Task Completion and Allocation of Task Assignment to Multi-Tasking Robots Using Ifk-Phg

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Abstract—An extension of a graph in which hyperedges can join subsets of vertices instead of simply pairs is called a hypergraph. The edges in a hypergraph are arbitrary, non-empty sets of vertices. Similar to traditional graphs, hypergraphs facilitate higher-order interactions in communication and social networks. In this paper, we explore the structural properties of Intuitionistic Fuzzy k-partite Hypergraphs (IFk-PHGs). Specifically, we analyze the lower and upper truncation of IFk-PHG, as well as different types of intersections, including intersecting IFk-PHG, K-intersecting IFk-PHG and strongly intersecting IFk-PHG. Furthermore, as long as an IFk-PHG is K-intersecting, we can conclude that it is strongly intersecting. A multi-tasking robot is a type of robotic system that is capable of performing multiple tasks or functions without the need for manual intervention or reprogramming between tasks. These robots are designed to be skilled and flexible allowing them to switch between different tasks seamlessly. Intuitionistic Fuzzy parameters enable flexible, advanced representation of multi-tasking robot systems, guiding decision-making processes like task allocation, task completion and coordination in uncertain situations addressing uncertainty in robot capabilities and task suitability. This work proposes how task is allocated to robots using IFk-PHG and which robot can complete the task quickly. The task completion of the robots can be done using score function formula.

I. INTRODUCTION:

In 1999 (1), K. T. Atanassov presented the concepts of Intuitionistic Fuzzy Sets (IFS) as an expansion of fuzzy sets. These collections contain the degree of ambiguity as well as Membership (Ms) and Non-membership (Nms). To expand the application section, the idea of a graph was widened to incorporate a hypergraph. In 1976, the author in (2) popularized the thoughts of graphs and hypergraphs. Article (3) includes fuzzy hypergraphs and its related extension. The authors in (4, 5, 8, 9) introduced and studied the various properties related to fuzzy directed hypergraphs that are intuitionistic. K.K. Myithili and R. Keerthika (6, 7) defined the thoughts of IFk-PHG and expanded the related concept.

This article deals with the ideas of intersecting, essentially intersecting, strong intersecting etc. If $\mathcal{H}$ is K-intersecting, then it is demonstrated to be strongly intersecting. Also, an algorithm for decision making is given. Finally, the last part deals with the application of IFk-PHG in car manufacturing plant.

Keywords: Intersecting IFk-PHG, K-intersecting IFk-PHG, chromatic number, non-trivial, multi-tasking robot.

AMS Classification: 05C65, 05C72.

II. METHODOLOGY

Definition The IFk-PHG (7) is an arranged triplet $\mathcal{H}$ in which

- $V = \{v_1, v_2, v_3, \cdots, v_n\}$ is a limited collection of vertices,
- V has intuitionistic fuzzy subsets of the group $E = \{E_1, E_2, E_3, E_m\}$,
- $E_j = \left\{\left(v_i, \mu_j(v_i), \nu_j(v_i)\right) : \mu_j(v_i), \nu_j(v_i) \geq 0, \mu_j(v_i) + \nu_j(v_i) \leq 1\right\},\ 1 \leq \quad j \leq m$,
- $E_j \neq \emptyset, 1 \leq j \leq m$,
- $\bigcup_j \operatorname{supp}(E_j) = V,\ 1 \leq j \leq m$.
- $\forall\, v_i \in E$ there are sets with k-disjoints $\psi_i, i = 1,2,\cdots,k$ and within the same set, no two vertices are adjacent $\ni E_k = \bigcap_{i=1}^{k} \psi_i = \emptyset$.

Definition Consider an IFk-PHG to be $\mathcal{H} = (\mathrm{V}, \mathrm{E}, \psi)$. The IFk-PHG height (7) is determined by $h(\mathcal{H}) = \{\max(\min(\mu_{k_{ij}})), \max(\max(\nu_{k_{ij}}))\}\ \forall\ 1 \le i \le m$ and $1 \le j \le n$. For the k-partite hyperedge ψ_{ij}, the Ms and Nms values are represented by $\mu_{k_{ij}}$ and $\nu_{k_{ij}}$ respectively.

Definition Let $\mathcal{H}$ be the IFk-PHG, $c(\mathcal{H}) = \{\mathcal{H}^{a_1,b_1}, \mathcal{H}^{a_2,b_2}, \mathcal{H}^{a_k,b_k}\}$. If $c(\mathcal{H})$ is ordered, then $\mathcal{H}$ is ordered (i.e) $\mathcal{H}^{a_1,b_1} \subset \mathcal{H}^{a_2,b_2} \subset \cdots \subset \mathcal{H}^{a_k,b_k}$. When $\{\mathcal{H}^{a_i,b_i} | i = 1, \cdot, k\}$ is simply ordered, the IFk-PHG is referred to as simply ordered (7), (i.e) if it's ordered and if $\psi \in \mathcal{H}^{a_{i+1},b_{i+1}} \backslash \mathcal{H}^{a_i,b_i}$ then $\psi \notin \mathcal{H}^{a_i,b_i}$.

III. RESULTS AND DISCUSSION

Definition The spike reduction of $\psi_i \in \mathcal{IF}_{\mathcal{P}}(\mathrm{V})$, which is denoted as $\widetilde{\psi_i}$ is defined by

$$\widetilde{\psi_i}^{\langle a_i,b_i\rangle} = \begin{cases} {\psi_i}^{\langle a_i,b_i\rangle} & \text{if} \quad \left|{\psi_i}^{\langle a_i,b_i\rangle}\right| \ge 2 \\ \emptyset & \text{if} \quad \left|{\psi_i}^{\langle a_i,b_i\rangle}\right| < 2 \end{cases}$$

where $a_i = \min\{\mu_{k_i}(v_i)\} \in (0,1]$ and $b_i = \max\{\nu_{k_i}(v_i)\} \in [0,1)$.

Definition Assume that an IFk-PHG is $\mathcal{H} = (\mathrm{V}, \mathrm{E}, \psi)$. $\widetilde{\mathcal{H}} = (\widetilde{\mathrm{V}}, \widetilde{\mathrm{E}}, \widetilde{\psi})$ defines the spike reduced IFk-PHG of $\mathcal{H}$, represented by $\widetilde{\mathcal{H}}$ where $\widetilde{\psi} = \{\widetilde{\psi_i} | \psi_i \in \psi\}$; $\widetilde{\mathrm{V}} = \cup_{\widetilde{\psi_i} \in \widetilde{\psi}} \operatorname{supp}(\widetilde{\psi})$ and

$$\langle \mu_{k_i}(\widetilde{v_i}), \nu_{k_i}(\widetilde{v_i})\rangle = \begin{cases} \langle a_i, b_i\rangle & \text{if } \widetilde{v_i} \in \operatorname{supp}(\widetilde{\psi_i}) \\ \langle 0, 1\rangle & \text{if } \widetilde{v_i} \notin \operatorname{supp}(\widetilde{\psi_i}) \end{cases}$$

Example

Consider an IFk-PHG $\mathcal{H}$ with $\mathrm{V} = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$, $\mathrm{E} = \{\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3\}$ and $\psi = \{\psi_1, \psi_2, \psi_3, \psi_4\}$whose incidence matrix is shown below:

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccc} \psi_1 & \psi_2 & \psi_3 & \psi_4 \end{array} \\ \begin{pmatrix} \langle 0.3, 0.5\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.4, 0.2\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.2, 0.6\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.3, 0.5\rangle \\ \langle 0, 1\rangle & \langle 0.8, 0.1\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0.1, 0.7\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.3, 0.4\rangle & \langle 0, 1\rangle \\ \langle 0.6, 0.3\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \end{pmatrix} \end{array}$$

Then the incidence matrix of $\widetilde{\mathcal{H}} = (\widetilde{\mathrm{V}}, \widetilde{\mathrm{E}}, \widetilde{\psi})$ where $\widetilde{\mathrm{V}} = \{\widetilde{v_1}, \widetilde{v_2}, \widetilde{v_3}, \widetilde{v_5}, \widetilde{v_6}, \widetilde{v_7}, \widetilde{v_8}\}$, $\widetilde{\mathrm{E}} = \{\widetilde{\mathrm{E}_1}, \widetilde{\mathrm{E}_2}, \widetilde{\mathrm{E}_3}\}$ and $\widetilde{\psi} = \{\widetilde{\psi_1}, \widetilde{\psi_2}, \widetilde{\psi_3}\}$ is as follows:

$$\begin{array}{c} \\ \widetilde{v_1} \\ \widetilde{v_2} \\ \widetilde{v_3} \\ \widetilde{v_5} \\ \widetilde{v_6} \\ \widetilde{v_7} \\ \widetilde{v_8} \end{array} \begin{array}{c} \begin{array}{ccc} \widetilde{\psi_1} & \widetilde{\psi_2} & \widetilde{\psi_3} \end{array} \\ \begin{pmatrix} \langle 0.1, 0.7\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.4, 0.2\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.2, 0.6\rangle \\ \langle 0, 1\rangle & \langle 0.4, 0.2\rangle & \langle 0, 1\rangle \\ \langle 0.1, 0.7\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.2, 0.6\rangle \\ \langle 0.1, 0.7\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \end{pmatrix} \end{array}$$

The graph of $\mathcal{H}$ and $\widetilde{\mathcal{H}}$ is depicted in Figure 1 and Figure 2:

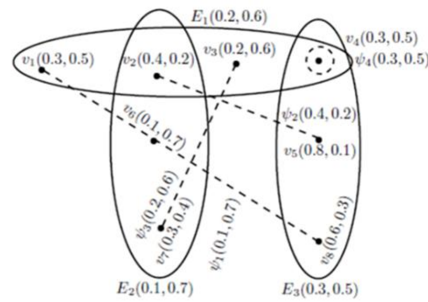

Figure 1: $\mathcal{H}$

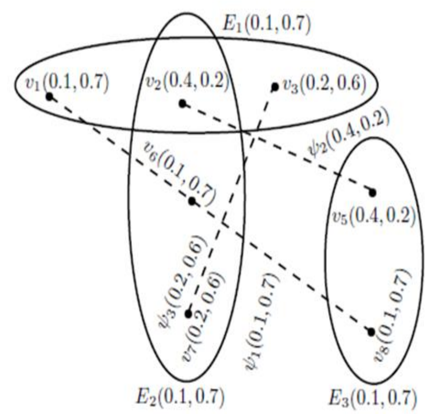

Figure 2: $\widetilde{\mathcal{H}}$

Note: The following modifications were made to $\widetilde{\mathcal{H}}$

(i) Spike is diminished.

(ii) Ms and Nms degree of the vertices have been changed.

Definition Consider an IFk-PHG, $\mathcal{H}$. A primitive r-coloring $\mathcal{P}$ of $\mathcal{H}$ is a separation $\{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \cdot, \mathcal{P}_r\}$ of V into r-subsets(colors) such that at least two colors of $\mathcal{P}$ cross the support of each intuitionistic fuzzy k-partite hyperedge of $\mathcal{H}$, with the exception of spike hyperedges.

Definition Assume an IFk-PHG be $\mathcal{H}$ and suppose that $c(\mathcal{H}) = \{\mathcal{H}^{a_1,b_1}, \mathcal{H}^{a_2,b_2} \cdot, \mathcal{H}^{a_k,b_k}\}$.

A Q-coloring $\mathcal{P}$ of $\mathcal{H}$ is a partition $\{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, , \mathcal{P}_r\}$ of V into r-subsets(colors) $\ni \mathcal{P}$ causes a coloring for every core hypergraph $\mathcal{H}^{a_i,b_i}$ of $\mathcal{H}$ with $\mathcal{H}^{a_i,b_i} = (V_i, E_i, \psi_i)$ wherein $V_i \subset V$, $E_i \subset E$ and $\psi_i \subset \psi$. The limitation of $\mathcal{P}$ to V_i, $\{\mathcal{P}_1 \cap V_i, \mathcal{P}_2 \cap V_i, \mathcal{P}_3 \cap V_i, \cdots, \mathcal{P}_s \cap V_i\}$ is a coloring of $\{\mathcal{H}^{a_i,b_i}\}$.(Recognize that color set $\mathcal{P}_i$ is empty).

Definition The minimum number $\chi_r(\mathcal{H})$ of colors required to provide a primitive coloring of $\mathcal{H}$ is known as the r-chromatic number of $\mathcal{H}$. An IFk-PHG's chromatic number is the smallest number $\chi(\mathcal{H})$ of colors required to produce a Q-coloring of $\mathcal{H}$.

Definition Let an IFk-PHG be $\mathcal{H}$. The lower truncation $\mathcal{H}_l$ of $\mathcal{H}$ at $\langle a_l, b_l\rangle$-level, $0 < a_l \le h_\mu(\mathcal{H}), 0 < b_l \le h_\nu(\mathcal{H})$ where $a_l < \mu_{k_i}$, $b_l < \nu_{k_i}\ \forall v_i$ is an IFk-PHG. $\mathcal{H}_l = \langle V_t, E_t, \psi_t, \mu_{t_l}(\Psi_{k_{ij}}), \nu_{t_l}(\Psi_{k_{ij}})\rangle$ where $V_t \subset V, E_t \subset E$ and $\psi_t \subset \psi$ represent the set of vertices, hyperedges and k-partite hyperedges of truncated IFk-PHG respectively and

$$\mu_{t_l}\left(\Psi_{k_{ij}}\right) = \begin{cases} \mu_{k_{ij}} & \text{if } \mu_{k_{ij}} \ge a_l \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{t_l}\left(\Psi_{k_{ij}}\right) = \begin{cases} \gamma_{k_{ij}} & \text{if } \nu_{k_{ij}} \le b_l \\ 0 & \text{otherwise} \end{cases},$$

are the Ms and Nms values of the k-partite hyperedge $\Psi_{k_{ij}}$.

Definition The upper truncation $\mathcal{H}_u$ of $\mathcal{H}$ at $\langle a_u, b_u\rangle$-level, $0 < a_u \le h_\mu(\mathcal{H})$,$0 < b_u \le h_\nu(\mathcal{H})$ where $a_u < \mu_{k_i}, b_u < \nu_{k_i}\ \forall v_i$ is an IFk-PHG, $\mathcal{H}_u = \langle V_t, E_t, \psi_t, \mu_{t_u}(\Psi_{k_{ij}}), \nu_{t_u}(\Psi_{k_{ij}})\rangle$, where $V_t \subset V, E_t \subset E$ and $\psi_t \subset \psi$ represent the set of vertices, hyperedges and k-partite hyperedges of truncated IFk-PHG respectively and

$$\mu_{t_u}\left(\Psi_{k_{ij}}\right) = \begin{cases} \mu_{k_{ij}} & \text{if } \mu_{k_{ij}} \le a_u \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{t_u}\left(\Psi_{k_{ij}}\right) = \begin{cases} \nu_{k_{ij}} & \text{if } \nu_{k_{ij}} \ge b_u \\ 0 & \text{otherwise} \end{cases}$$

are the Ms and Nms values of the k-partite hyperedge $\Psi_{k_{ij}}$.

Note: Lower and upper truncation Ms values are represented by μ_{t_l}, μ_{t_u} and Nms values by ν_{t_l}, ν_{t_u} respectively.

Definition The term IFk-PHG $\mathcal{H}$ refers to an intersecting IFk-PHG if each IF k-partite hyperedge pair $\{\psi_i, \psi_j\} \subseteq \psi$, $\psi_i \cap \psi_j \ne \emptyset$.

Definition Let an IFk-PHG be $\mathcal{H}$ and $c(\mathcal{H}) = \{\mathcal{H}^{a_1,b_1}, \mathcal{H}^{a_2,b_2}, \cdots, \mathcal{H}^{a_k,b_k}\}$, if $\mathcal{H}^{a_i,b_i}$ is an intersecting IFk-PHG for every $i = 1,\cdots,k$ so $\mathcal{H}$ is K-intersecting IFk-PHG.

Definition It is considered that an IFk-PHG is strongly intersecting, if any two k-partite hyperedges ψ_i and ψ_j possess a usual spike of height $h = h(\psi_i) \wedge h(\psi_j)$.

Theorem Assume $\mathcal{H}$ is ordered, intersecting IFk-PHG, then every IF k-partite hyperedge $\mathcal{T}$ of $\mathcal{H}$ includes a representative of $\mathcal{TR}(\mathcal{H}_{h(\mathcal{T})})$, in which $\mathcal{H}_{h(\mathcal{T})}$ is the upper truncation of $\mathcal{H}$ at level $h(\mathcal{T})$. $\mathcal{T}$ is specifically an Intuitionistic Fuzzy Transversal (IFT) of $\mathcal{H}_{h(\mathcal{T})}$.

Proof: Suppose $c(\mathcal{H}) = \{\mathcal{H}^{a_i,b_i} = (V^{a_i,b_i}, E^{a_i,b_i}, \psi^{a_i,b_i}) | i = 1,2,\cdots,k\}$ and suppose $\mathcal{T}_j \in \psi$. Assume that, $\langle a_1, b_1\rangle = h(\mathcal{T})$. As $\mathcal{H}$ is ordered and $\mathcal{T}^{a_i,b_i} \ne \emptyset$. Also, because $\mathcal{H}$ intersects this implies that $\mathcal{H}^{a_k,b_k}$ is also intersecting. Hence, $\mathcal{T}^{a_1,b_1}$ is an IFT of $\mathcal{H}^{a_n,b_n}$. Consider $\mathcal{T}_1$ be a minimal IFT of $\mathcal{H}^{a_k,b_k} \subseteq \mathcal{T}^{a_1,b_1}$. Because $\mathcal{H}$ is ordered, there is a structured sequence of sets

$$\mathcal{T}_k \supseteq \cdots \mathcal{T}_i \supseteq \cdots \supseteq \mathcal{T}_1$$

$\ni \mathcal{T}_i$ is a minimal IFT of $\mathcal{H}^{a_i,b_i}$ for each $\langle a_1, b_1\rangle \in F_k(\mathcal{H})$. The elementary IF subset with support $\mathcal{T}'$ is denoted by Θ_i and height $\langle a_i, b_i\rangle$ for $i = 1,2,\cdots,k$. Then, $\bigcup_{i=1}^{n} \Theta_i \in \mathcal{TR}(\mathcal{H})$ and $\mathcal{T}' \subseteq \mathcal{T}$. Hence, every IF k-partite hyperedge $\mathcal{T}$ of $\mathcal{H}$ contains a member of $\mathcal{TR}(\mathcal{H}_{h(\mathcal{T})})$.

Theorem Suppose that $\mathcal{H}$ is ordered, intersecting IFk-PHG with $c(\mathcal{H}) = \{\mathcal{H}^{a_i,b_i} = (V^{a_i,b_i}, E^{a_i,b_i}, \psi^{a_i,b_i}) | i = 1,2,\cdots,k\}$. Let $\chi(\mathcal{H}^{a_i,b_i}) > 2$ and $\mathcal{H}^{a_k,b_k}$ is simple. Then for each $\langle a_i, b_i\rangle \in F_k(\mathcal{H})$, $\mathcal{TR}(\mathcal{H}^{a_i,b_i}) = \{\Theta(\mathcal{T}, \langle a_i, b_i\rangle) | \mathcal{T} \in \mathcal{H}^{a_i,b_i}\}$ where $\Theta(\mathcal{T}, \langle a_i, b_i\rangle)$ is an elementary IF subset utilizing support ψ and height $\langle a_i, b_i\rangle$.

Proof: According to the hypothesis, $\mathcal{H}^{a_i,b_i}$ is intersecting, simple and $\chi(\mathcal{H}^{a_i,b_i}) > 2$ for every $\mathcal{H}^{a_i,b_i} \in c(\mathcal{H})$. $\mathcal{T}$ is the collection of all $\mathcal{H}$'s minimal transversals. Consequently, the collection of $\mathcal{H}^{a_i,b_i} = \mathcal{TR}(\mathcal{H}^{a_i,b_i})$, for every $\langle a_i, b_i\rangle$ belongs to $F_k(\mathcal{H})$.

Definition When $\widetilde{\mathcal{H}}$ intersects, then an IFk-PHG $\mathcal{H}$ is known as essentially intersecting.

Also, if $\widetilde{\mathcal{H}}$ is strongly intersecting then $\mathcal{H}$ is known as essentially strongly intersecting.

Definition $\mathcal{H}$ is said to be non-trivial if it has atleast one edge $\psi \ni |supp(\psi)| \ge 2$.

Theorem Let $\mathcal{H}$ be an IFk-PHG. Then $\mathcal{H}$ is strongly intersecting $\Leftrightarrow$ $\mathcal{H}$ is K-intersecting.

Proof: Necessary Part:
Assume $\mathcal{H}$ has a strong intersection, consider ψ_i, ψ_j be the edges of $\mathcal{H}^{a_i,b_i} \in c(\mathcal{H})$. There are then two edges ψ_1 and ψ_2 of $\mathcal{H} \ni {\psi_1}^{\langle a_i,b_i\rangle} = \psi_1$ and ${\psi_2}^{\langle a_i,b_i\rangle} = \psi_2$. Both ψ_1 and ψ_2 have the common spike Θ_{υ_i} where $0 \le a_i \le h_\mu(\Theta_{v_i})$ and $0 \le b_i \le h_\nu(\Theta_{v_i})$, because $\mathcal{H}$ is strongly intersecting. Thus, $(\Theta_{v_i}) = \{\upsilon_i\} \subseteq \psi_i \cap \psi_j$. Hence $\mathcal{H}^{a_i,b_i}$ is intersecting and $\mathcal{H}$ is K-intersecting.

Sufficient Part:
Let $\mathcal{H}$ be K-intersecting and assume ζ_i and ζ_j be its hyperedges and consider $\langle a_i, b_i\rangle = h(\zeta_i) \wedge h(\zeta_j)$ and let $\psi_i = {\zeta_i}^{a_i,b_i}, \psi_j = {\zeta_j}^{a_i,b_i}$ then both $\psi_i, \psi_j \in \mathcal{H}^{a_i,b_i} = \mathcal{H}^{a_j,b_j}$ where $a_{j+1} < a_i \le a_j, b_{j+1} < b_i \le b_j$.Let$\langle a_{k+1}, b_{k+1}\rangle = \langle 0,1\rangle$, since $\mathcal{H}^{a_i,b_i}$ is intersecting $\exists$ a vertex $\upsilon_i \in \psi_i \cap \psi_j$. A spike $\Theta_{v_i}$with height $\langle a_i, b_i\rangle$ and support $\{v_i\}$ is present in both ζ_i and ζ_j. So, $\mathcal{H}$ therefore intersects strongly.

Theorem Assume $\mathcal{H}$ be an IFk-PHG and let $c(\mathcal{H}) = \{\mathcal{H}^{a_i,b_i} = (V^{a_i,b_i}, E^{a_i,b_i}, \psi^{a_i,b_i}) | i = 1,\cdots,k\}$. So, $\mathcal{H}$ is intersecting if and only if $\mathcal{H}^{a_k,b_k} = (V^{a_k,b_k}, E^{a_k,b_k}, \psi^{a_k,b_k})$ is intersecting.
Proof: $\mathcal{H}$ is intersecting $\Leftrightarrow$ $\text{supp}(\mathcal{H}) = \{\text{supp}(\psi_j) | \psi_j \in \psi\}$ is intersecting. Similarly, each pair of intuitionistic fuzzy k-partite hyperedg $\{\psi_1, \psi_2, \cdots, \psi_k\} \subseteq \psi$, $\mathcal{H}^{a_1,b_1}, \mathcal{H}^{a_2,b_2}, \cdots, \mathcal{H}^{a_k,b_k}$ are intersecting.
Conversely, let $\mathcal{H}^{a_k,b_k} = (V^{a_k,b_k}, E^{a_k,b_k}, \psi^{a_k,b_k})$ is intersecting.
Since, $\text{supp}(\mathcal{H}) = \{\text{supp}(\psi_j) | \psi_j \in \psi\}$ is intersecting, so $\mathcal{H}$ is also intersecting.

IV. DECISION METHOD ALGORITHM

Step 1: Input the set of vertices as $V = \{\upsilon_1, \upsilon_2, \dots, \upsilon_r\}$ where $\upsilon_1, \upsilon_2, \dots, \upsilon_r$ represent tasks of the plant.
Step 2: Make group of related tasks where $E = \{E_1, E_2, \dots, E_n\}$ is the set of different tasks.
Step 3: Construct IFk-PHG by assuming the partite set as robots $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \dots, \mathcal{R}_p\}$.
Step 4: Apply the score function $\mathcal{S}_k(\mathcal{R}_i) = \langle \frac{1-\nu}{2-\mu-\nu}, 1 - \frac{1-\nu}{2-\mu-\nu}\rangle$ to evaluate the score values of each robot.
Step 5: The decision is $\Gamma = \langle \max\{\mathcal{S}_k(\mathcal{R}_i)\}, \min\{\mathcal{S}_k(\mathcal{R}_i)\}\rangle$.

An application of IFk-PHG in Car Manufacturing Plant
In today's rapidly advancing technological landscape, robotics plays a vital role in shaping industries and revolutionizing the way we work. Among the remarkable developments in robotics, multi-tasking robots stand out as versatile and adaptable machines that can perform a multitude of tasks within a single system. These robots are designed to handle various functions seamlessly, offering increased efficiency, cost-effectiveness and flexibility in a wide range of applications.
A multi-tasking robot combines the capabilities of several specialized robots into a single united system. Equipped with sophisticated sensors, actuators and artificial intelligence these robots can perceive their environment, make decisions based on real-time data and autonomously switch between different tasks. This creativity enables them to excel in industries such as manufacturing, logistics, healthcare, agriculture and more.
The key features of multi-tasking robots include their ability to efficiently switch between tasks, adapt to changing requirements and collaborate with human operators. By reducing the need for multiple specialized robots, they optimize floor space and reduce operational costs making them a valuable asset for business aiming to enhance productivity and competitiveness.
As technological advancements continue to drive innovation in robotics, multi-tasking robots are likely to become even more sophisticated, offering endless possibilities for the future of automation. This introduction sets the stage for exploring the vast potential of multi-tasking robots and their transformative impact on various industries empowering us to embrace a new era of efficiency and automation.

Description of the Proposed Model
Imagine a car manufacturing plant produces different models of cars. There are variety of tasks such as frame welding, body painting, interior installation,

automated inspection, final assembly, quality control, packaging and loading, logistics and transportation etc. are performed by humans. Instead of humans a multi-tasking robot is used and by assigning various tasks to robots in different ways, the manufacturing plant has to decide which robot can complete the task fast.

Assume that $V = \{\upsilon_1, \upsilon_2, \dots, \upsilon_{12}\}$ are various tasks of the car manufacturing plant, $E = \{E_1, E_2, E_3, E_4\}$ are group of related tasks for the different models of cars. This scenario is modeled as an IF hypergraph. For this IF-hypergraph, we can formulate IFk-PHG in which the partite sets $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \mathcal{R}_4\}$ are contemplated as multi-tasking robots, which defines the degree of Ms and Nms of robot capability of completing task and robot capability of not completing task respectively. Figure 3 shows the pattern of IF hypergraph.

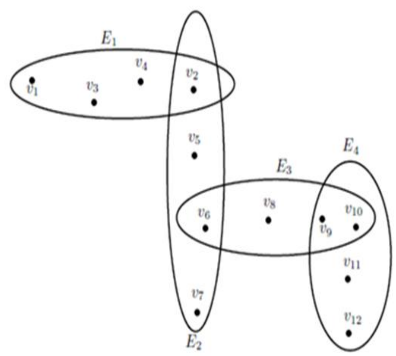

Figure 3: Modeled Intuitionistic Fuzzy Hypergraph

The vertex values are tabulated as shown in Table 1.

$\upsilon_1\langle 0.59,0.15\rangle$ $\upsilon_2\langle 0.90,0.01$ $\upsilon_3\langle 0.65,0.21\rangle$
$\upsilon_4\langle 0.81,0.11\rangle$ $\upsilon_5\langle 0.86,0.10\rangle$ $\upsilon_6\langle 0.71,0.14\rangle$
$\upsilon_7\langle 0.92,0.01\rangle$ $\upsilon_8\langle 0.88,0.04$ $\upsilon_9\langle 0.77,0.13\rangle$
$\upsilon_{10}\langle 0.90,0.04\rangle$ $\upsilon_{11}\langle 0.93,0.02\rangle$ $\upsilon_{12}\langle 0.54,0.28\rangle$
Consider E_1 as $\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$, E_2 as $\{\upsilon_2, \upsilon_5, \upsilon_6, \upsilon_7\}$, E_3 as $\{\upsilon_6, \upsilon_8, \upsilon_9, \upsilon_{10}\}$ and E_4 as $\{\upsilon_9, \upsilon_{10}, \upsilon_{11}, \upsilon_{12}\}$. Assignment of tasks to robots can be done in the following cases:

Case (i):

Consider robot $\mathcal{R}_1$ has the tasks $\{\upsilon_1, \upsilon_5, \upsilon_9\}$, $\mathcal{R}_2$ has the tasks $\{\upsilon_2, \upsilon_8, \upsilon_{11}\}$, $\mathcal{R}_3$ has the tasks $\{\upsilon_3, \upsilon_6, \upsilon_{12}\}$ and $\mathcal{R}_4$ has the tasks $\{\upsilon_4, \upsilon_7, \upsilon_{10}\}$. Its Ms and Nms value is shown in Table 2.

Table 2: Robot values

Robots $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle$
$\mathcal{R}_1$ $\langle 0.59,0.15\rangle$
$\mathcal{R}_2\langle 0.88,0.04\rangle$
$\mathcal{R}_3$ $\langle 0.54,0.28\rangle$
$\mathcal{R}_4$ $\langle 0.81,0.11\rangle$

Applying the score function $\mathcal{S}_k(\mathcal{R}_i) = \langle \frac{1-\nu}{2-\mu-\nu}, 1 - \frac{1-\nu}{2-\mu-\nu} \rangle$ to calculate the score values of robots.

$$\mathcal{S}_k(\mathcal{R}_1) = \langle \frac{1-0.15}{2-0.59-0.15}, 1 - \frac{1-0.15}{2-0.59-0.15} \rangle = \langle \frac{0.85}{1.26}, 1 - \frac{0.85}{1.26} \rangle = \langle 0.67, 0.33 \rangle$$

$$\mathcal{S}_k(\mathcal{R}_2) = \langle \frac{1-0.04}{2-0.88-0.04}, 1 - \frac{1-0.04}{2-0.88-0.04} \rangle = \langle \frac{0.96}{1.08}, 1 - \frac{0.96}{1.08} \rangle = \langle 0.89, 0.11 \rangle$$

$$\mathcal{S}_k(\mathcal{R}_3) = \langle \frac{1-0.28}{2-0.54-0.28}, 1 - \frac{1-0.28}{2-0.54-0.28} \rangle = \langle \frac{0.72}{1.18}, 1 - \frac{0.72}{1.18} \rangle = \langle 0.61, 0.39 \rangle$$

$$\mathcal{S}_k(\mathcal{R}_4) = \langle \frac{1-0.11}{2-0.81-0.11}, 1 - \frac{1-0.11}{2-0.81-0.11} \rangle = \langle \frac{0.89}{1.08}, 1 - \frac{0.89}{1.08} \rangle = \langle 0.82, 0.18 \rangle$$

Then the decision to select which robot can complete the task fast is

$\Gamma = \langle \max\{\mathcal{S}_k(\mathcal{R}_i)\}, \min\{\mathcal{S}_k(\mathcal{R}_i)\} \rangle = \langle 0.89, 0.11 \rangle$

Hence it is concluded that among four robots second robot ($\mathcal{R}_2$) will complete the task quickly

Case (ii):

Let us consider the robot $\mathcal{R}_1$ has the tasks $\{\upsilon_3, \upsilon_7, \upsilon_{10}\}$, $\mathcal{R}_2$ has the tasks $\{\upsilon_4, \upsilon_6, \upsilon_{11}\}$, $\mathcal{R}_3$ has the tasks $\{\upsilon_1, \upsilon_5, \upsilon_9\}$ and $\mathcal{R}_4$ has the tasks $\{\upsilon_2, \upsilon_8, \upsilon_{12}\}$. Its Ms and Nms value is shown in Table 3.

Table 3: Values of the robot

Robots $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle$
$\mathcal{R}_1$ $\langle 0.65,0.21\rangle$
$\mathcal{R}_2$ $\langle 0.71,0.14\rangle$
$\mathcal{R}_3$ $\langle 0.59,0.15\rangle$
$\mathcal{R}_4$ $\langle 0.54,0.28\rangle$

The score values of each robot is $\mathcal{S}_k(\mathcal{R}_1) = \langle 0.69,0.31\rangle$, $\mathcal{S}_k(\mathcal{R}_2) = \langle 0.75,0.25\rangle$, $\mathcal{S}_k(\mathcal{R}_3) = \langle 0.67,0.33\rangle$, $\mathcal{S}_k(\mathcal{R}_4) = \langle 0.61,0.39\rangle$.

Then the decision to select which robot can complete the task fast is
$\Gamma = \langle \max\{\mathcal{S}_k(\mathcal{R}_i)\}, \min\{\mathcal{S}_k(\mathcal{R}_i)\}\rangle = \langle 0.75, 0.25\rangle$
Hence it is concluded that among four robots second robot $(\mathcal{R}_2)$ will complete the task quickly.

Case (iii):
Assume the tasks given to robot $\mathcal{R}_1$ is $\{\upsilon_4, \upsilon_6, \upsilon_{11}\}$, $\mathcal{R}_2$ is $\{\upsilon_1, \upsilon_5, \upsilon_{10}\}$, $\mathcal{R}_3$ is $\{\upsilon_2, \upsilon_8, \upsilon_{12}\}$ and $\mathcal{R}_4$ is $\{\upsilon_3, \upsilon_7, \upsilon_9\}$. Table 4 shows its Ms and Nms value.

Table 4: Values of Ms and Nms

Robots $\langle \mu_{k_{ij}}, \nu_{k_{ij}}\rangle$
$\mathcal{R}_1$ $\langle 0.71, 0.14\rangle$
$\mathcal{R}_2$ $\langle 0.59, 0.15\rangle$
$\mathcal{R}_3$ $\langle 0.54, 0.28\rangle$
$\mathcal{R}_4$ $\langle 0.65, 0.21\rangle$

The score values of each robot is $\mathcal{S}_k(\mathcal{R}_1) = \langle 0.75, 0.25\rangle$, $\mathcal{S}_k(\mathcal{R}_2) = \langle 0.67, 0.33\rangle$, $\mathcal{S}_k(\mathcal{R}_3) = \langle 0.61, 0.39\rangle$, $\mathcal{S}_k(\mathcal{R}_4) = \langle 0.69, 0.31\rangle$.
Then the decision to select which robot can complete the task fast is $\Gamma = \langle \max\{\mathcal{S}_k(\mathcal{R}_i)\}, \min\{\mathcal{S}_k(\mathcal{R}_i)\}\rangle = \langle 0.75, 0.25\rangle$
Hence it is concluded that among four robots second robot $(\mathcal{R}_1)$ will complete the task quickly.
For the above said example there exists 576 number of combinations. The variety of tasks assigned for four robots indicates the combinations. In each case, the smart work of the robot can be identified, so that it can be concluded that the assignment of robots to the tasks shall be fixed to obtain the better result.

V. CONCLUSION

This paper introduces the concepts of upper and lower truncation, chromatic number, essential intersection, and intersection in IFk-PHG. Additionally, an application of IFk-PHG in robotics within a car manufacturing plant is presented, offering a novel approach to representing k-partite sets as robots. Future research will focus on extending IFk-PHG to new applications.

VI. ABBREVIATIONS

IFk-PHG - Intuitionistic Fuzzy k-partite Hypergraph
IFS - Intuitionistic Fuzzy Set
Ms - Membership
Nms - non-membership
IFT - Intuitionistic Fuzzy Transversal

VII. ACKNOWLEDGEMENT

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