Optimization of an Inventory System with Quartic Polynomial Demand, Constant Deterioration Rate, and Backordering

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Abstract- This paper develops a deterministic inventory control model for deteriorating items subject to complex, time-varying demand. Demand is modeled using a quartic polynomial function to capture non-linear market dynamics that are inadequately represented by conventional linear or quadratic forms. A constant deterioration rate is assumed over the product life cycle, and planned shortages with complete backordering are permitted. The inventory evolution is described by first-order differential equations, yielding closed-form analytical expressions for inventory levels and optimal replenishment decisions. The optimal policy is obtained by minimizing the total relevant cost with respect to the cycle length and shortage duration. Numerical experiments and parametric sensitivity analyses validate the model and demonstrate the effects of demand coefficients and deterioration parameters on cost-efficient inventory strategies.

Keywords: Quartic polynomial demand; Constant deterioration rate; Planned shortages; Complete backordering; Inventory optimization; Nonlinear demand.

I. INTRODUCTION

Inventory management is a central discipline within operations research, providing analytical tools for determining optimal replenishment policies in complex supply chain systems. However, many classical inventory models remain limited in applicability due to simplifying assumptions of constant demand and non-deteriorating items. In practical environments, product deterioration and evolving consumer demand introduce significant dynamics that render static formulations inadequate for effective decision-making.

The deterioration of physical goods—common in sectors such as pharmaceuticals, chemicals, and high-technology products—has a pronounced impact on inventory holding costs and service performance. Since the seminal contribution of Ghare and Schrader (1963), which first incorporated deterioration into inventory theory, substantial efforts have been made to model product decay more realistically. Among these, the Weibull distribution has emerged as a powerful and flexible representation capable of capturing time-dependent deterioration behavior.

Concurrently, demand patterns in modern markets often exhibit complex growth, saturation, and decline phases that cannot be adequately described by linear or quadratic functions. To address this limitation, the present study adopts a quartic polynomial demand function, offering enhanced flexibility to represent non-linear demand evolution over finite planning horizons.

Integrating these features, this paper develops a deterministic inventory optimization model that combines fourth-order demand dynamics with constant deterioration, while permitting planned shortages under a complete backlogging policy. Closed-form analytical expressions for inventory levels and total cost functions are derived using differential equation. Numerical illustrations and sensitivity analyses are conducted to demonstrate the model's effectiveness and to highlight the influence of key parameters on optimal inventory decisions in volatile conditions.

II. LITERATURE REVIEW

Research on inventory systems for deteriorating items was initiated by the seminal work of Ghare and Schrader (1963), who first quantified the economic implications of exponential decay in inventory levels. This foundational model was later extended by Covert and Philip (1973) through the introduction of the Weibull distribution, a significant advancement that enabled deterioration to be modeled as a time-dependent process capable of capturing increasing or decreasing failure rates over a product's life cycle.

As inventory theory evolved, increasing attention was given to the realistic treatment of shortages. Wee (1993) incorporated backordering into deteriorating inventory systems, establishing its importance in practical decision-making. Subsequent studies by Ouyang, Wu, and Cheng (2005) and Dye, Ouyang, and Hsieh (2007) further advanced this stream by considering partial backlogging and the time value of money. These contributions emphasized that inventory policies are intrinsically linked to financial parameters and customer waiting behavior.

Another major development in inventory modeling was the relaxation of the constant demand assumption. Chakrabarty, Giri, and Chaudhuri (1998) combined Weibull deterioration with trended demand, while Tripathy and Mishra (2010) and Mishra (2012) investigated models with time-dependent holding costs. The interaction between pricing strategies and deterioration effects was later examined by Shaikh (2014), highlighting the interdependence between demand dynamics and inventory decay.

However, linear demand patterns were found to be insufficient for representing the volatility observed in contemporary markets. This limitation prompted the adoption of polynomial demand functions. Ganesh Kumar et al. (2016) demonstrated that cubic demand functions provide improved flexibility for fluctuating demand environments. Further extensions were explored by Singh and Saxena (2018) and Tiwari et al. (2019), who analyzed quadratic demand models incorporating trade credit policies and sustainability considerations.

More recently, the need for higher-order demand representations has gained prominence. Lin, Chang, and Dye (2021) and Sivashankari and Ramachandran (2021) introduced quartic polynomial demand functions, establishing their effectiveness in capturing highly volatile, seasonal, or multi-phase demand cycles. Comprehensive discussions on polynomial demand were presented by Panda, Sana, and Chaudhuri (2022), while alternative modeling approaches, including fuzzy-based inventory systems, were examined by Rani and

Research Gap

Despite advances in inventory modeling, there is a lack of deterministic frameworks that integrate quartic polynomial demand, a constant deterioration rate, and complete backlogging while maintaining analytical tractability. Existing models focus on lower-order demand or time-dependent deterioration, often requiring numerical solutions.

III. MODEL DESCRIPTION AND ASSUMPTIONS

The model considers a single-item deterministic inventory system operating over a finite replenishment cycle. The following assumptions are adopted to reflect realistic inventory behavior and to extend existing deterioration-based models.

I. High-Order Demand Modeling

To better represent complex market dynamics, the demand rate $Q(t)$ is modeled as a quartic polynomial function of time rather than using restrictive linear or quadratic trends.

$$Q(t) = p + qt^2 + rt^4; p, q, and\ r$$

are constants.

II. Constant Deterioration Rates

The physical degradation of inventory is assumed to occur at a constant rate throughout the replenishment cycle, reflecting products with uniform spoilage characteristics.

$$f(t) = \theta t$$

Where θ represents the constant proportion of inventory deteriorating per unit time.

III. Replenishment is instantaneous

Replenishment is assumed to be instantaneous, with zero lead time, consistent with classical deterministic inventory models.

IV. Planned Shortages and backlogging;

Shortages are permitted and fully backlogged, meaning that all unmet demand during stock-out periods is satisfied in subsequent replenishments.

V. Inventory-related costs

The model explicitly incorporates all relevant cost components, including holding costs, shortage costs, ordering costs, and deterioration costs, enabling a comprehensive evaluation of the system's total economic performance.

These assumptions collectively provide a foundation for an analytically tractable inventory model that extends classical deterioration frameworks by explicitly incorporating nonlinear (quartic) demand dynamics, constant deterioration, planned shortages, and comprehensive cost considerations, thereby enhancing both theoretical rigor and practical applicability.

IV. MATHEMATICAL FORMULATION

During the positive inventory period $[0, t_1]$, the inventory level is governed by a first-order differential equation that accounts for both demand and constant deterioration. In the shortage period $[t_1, T]$, the inventory is negative, representing backorders, and depletion occurs solely due to demand, as no stock is available to deteriorate.

Formally, the inventory dynamics can be expressed as:

$$\frac{dI'_1(t)}{dt} + \theta t I'_1(t) = -(p + qt^2 + rt^4); 0 \le t \le t_1 \qquad (1)$$

And during the interval $[t_1, T]$, the shortage occurs, so the differential equation is given by

$$\frac{dI'_2(t)}{dt} = -(p + qt^2 + rt^4); \ t_1 \le t < T \qquad (2)$$

With the boundary conditions: $t = 0, I(0) = W$,

$t = t_1 ; \ I'(t_1) = 0$

$t = T ; \ I'(T) = S$

Now, by solving above equations (1), we get:

$$I.F. = e^{\int \theta t dt} = e^{\frac{\theta t^2}{2}} = 1 + \frac{\theta t^2}{2} + \cdots \qquad \text{(By ignoring higher terms values)}$$

$$I.F. = 1 + \frac{\theta t^2}{2}$$

$$I'_1(t)\left(1 + \frac{\theta t^2}{2}\right) = -\int \left(1 + \frac{\theta t^2}{2}\right)(p + qt^2 + rt^4)dt$$

$$I'_1(t)\left(1+\frac{\theta t^2}{2}\right) = -\left[pt+\frac{qt^3}{3}+\frac{rt^5}{5}+\frac{p\theta t^3}{6}+\frac{q\theta t^5}{10}+\frac{r\theta t^7}{14}\right]+D$$

Using condition at $t = t_1$, $I_1(t) = 0$

$$D = pt_1+\frac{qt_1^3}{3}+\frac{rt_1^5}{5}+\frac{p\theta t_1^3}{6}+\frac{q\theta t_1^5}{10}+\frac{r\theta t_1^7}{14}$$

By putting value of D in equation, we get

$$I'_1(t)\left(1+\frac{\theta t^2}{2}\right) = \left[p(t_1-t)+\frac{q}{3}(t_1^3-t^3)+\frac{r}{5}(t_1^5-t^5)+\frac{p\theta}{6}(t_1^3-t^3)+\frac{q\theta}{10}(t_1^5-t^5)+\frac{r\theta}{14}(t_1^7-t^7)\right]$$

$$I'_1(t) = \left\{p(t_1-t)+\frac{q}{3}(t_1^3-t^3)+\frac{r}{5}(t_1^5-t^5)+\frac{p\theta}{6}(t_1^3-t^3)+\frac{q\theta}{10}(t_1^5-t^5)+\frac{r\theta}{14}(t_1^7-t^7)-\left[\frac{p\theta}{2}(t_1t^2-t^3)+\frac{q\theta}{6}(t_1^3t^2-t^5)+\frac{r\theta}{10}(t_1^5t^2-t^7)+\frac{p\theta^2}{12}(t_1^3t^2-t^5)+\frac{q\theta^2}{20}(t_1^5t^2-t^7)+\frac{r\theta^2}{28}(t_1^7t^2-t^9)\right]\right\}$$
(3)

Now applying boundary conditions i.e., at $t = 0, I'_1(t) = W$

$$W = pt_1+\frac{qt_1^3}{3}+\frac{rt_1^5}{5}+\frac{p\theta t_1^3}{6}+\frac{q\theta t_1^5}{10}+\frac{r\theta t_1^7}{14} \quad (4)$$

By solving equation (2),

$$I'_2(t) = -\int(p+qt^2+rt^4)dt$$

$$I'_2(t) = -\left(pt+\frac{qt^3}{3}+\frac{rt^5}{5}\right)+E$$

By using condition at $t = t_1$, $I_2(t) = 0$, we get

$$E = pt_1+\frac{qt_1^3}{3}+\frac{rt_1^5}{5}$$

$$I'_2(t) = \left[p(t_1-t)+\frac{q}{3}(t_1^3-t^3)+\frac{r}{5}(t_1^5-t^5)\right] \quad (5)$$

Again by applying boundary condition, at $t = T, I_2(t) = -S$

$$S = -\left[p(t_1-T)+\frac{q}{3}(t_1^3-T^3)+\frac{r}{5}(t_1^5-T^5)\right] \quad (6)$$

The order quantity per cycle is: $Q = W + S$

$$Q = pt_1+\frac{qt_1^3}{3}+\frac{rt_1^5}{5}+\frac{p\theta t_1^3}{6}+\frac{q\theta t_1^5}{10}+\frac{r\theta t_1^7}{14}-\left[p(t_1-T)+\frac{q}{3}(t_1^3-T^3)+\frac{r}{5}(t_1^5-T^5)\right] \quad (7)$$

Holding cost per unit per unit time is:

$$C_H = C'_{CH}\int_0^{t_1} I'_1(t)dt$$

$$C_H = C'_{CH}\Big[p\left(t_1 t - \frac{t^2}{2}\right) + \frac{q}{3}\left(t_1^3 t - \frac{t^4}{4}\right) + \frac{r}{5}\left(t_1^5 t - \frac{t^6}{6}\right) + \frac{p\alpha}{\beta+1}\left(t_1^{\beta+1} t - \frac{t^{\beta+2}}{\beta+2}\right) + \frac{q\alpha}{\beta+3}\left(t_1^{\beta+3} t - \frac{t^{\beta+4}}{\beta+4}\right) + \frac{r\alpha}{\beta+5}\left(t_1^{\beta+5} t - \frac{t^{\beta+5}}{\beta+5}\right) - p\alpha\left(\frac{t_1 t^{\beta+1}}{\beta+1} - \frac{t^{\beta+2}}{\beta+2}\right) - \frac{q\alpha}{3}\left(\frac{t_1^3 t^{\beta+1}}{\beta+1} - \frac{t^{\beta+4}}{\beta+4}\right) - \frac{r\alpha}{5}\left(\frac{t_1^5 t^{\beta+1}}{\beta+1} - \frac{t^{\beta+6}}{\beta+6}\right) - \frac{p\alpha^2}{\beta+1}\left(\frac{t_1^{\beta+1} t^{\beta+1}}{\beta+1} - \frac{t^{2\beta+2}}{2\beta+2}\right) - \frac{q\alpha^2}{\beta+3}\left(\frac{t_1^{\beta+3} t^{\beta+1}}{\beta+1} - \frac{t^{2\beta+4}}{2\beta+4}\right) - \frac{r\alpha^2}{\beta+5}\left(\frac{t_1^{\beta+5} t^{\beta+1}}{\beta+1} - \frac{t^{2\beta+6}}{2\beta+6}\right)\Big]_0^{t_1}$$

$$C_{HC} = C'_{CH}\left[\frac{pt_1^2}{2} + \left(\frac{q}{4} + \frac{p\theta}{12}\right)t_1^4 + + \left(\frac{r}{6} + \frac{q\theta}{18} - \frac{p\theta^2}{72}\right)t_1^6 + + \left(\frac{r\theta}{24} - \frac{qp}{96}\right)t_1^8 - \frac{r\theta^2 t_1^{10}}{120}\right] \quad (8)$$

Shortage cost per unit per unit time is:

$$C_S = -C'_{CS}\int_{t_1}^{T} I'_2(t)\,dt$$

$$C_S = -C'_{CS}\left[p\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + q\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + r\left(\frac{t_1^5 T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] \quad (9)$$

Ordering cost per unit per unit time is:

$$C_O = -C'_{CO} \quad (10)$$

Deterioration cost per unit per unit time is:

$$C_D = C'_{CD}\left[W - \int_0^{t_1} Q(t)dt\right]$$

$$C_{DC} = C'_{CD}\left[\frac{p\theta t_1^3}{6} + \frac{q\theta t_1^5}{10} + \frac{r\theta t_1^7}{14}\right] \quad (11)$$

Total Inventory Cost (TIC) given by:

$$T_{IC} = \frac{1}{T}(C_H + C_S + C_O + C_D)$$

$$T_{IC} = \frac{1}{T}\Big\{C'_{CH}\left[\frac{pt_1^2}{2} + \left(\frac{q}{4} + \frac{p\theta}{12}\right)t_1^4 + + \left(\frac{r}{6} + \frac{q\theta}{18} - \frac{p\theta^2}{72}\right)t_1^6 + + \left(\frac{r\theta}{24} - \frac{qp}{96}\right)t_1^8 - \frac{r\theta^2 t_1^{10}}{120}\right] + C'_{CS}\left[p\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + q\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + r\left(\frac{t_1^5 T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + C'_{CO} + C'_{CD}\left[\frac{p\theta t_1^3}{6} + \frac{q\theta t_1^5}{10} + \frac{r\theta t_1^7}{14}\right]\Big\} \quad (12)$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{d(T_{IC})}{dT} = -\frac{1}{T^2}\Big\{C'_{CH}\left[\frac{pt_1^2}{2} + \left(\frac{q}{4} + \frac{p\theta}{12}\right)t_1^4 + + \left(\frac{r}{6} + \frac{q\theta}{18} - \frac{p\theta^2}{72}\right)t_1^6 + + \left(\frac{r\theta}{24} - \frac{qp}{96}\right)t_1^8 - \frac{r\theta^2 t_1^{10}}{120}\right] + C'_{CS}\left[p\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) + q\left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4}\right) + r\left(\frac{t_1^5 T}{5} - \frac{T^6}{30} - \frac{t_1^6}{6}\right)\right] + C'_{CO} + C'_{CD}\left[\frac{p\theta t_1^3}{6} + \frac{q\theta t_1^5}{10} + \frac{r\theta t_1^7}{14}\right]\Big\} - \frac{1}{T}\Big\{C'_{SC}\left[p(t - T) + + q\left(\frac{t^3}{3} - \frac{T^3}{3}\right) + r\left(\frac{t^5}{5} - \frac{T^5}{5}\right)\right]\Big\} \quad (13)$$

$$\frac{d(T_{IC})}{dt_1} = \frac{1}{T}\Big\{C'_{CH}\left[pt_1 + 4\left(\frac{q}{4} + \frac{p\theta}{12}\right)(t_1^3) + 6\left(\frac{e}{6} + \frac{q\theta}{18} - \frac{p\theta^2}{72}\right)(t_1^5) - 8\left(\frac{r\theta}{24} - \frac{cq}{96}\right)t_1^7 - \frac{r\theta^2 t_1^9}{12}\right] - C'_{SC}[\,p(T - t_1) + q(t_1^2 T - t_1^3) + r(t_1^4 T - t_1^5)] + C'_{CD}\left[\frac{p\theta t_1^2}{2} + \frac{q\theta\, t_1^4}{2} + \frac{r\theta\, t_1^6}{2}\right]\Big\} \quad (14)$$

The cycle parameters t_1 and T are determined by solving Equations (13) and (14) using MAPLE 15, which facilitates precise computation of inventory depletion and shortage durations under the proposed model.

V. NUMERICAL EXAMPLE

Next, we present a numerical example to verify the optimality of the proposed solution. The example is solved using MAPLE 15. To illustrate the model numerically, the parameters of the inventory system are defined as follows:

$$p = 10, q = 4,, r = 1, C'_{CH} = 4, C'_{CS} = 15, C'_{CO} = 100, C'_{CD} = 10, \theta = 0.01$$

Based on the specified parameter values, the computational results obtained using *Maple 15* reveal that the optimal shortage period is $t_1 = 1.037414372$ per unit time. Furthermore, the optimal length of the ordering cycle is determined to be $T = 1.318536282$ time units. The corresponding total inventory cost associated with these optimal values is calculated as T_{IC}.

Sensitivity analysis is then conducted by varying key parameters—such as demand coefficients, holding cost, shortage cost, ordering cost, deterioration cost, and deterioration rate—within ±10%, ±20% ranges.

2D Representation

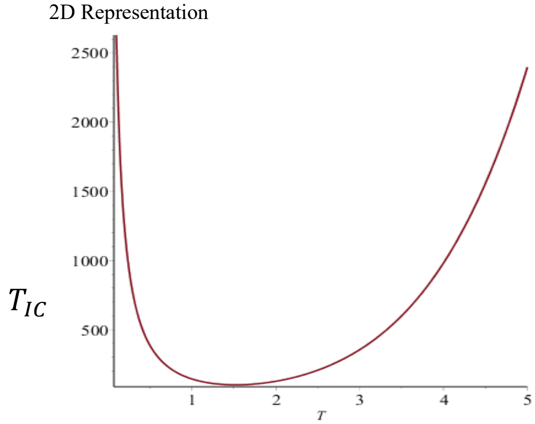

Figure 1 (Total Inventory Cost vs Time)

3D Representation

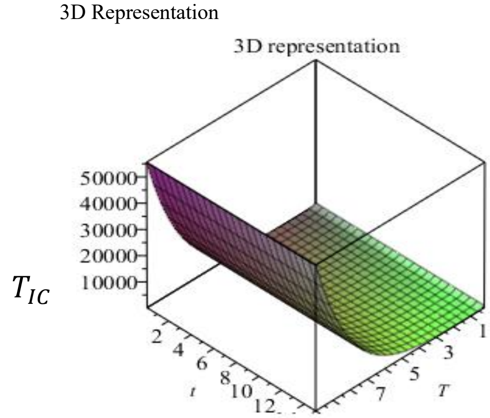

Figure 2 (3D Representation of Total Inventory Cost vs Time)

VI. SENSITIVE ANALYSIS OF PARAMETERS

In this section, we examine the effect of variations in the model parameters $p, q, r, C'_{CH}, C'_{CS}, C'_{CO}, C'_{CD}$ *and* θ on the optimal order quantity and the optimal total inventory cost per unit time. Sensitivity analysis is performed by varying each parameter individually while keeping others constant at their baseline values. The resulting changes in T_{IC} *and* Q are summarized in below table, which highlights the degree of responsiveness of the system to each parameter.

Parameter	% change	Change in			
		T	t_1	Q	T_{IC}
p	+20%	1.489919263	1.171731403	23.80048611	106.6471721
	+10%	1.505043893	1.183590044	22.68750103	104.2601629
	-10%	1.535655925	1.207597845	20.39690417	99.41249624
	-20%	1.551123913	1.219732935	19.21864856	96.95145676
	+20%	1.488110542	1.170324722	21.69902845	103.5925812

q	+10%	1.503792335	1.182615346	21.63517758	102.7346573
	-10%	1.537684403	1.209179652	21.45160872	100.9327540
	-20%	1.556049282	1.223575061	21.32959039	99.98499156
r	+20%	1.503464385	1.182411197	21.44910565	102.3984937
	+10%	1.511675987	1.188821929	21.49946739	102.1274082
	-10%	1.529356115	1.202621408	21.61032875	101.5644570
	-20%	1.538915163	1.210080442	21.67163338	101.2658760
C'_{CH}	+20%	1.474630644	1.113110001	20.44774467	106.5701028
	+10%	1.496095804	1.152435523	20.96056793	104.3002535
	-10%	1.547859929	1.243200311	22.24741249	99.18598341
	-20%	1.579672229	1.296404212	23.07516401	96.27448595
C'_{CS}	+20%	1.495091027	1.219049449	20.94475435	103.6174484
	+10%	1.506655010	1.208239305	21.22183460	102.7997731
	-10%	1.536618404	1.180436325	21.95612951	100.7284180
	-20%	1.556517383	1.162138203	22.45717733	99.38920256
C'_{CO}	+20%	1.595905168	1.254763389	23.49787920	114.6788854
	+10%	1.559638059	1.226363089	22.54521095	108.3414984
	-10%	1.477259234	1.161838701	20.51472391	95.17760308
	-20%	1.429723836	1.124595954	19.42123544	88.29897486
C'_{CD}	+20%	1.519388707	1.194095291	21.53068679	101.9021660
	+10%	1.519839928	1.194820538	21.54186483	101.8754445
	-10%	1.520746817	1.196277224	21.56434762	101.8218439
	-20%	1.521202503	1.197008688	21.57565293	101.7949639
θ	+20%	1.519158086	1.193733155	21.53307975	101.9135328
	+10%	1.519724175	1.194638849	21.54306025	101.8811433
	-10%	1.520863462	1.196460164	21.56315442	101.8161139
	-20%	1.521436694	1.197375831	21.57326877	101.7834728

Table 1: In an inventory system with deterioration and shortages, variations in demand rate, cost parameters, and deterioration rate significantly influence the optimal cycle time, shortage initiation time, order quantity, and total inventory cost.

The table above summarizes the sensitivity analysis outcomes, highlighting how variations in demand-related coefficients, cost factors, and deterioration parameters influence the optimal inventory decisions. The results demonstrate that holding and ordering costs play a dominant role in determining system performance, while the effects of deterioration-related parameters are relatively less pronounced.

VII. CONCLUSION

The sensitivity analysis indicates that, under quartic time-dependent demand, deterioration significantly affects the optimal order quantity (Q) and total inventory cost (T_{IC}) throughout the replenishment cycle. Increases in demand parameters (p, q, r) lead to higher Q and T_{IC}, while increases in holding, shortage, ordering, and deterioration costs, as well as the deterioration rate, result in a reduced optimal order quantity. The results confirm that both demand and cost parameters strongly influence the system's cost structure, and the proposed model effectively identifies the minimum total inventory cost under nonlinear demand, demonstrating robustness and practical applicability.

VIII. FUTURE WORK

Future research may consider incorporating stochastic demand, partial backlogging, or sustainability-related factors such as carbon emissions and green investment policies to further enhance the applicability of the model.

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