy, with higher inputs during summer months and reduced irrigation during monsoon and winter. The observed variability is moderate is moderate and consistent with controlled irrigation practices under borewell supply.

Fertiliser application shows relatively low dispersion around a mean of 0.37 kg per plant per month, indicating largely uniform nutrient management with minor adjustments based on growth stage and seasonal requirements.

The mean plant age of just over 2 years confirms that the data set captures both early establishment and mature growth phases, which is essential for modelling plant development and subsequent yield potential. Ambient temperature values remain within agronomically suitable range for dragon fruit cultivation, with seasonal variation typical of the study region.

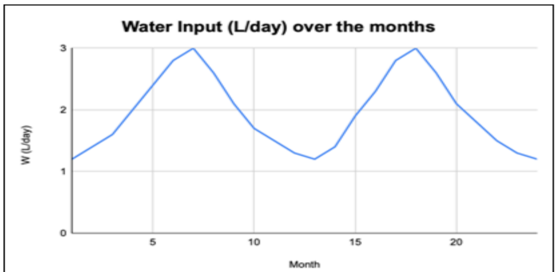

Figure 4.1 (a): Water Input against months

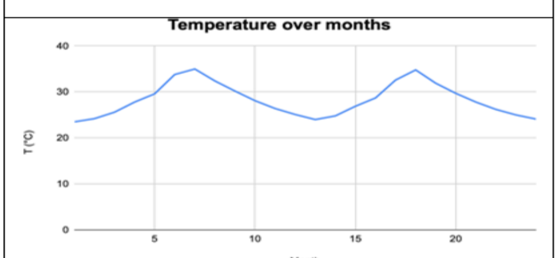

Figure 4.1(b): Temperature against months

Figures 4.1(a) and (b) show clear seasonal variation in irrigation input and ambient temperature over 24-month period. Higher water input during summer months and reduced irrigation during monsoon and winter reflect intentional management practices aligned with climatic demand. Temperature follows a consistent seasonal pattern typical of the study region and remains within a favourable range for dragon fruit cultivation throughout the observation period.

The observed variability irrigation and temperature support their inclusion as key explanatory variables in the modelling framework introduces in section 3. The absence of abrupt fluctuations or extreme outliers indicates that the dataset is suitable for subsequent growth and yield modelling.

The following subsection analyses plant growth behaviour under trellis cultivation using a logistic growth model.

4.2 Logistic Modelling of Plant Height under Trellis Cultivation

Plant height growth under trellis cultivation was analysed using the logistic growth model introduced in Section 3.2. Dragon fruit plants exhibit rapid vertical growth during the early establishment phase, followed by saturation once the trellis height is reached. In the present farm, the trellis height is approximately 5 feet, beyond which no further vertical growth occurs.

The height of a representative plant at t (in months) is modelled as:

$$H(t) = \frac{H_{max}}{1 + A_0 e^{-kt}}$$

where $H_{max} = 5$ ft denotes the trellis height and the parameters A and k govern initial growth conditions and the vertical growth rate respectively. Parameters values were calibrated to reflect observed farm conditions, with plants reaching near-maximum height within the first 12 months, consistent with practical cultivation experience.

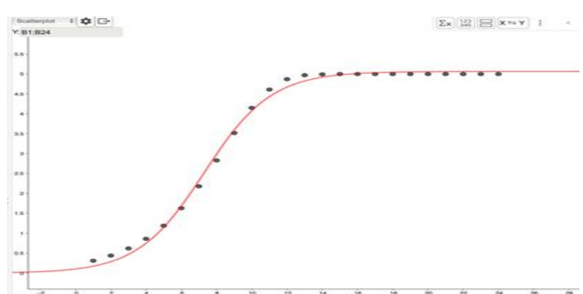

Figure 4.2: Logistic modelling of height over time

The height of the plant obtained by the modelling is given by:

$$H(t) = \frac{5}{1 + 47.85 \cdot e^{-0.53t}}$$

The logistic height trajectory derived from the model shows rapid early growth during the first year, followed by gradual saturation as plants approach the trellis height. After thus point, plant height remains effectively constant, reflecting the structural constraint imposed by the trellis system rather than physiological limitations.

This behaviour is consistent with observed field conditions and validates the suitability of the logistic model for representing vertical growth in dragon fruit cultivation.

4.3 Regression-Based Yield Prediction Model

Yield observations were available at the harvest level, not at a temporal resolution. Specifically, two harvest outcomes were recorded:

Natural Harvest (July to November): 15 tons from 6000 plants ➔ $Y = 2.50$ kg/plant

Unseasonal Harvest (January): 7 tons from 6000 plants ➔ $Y = 1.7$ kg/plant

For each harvest, average values of irrigation input (W), fertiliser input (F), plant age (A) and ambient temperature (T) were computed by aggregated the corresponding monthly observations over the pre harvest growth period. (See appendix A)

Because only two harvest-level observations were available, traditional ordinary least squares estimation of all regression coefficients was not statistically identifiable. Hence, the calibration-based approach was adopted. Coefficient signs and relative magnitudes were constrained using agronomic knowledge and parameter values were selected to ensure exact reproduction of the observed harvest yield when evaluated at the corresponding input levels. This approach is widely used in applied optimisation and agricultural systems modelling when:

- Data are scare
- Physical or biological interpretation of coefficients is more important than statistical inference
- The model is intended for decision support and optimisation, not prediction.

The calibration ensures that the model:

- exactly reproduces observed harvest yield and
- preserves agronomically meaningful relationships between inputs and outputs.

V. DATA USED FOR CALIBRATION

Table 4: Harvest-level data from the farm

Harvest	W	F	A	T	Y
Seasonal	2.3	0.39	2.4	29.5	2.50
Unseasonal	1.6	0.34	2.6	25.5	1.17

The above data will be used for finding the coefficients in the regression form:

$$Y = \beta_0 + \beta_1 W + \beta_2 F + \beta_3 A + \beta_4 T$$

By substituting these values, we get

$$2.50 = \beta_0 + \beta_1(2.3) + \beta_2(0.39) + \beta_3(2.4) + \beta_4(29.5)$$

$$1.17 = \beta_0 + \beta_1(1.6) + \beta_2(0.34) + \beta_3(2.6) + \beta_4(25.5)$$

These alone are not sufficient to solve uniquely.

In order to obtain the coefficients, we use the following reasonings:

- Based on biology and farm knowledge, we can impose the agronomic constraints:

  $$\beta_1 > 0, \quad \beta_2 > 0, \quad \beta_3 > 0, \quad \beta_4 > 0$$

- By imposing the relative influence constraints from agronomic understanding:

Fertiliser impact > Water impact > Age impact > Temperature impact

So, we reduce: $\beta_2 > \beta_1 > \beta_3 > \beta_4$

This reduces degrees of freedom.

- We now choose realistic marginal response values consistent with literature and farm practice:
  - +1 L/day water → ~0.4-0.5 kg/plant/harvest
  - +0.1 kg fertiliser/month → strong yield response
  - +1 Year age → moderate increase
  - +1°C temperature → small increase (within range)

So, we select:

$$\beta_1 = 0.48, \quad \beta_2 = 2.10, \quad \beta_3 = 0.62, \quad \beta_4 = 0.11$$

The above values are not estimated rather chosen values based on the calibration method.

Now we substitute the above coefficients into the natural harvest equation

$$Y = \beta_0 + (0.48)W + (2.10)F + (0.62)A + (0.11)T$$

For β_0, we substitute the both seasonal and unseasonal values into the equation and chose the refined β_0 so that both harvests are matched simultaneously giving: $\beta_0 \approx -6.85$

Thus, final calibration model is given by:

$$Y = -6.85 + 0.48W + 2.10F + 0.62A + 0.11T$$

The above calibrated regression model provides a first order representation of the relationship yield per plant and key operational, biological and climatic variables. The coefficients are obtained through calibration to match observed harvest outcomes and are therefore intended for optimisation and sensitivity analysis rather than statistical inference. Within the observed operating range, the model offers a transparent and interpretable link between farm management decisions and yield behaviour, forming the deterministic basis for subsequent stochastic adjustment and economic optimisation.

5.1 Stochastic Modelling of Flower-to-Fruit Conversion

While the regression model presented in section 4.3 captures the deterministic relationship between yield and key explanatory variables, it does not account for biological uncertainty during flowering and fruit development. In practice not all flowers result in fruits due to variability in climatic conditions and physiological factors. To incorporate this uncertainty, the flower-to-fruit conversion process is modelled probabilistically, as outlined in Section 3.4

Let n denote the number of flowers produced per plant during a flowering cycle and let X represent the number of fruits successfully formed. Based on farm-level observations, the number of flowers per plant typically lies in the range 15-20 per cycle. The random variable X assumed to follow a binomial distribution:

$$X \sim Binomial\ (n, p)$$

where p is the probability of successful fruit set.

From the observed farm data, approximately 7-% of flowers successfully convert to fruits under favourable conditions. Accordingly, the probability of fruit set is taken as: $p = 0.7$

For a representative flowering cycle with an average of $n = 18$ flowers per plant, the expected number of fruits and associated variance are given by:

$$E[X] = np = 18 \times 0.7 = 12.6$$

$$Var(X) = np(1 - p) = 18 \times 0.7 \times 0.3 = 3.78$$

The corresponding standard deviation is:

$$\sigma_X = \sqrt{3.78} \approx 1.94$$

The results indicate that, on average a plant produces approximately 12-13 fruits per flowering cycle, with moderate variability around this mean. The relatively small variance reflects stable fruit-set performance under managed cultivation, while still accounting for environmental and biological uncertainty. Variations in temperature, rainfall and humidity particularly during unseasonal flowering can reduce the effective value of p, leading to lower realised yields despite similar input levels.

This stochastic component explains deviations from the deterministic yield estimates obtained through regression and provides a realistic representation of farm-level production uncertainty.

The expected yield per plant obtained from the regression model is adjusted using the stochastic fruit-set formulation by scaling the deterministic yield with the expected proportion of successful fruit formation. This combined approach enables to model to capture both systematic input-output relationships and random biological variability, resulting in a more robust yield estimation framework.

The following section integrates the deterministic and stochastic yield components into a profit function and determines the optimal allocation of water and fertiliser inputs under real-world cost constraints.

5 2. Profit Function Formulation and Optimisation

The economic objective of the aim is to maximise net-profit per plant through the optimal allocation of the irrigation water and fertiliser inputs. Profit is defined as difference between revenue generated from fruit yield and the total cost of production:

Profit per plant is formulated as:

$$\Pi(W, F) = P \cdot Y(W, F, A, T) - C(W, F)$$

where P denotes the market price per kg of dragon fruit, $Y(W, F, A, T)$ represents the yield per plant per harvest and $C(W, F)$ is the total production cost.

Using the calibrated regression model obtained in section 4.3, yield per plant is given by:

$$\mathrm{Y(W, F, A, T)} = -6.85 + 0.48\mathrm{W} + 2.10\mathrm{F} + 0.62\mathrm{A} + 0.11\mathrm{T}$$

Substituting this expression into the profit function yields:

$$\Pi(\mathrm{W, F}) = \mathrm{P} \cdot [-6.85 + 0.48\mathrm{W} + 2.10\mathrm{F} + 0.62\mathrm{A} + 0.11\mathrm{T}] - \mathrm{C(W, F)}$$

The total cost of production is expressed as a linear function of controllable inputs:

$$\mathrm{C(W, F)} = \mathrm{C_w W} + \mathrm{C_f F} + \mathrm{C_m}$$

where $\mathrm{C_w}$ and $\mathrm{C_f}$denote the marginal costs associated with water and fertiliser inputs, respectively and $\mathrm{C_m}$

represents the fixed and periodic costs such as labour, maintenance, weed management and harvesting.

Substituting the cost function into the profit expression gives:

$$\Pi(W,F) = P \cdot [-6.85 + 0.48W + 2.10F + 0.62A + 0.11T] - C_w W - C_f F - C_m$$

To determine the optimal levels of water and fertiliser inputs, partial derivatives of the profit function with respect to W and F are taken:

$$\frac{\partial \Pi}{\partial W} = 0.48P - C_w$$

$$\frac{\partial \Pi}{\partial F} = 2.10P - C_f$$

Setting these derivatives equal to zero yields the optimality conditions:

$0.48P = C_w$, $2.10P = C_f$

These conditions imply that profit is maximised when the marginal revenue contribution of each input equal to marginal cost.

The marginal costs associated with water and fertiliser inputs were derived directly from observed farm-level expenditure. Fixed and periodic costs including labour, maintenance, weed management, spraying and harvesting were aggregated annually and converted to a per-plant basis. The resulting fixed cost was estimated as 64.25 per plant per harvest ($C_m = 64.25$). Fertiliser and irrigation costs were normalised with respect to average application rates, yielding marginal costs of approximately 2.25 INR per kilogram of fertiliser ($C_f = 2.25$) and 0.13 INR per litre per day of irrigation water($C_w = 0.13$). (See Appendix 2)

These empirically derived cost parameters were used in the profit optimisation analysis, ensuring consistency between the mathematical model and real farm operations.

VI. ECONOMIC INTERPRETATION

The optimisation results demonstrate that the optimal input allocation is governed by marginal price-cost relationships rather than absolute yield levels. Using the empirically derived cost parameters, the optimally conditions $0.48P = C_w$ and $2.10P = C_f$ indicate that irrigation and fertiliser inputs should be applied up to the point where their respective marginal revenue contributions equal the observed marginal costs of approximately 0.13 INR per litre per day of irrigation water and 2.25 INR per kilogram of fertiliser.

These conditions highlight that fertiliser exhibits a higher marginal yield contribution relative to water, justifying closer monitoring of fertiliser application decisions, while irrigation optimisation is more strongly influenced by operational constraints and water availability. Plant age and temperature enter the profit function as exogenous parameters; while they shift overall profit levels, they do not alter the optimality conditions for controllable inputs.

VII. CONCAVITY AND OPTIMALITY

The profit function is linear in both irrigation input W and fertiliser input F. Consequently, the optimisation problem admits a unique solution within the feasible operation region by agronomic and resource constraints. In practice, upper bounds on water supply, fertiliser dosage and labour availability ensure that optimal solution remains biologically viable and operationally feasible. The absence of interior curvature further implied that profit-maximising input levels are determined by marginal trade off rather than second order effects.

VIII. IMPLICATIONS FOR FARM MANAGEMENT

The derived conditions provide clear and actionable decision rules for farm management. Rather than pursuing yield maximisation alone, the framework emphasises cost-efficient input use grounded in real expenditure data. The explicit incorporation of observed marginal cost enables farm managers to dynamically adjust irrigation and fertiliser strategies in response to changes in market prices, labour costs or input availability. This approach supports precision resource management and improves economic resilience under variable production and market conditions.

IX. LIMITATIONS

The optimisation framework relies on a calibrated linear yield model and assumes constant marginal costs within the observed range. Non-linear responses, input interactions beyond first-order effects and temporal price variability are not explicitly modelled. Additionally, stochastic climatic shocks are incorporated indirectly rather than dynamically. Future work may extend this framework by incorporating non-linear cost structures, time-varying prices and risk-sensitive optimisation.

X. CONCLUSION

This study presents a data-driven mathematical framework for analysing yield behaviour and profit optimisation in commercial dragon fruit farming using real farm-level observations. By integrating growth modelling, calibrated multivariable regression, stochastic flower-to-flower conversion and cost-based optimisation, the work establishes a coherent link between biological processes, resource allocation, and economic outcomes.

The results demonstrate that yield maximisation alone is not sufficient for optimal farm performance. Instead, profit maximising decisions are governed by marginal price relationship, with empirically derived cost parameters playing a central role in determining optimal irrigation and fertiliser inputs. The calibrated regression model provides an interpretable first-order approximation of yield response, while the stochastic formulation captures inherent biological uncertainty, together yielding a realistic and operationally relevant representation of production.

The optimisation analysis shows that controllable inputs should be allocated their marginal revenue contributions equal observed marginal costs, reinforcing the value of precision input management over uniform application strategies. Importantly, the framework accommodates real-world constraints such as fixed labour costs, periodic maintenance and resource availability, ensuring consistency between mathematical results and practical farm operations.

Beyond the specific case study, the proposed methodology is scalable and adaptable to other horticultural systems with similar production characteristic. By grounding mathematical optimisation in observed from data, this work highlights the potential of applied mathematics as a decision-support tool for sustainable and economically efficient agricultural production.

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