

On the Construction of Symmetrical Factorial Experiments with Partial Confounding

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Abstract- Only one element may be examined at a time in the basic designs. Factorial experiments are used to examine several factors at once. There will be more treatment combinations if the factorial design's variables and levels are increased. Only a portion of the whole factorial experiment may yield information on the main effects and lower order interactions. The number of experimental units in a single replicate itself grows significantly as the number of variables and levels rises concurrently. This results in an excessively high block size, which rules out the blocks' homogeneity. So, the experimental error will be increases and the precision of the design will decrease. This limitation of factorial experiment can be overcome by the technique of reducing the block size. This is possible when the interaction contrast and the contrast between the blocks are identical they are given by the same fraction. As a result, the block effects and interaction effects are combined and cannot be distinguished from one another. Blocks have confused the impact of interactions. Information on the muddled effects is obviously lost. By reducing the block size, it is now possible to estimate the major impacts and additional interactions more precisely. In this paper, introducing the new method of construction of 2^5 factorial experiments for $F_1F_4F_5$ and $F_2F_3F_5$ are partially confounded in four blocks through application.

Index Terms- ANOVA, Factorial Experiments, Partial Confounding, RBD, Replication.

I. INTRODUCTION

A precision of a basic design of experiments may be increased by means of replications, blocking, analysis of co-variance, incomplete blocks, etc., following these measures another technique of increasing the precision of experimental design is called confounding. Even though the factorial experiments superior to simple experiments like *CRD*, *RBD* and *LSD*. Then the factorial experiment is having their own limitations. One

among them is this complexity of the design. The number of factors and the number of levels increases simultaneously then the number of experimental units in a single replicate itself becomes very large. This leads to block size to be too large and hence the homogeneity of the blocks ruled out. So, the experimental error will be increases and the precision of the design will decrease. This limitation of factorial experiment can be overcome by the technique of reducing the block size. This is possible when the interaction contrast and the contrast between the blocks are identical they are given by the same fraction. As a result, the block effects and interaction effects are combined and cannot be distinguished from one another. Blocks have confused the impact of interactions. Information on the muddled effects is obviously lost. By reducing the block size, it is now possible to estimate the major impacts and additional interactions more precisely. Yates F. introduced the idea of confounding in factorial experiments in 1937. Bose and Kishen created the principle of symmetrical factorial trials in 1940. Kempthorne (1947) addressed the use of partial confounding in factorial experiments. Cochran and Cox (1957) give a thorough review of factorial experiments, including symmetrical factorial trials with partial confounding. The issue of carrying out a mn factorial experiment in block size and the possibility of creating designs for all the values is examined by Sarah C. Cotter (1974). Cotter (1975) presented a novel partial confounding approach that combines imperfect block design and the confounding methodology. The construction of symmetrical factorial experiments with partial confounding is recently developed by Mukhopadhyay (2002). Das and Dey (2004) has developed a method for constructing symmetrical factorial experiments with partial confounding using different sets. The

method of trend free and blocked fractional factorial design by utilizing linear codes by Singh et.al (2016). When a typical fractional factorial design is employed to explore the main, Zhang et al. (2011) have taken into consideration the experimental conditions when complete information on two factor interactions is lost and interactions are frequently confused with blocks. In this paper, introducing the new method of construction of 2^5 factorial experiments for $F_1F_4F_5$ and $F_2F_3F_5$ are partially confounded in four blocks through application.

II. PRELIMINARIES

A. Definition of Confounding

Confounding is the process of making one or more interaction contrasts equal to the block contrast in order to reduce the block size. The factorial experiment confuses only the higher order interactions. Because it doesn't matter if knowledge about higher level interactions is lost. Typically, there shouldn't be as much confusion between the primary impact and the two-factor interaction as this.

B. Types of Confounding

When a factorial experiment has two or more replicates. After then, it is common to wonder if the same set of interactions is confused in every duplicate or if different sets of interactions are confused in various replicates. In factorial studies, this results in two forms of confounding: partial and total or full confounding. if the various sets of interactions in several replicates are confused. Partial confounding is the term used to describe the confounding that occurs in factorial studies. These replicates that are not confounded can be used to assess the confounded factorial effects in partial confounding. As a result, all of the factorial effects will be included in the ANOVA as they may be calculated in partial confounding.

C. Defining Contrast Rule

In confounding problem, the blocks can be sub-grouped into two or more incomplete blocks the most general method of their sub-group is called defining contrast rule. This method uses the linear combinations define by,

$$L = \sum_{i=1}^n p_i x_i \quad (1)$$

where p_i is the exponent (index) of the i^{th} factor in the confounded factorial effect, and x_i is the level of the i^{th} factor that appears in the specific

treatment combinations. It is obvious that n indicates how many elements there are in the design.

III. METHOD OF CONSTRUCTION FOR SYMMETRICAL FACTORIAL EXPERIMENTS WITH PARTIAL CONFOUNDING

Step 1: Consider a s^n symmetrical factorial experiments with n factors each with s levels.

Step 2: Form the s^n treatment combinations of symmetrical factorial experiments.

Step 3: Consider the number of blocks is more than two and the design is s^n then find the number of blocks size is $\left\{ \frac{\text{Design Size}}{\text{No. of Blocks}} \right\}$.

Step 4: Find the number of confounded interactions is $\left\{ \frac{\text{No. of Blocks} - 1}{\text{No. of Levels} - 1} \right\}$.

Step 5: To construct the linear treatment combinations of partial confounded factors blocks using the formula of defining contrast rule.

IV. APPLICATION

A. Construction of 2^5 Factorial Experiments for $F_1F_4F_5$ and $F_2F_3F_5$ are Partially Confounded in 2^2 Blocks

Consider the 2^5 symmetrical factorial experimental design with five factors F_1, F_2, F_3, F_4, F_5 , each with two levels namely 0 and 1. Then this will constitute 2^5 symmetrical factorial experimental design with 32 treatment combinations in each replicate are shown in table I.

The number of blocks is 4 and the design size is 2^5 it is follows that the block size is $= \frac{\text{Design size}}{\text{Number of blocks}} = \frac{32}{4} = 8$.

The number of confounded interactions = $\frac{\text{Number of blocks} - 1}{\text{Number of level} - 1} = 3$. In 2^5 factorial experiments in 2^2 blocks, clearly, 3 factor effects are confounded. Suppose the two factorial effects to be partially confounded with blocks are $F_1F_4F_5$ and $F_2F_3F_5$. Then the corresponding defining contrast will be

$$L_1 = x_1 + x_4 + x_5 \quad (F_1 F_4 F_5)$$

$$L_2 = x_2 + x_3 + x_5 \quad (F_2 F_3 F_5)$$

Table I. 2^5 symmetrical factorial experimental design with 32 treatment combinations

(1)	f_1	f_2	f_1f_2	f_3	f_1f_3	f_2f_3	$f_1f_2f_3$
f_1	f_1f_4	f_2f_4	$f_1f_2f_4$	f_3f_4	$f_1f_3f_4$	$f_2f_3f_4$	$f_1f_2f_3f_4$
f_5	f_1f_5	f_2f_5	$f_1f_2f_5$	f_3f_5	$f_1f_3f_5$	$f_2f_3f_5$	$f_1f_2f_3f_5$
f_4f_5	$f_1f_4f_5$	$f_2f_4f_5$	$f_1f_2f_4f_5$	$f_3f_4f_5$	$f_1f_3f_4f_5$	$f_2f_3f_4f_5$	$f_1f_2f_3f_4f_5$

Now, every treatment combination of 2^5 factorial experiment will give a particular value of L_1 (mod 2) and L_2 (mod 2) namely,

$$(L_1, L_2) = (1,1) (1,2) (2,1) (2,2)$$

The set of treatment combinations giving the same value of L_1 and L_2 are assigned. In the same block for example, the set of treatments giving the value of $L_1 = 0$ (mod 2) and $L_2 = 0$ (mod 2) are placed in the same block. This block is the key block or principle block. Similarly, the set of treatments giving the values of $L_1 = 0$ (mod 2) and $L_2 = 0$ (mod 2) are assigned in the same block. Thirdly, the set of treatments giving the values of $L_1 = 0$ (mod 2) and $L_2 = 0$ (mod 2) are assigned in another block. Lastly, the set of treatments giving the values of $L_1 = 1$ (mod 2) and $L_2 = 1$ (mod 2) are placed in the last block. In this study, ($F_1F_4F_5$ and $F_2F_3F_5$ are partially confounded). Find that $L_1 = 0$ (mod 2) and $L_2 = 0$ (mod 2) for the treatments.

$$L(1) = (1+1+1,1+1+1) = (1,1)$$

$$L(f_1) = (2+1+1,1+1+1) = (2,1)$$

$$L(f_2) = (1+1+1,2+1+1) = (1,2)$$

$$L(f_1f_2) = (2+1+1,2+1+1) = (2,2)$$

$$L(f_3) = (1+1+1,1+2+1) = (1,2)$$

$$L(f_1f_3) = (2+1+1,1+2+1) = (2,2)$$

$$L(f_2f_3) = (1+1+1,2+2+1) = (1,3) = (1,2)$$

$$L(f_1f_2f_3) = (2+1+1,2+2+1) = (2,3) = (2,1)$$

$$L(f_4) = (1+2+1,1+1+1) = (2,1)$$

$$L(f_1f_4) = (2+2+1,1+1+1) = (3,1) = (1,1)$$

$$L(f_2f_4) = (1+2+1,2+1+1) = (2,2)$$

$$L(f_1f_2f_4) = (2+2+1,2+1+1) = (3,2) = (1,2)$$

$$L(f_3f_4) = (1+2+1,1+2+1) = (2,2)$$

$$L(f_1f_3f_4) = (2+2+1,1+2+1) = (3,2) = (1,2)$$

$$L(f_2f_3f_4) = (1+2+1,2+2+1) = (2,3) = (1,2)$$

$$L(f_1f_2f_3f_4) = (2+2+1,2+2+1) = (3,3) = (1,1)$$

$$L(f_5) = (1+1+2,1+1+2) = (1,1)$$

$$L(f_1f_5) = (2+1+2,1+1+2) = (3,2) = (1,2)$$

$$L(f_2f_5) = (1+1+2,2+1+2) = (2,3) = (2,1)$$

$$L(f_1f_2f_5) = (2+1+2,2+1+2) = (3,3) = (1,1)$$

$$L(f_3f_5) = (1+1+2,1+2+2) = (2,3) = (2,1)$$

$$L(f_1f_3f_5) = (2+1+2,1+2+2) = (3,3) = (1,1)$$

$$L(f_2f_3f_5) = (1+1+2,2+2+2) = (3,2) = (1,2)$$

$$L(f_1f_2f_3f_5) = (2+1+2,2+12+2) = (3,4) = (1,2)$$

$$L(f_4f_5) = (1+2+2,1+1+2) = (3,2) = (1,2)$$

$$L(f_1f_4f_5) = (2+2+2,1+1+2) = (4,2) = (2,2)$$

$$L(f_2f_4f_5) = (1+2+2,2+1+2) = (3,3) = (1,1)$$

$$L(f_1f_2f_4f_5) = (2+2+2,2+1+2) = (4,3) = (2,1)$$

$$L(f_3f_4f_5) = (1+2+2,1+2+2) = (3,3) = (1,1)$$

$$L(f_1f_3f_4f_5) = (2+2+2,1+2+2) = (4,3) = (2,1)$$

$$L(f_2f_3f_4f_5) = (1+2+2,2+2+2) = (3,4) = (1,2)$$

$$L(f_1f_2f_3f_4f_5) = (2+2+2,2+2+2) = (4,4) = (2,2)$$

Constructing the $L_1 = 0$ (mod 2) and $L_2 = 0$ (mod 2) for the treatments are (1), f_2f_3 , f_2f_4 , $f_1f_2f_3f_4$, $f_1f_2f_5$, $f_1f_3f_5$, $f_2f_4f_5$, $f_3f_4f_5$. Similarly, find that $L_1 = 0$ (mod 2) and $L_2 = 1$ (mod 2) for the treatments $f_2, f_3, f_1f_2f_4, f_1f_3f_4, f_1f_5, f_1f_2f_3f_5, f_4f_5, f_2f_3f_4f_5$. Again, $L_1 = 1$ (mod 2) and $L_2 = 0$ (mod 2) for the treatments $f_1, f_1f_2f_3, f_4, f_2f_3f_4, f_2f_5, f_1f_2f_4f_5, f_1f_3f_4f_5$. Lastly, $L_1 = 1$ (mod 2) and $L_2 = 1$ (mod 2) for the treatments $f_1f_2, f_1f_3, f_2f_4f_3f_4, f_5, f_2f_3f_5, f_1f_4f_5, f_1f_2f_3f_4f_5$.

These treatment combinations are assigned to different blocks denoted by block I (Key block or principle block), block II, block III and block IV respectively. Thus, the construction of designs is showing in the following table II.

Since, the number of confounded interactions is 3 and the third confounded interactions other than $F_1F_4F_5$ and $F_2F_3F_5$ is generalized interactions of $F_1F_4F_5 * F_2F_3F_5 = F_1F_4 F_2F_3F_5^2 = F_1F_2F_3F_4$ (mod 2)

All the sum of squares is presented in the table III.

Table II. Construction of 2⁵ Factorial Experiments in 2² Blocks with F₁F₄F₅ and F₂F₃F₅ are Partially Confounding.

Block I	Block II	Block III	Block IV
(L1, L2) = (1,1)	(L1, L2) = (2,1)	(L1, L2) = (1,2)	(L1, L2) = (2,2)
(1)	f ₁	f ₂	f ₁ f ₂
f ₂ f ₃	f ₁ f ₂ f ₃	f ₃	f ₁ f ₃
f ₁ f ₄	f ₄	f ₁ f ₂ f ₄	f ₂ f ₄
f ₁ f ₂ f ₃ f ₄	f ₂ f ₃ f ₄	f ₁ f ₃ f ₄	f ₃ f ₄
f ₁ f ₂ f ₅	f ₂ f ₅	f ₁ f ₅	f ₅
f ₁ f ₃ f ₅	f ₃ f ₅	f ₁ f ₂ f ₃ f ₅	f ₂ f ₃ f ₅
f ₂ f ₄ f ₅	f ₁ f ₂ f ₄ f ₅	f ₄ f ₅	f ₁ f ₄ f ₅
f ₃ f ₄ f ₅	f ₁ f ₃ f ₄ f ₅	f ₂ f ₃ f ₄ f ₅	f ₁ f ₂ f ₃ f ₄ f ₅

Table III. ANOVA for 2⁵ Factorial Experiment with F₁F₄F₅ and F₂F₃F₅ F₁F₄F₅ Partially Confounded in 4 Blocks

SV	DF	SS	MSS	F-Ratio
Blocks	4r-1	Q _B	$M_{F_B} = \frac{Q_B}{4r-1}$	$F_B = \frac{M_{F_B}}{M_E} \sim F_{(4r-1), (28r-28)}$
Treatments	28	Q _T	-	-
Main Effects	1	Q _{F₁}	M _{F₁} = Q _{F₁}	$F_{F_1} = \frac{M_{F_1}}{M_E} \sim F_{1,28(r-1)}$
F ₂	1	Q _{F₂}	M _{F₂} = Q _{F₂}	$F_{F_2} = \frac{M_{F_2}}{M_E} \sim F_{1,28(r-1)}$
F ₃	1	Q _{F₃}	M _{F₃} = Q _{F₃}	$F_{F_3} = \frac{M_{F_3}}{M_E} \sim F_{1,28(r-1)}$
F ₄	1	Q _{F₄}	M _{F₄} = Q _{F₄}	$F_{F_4} = \frac{M_{F_4}}{M_E} \sim F_{1,28(r-1)}$
F ₅	1	Q _{F₅}	M _{F₅} = Q _{F₅}	$F_{F_5} = \frac{M_{F_5}}{M_E} \sim F_{1,28(r-1)}$
2-Factor Interactions	1	Q _{F₁F₂}	M _{F₁F₂} = Q _{F₁F₂}	$F_{F_1F_2} = \frac{M_{F_1F_2}}{M_E} \sim F_{1,28(r-1)}$
F ₁ F ₃	1	Q _{F₁F₃}	M _{F₁F₃} = Q _{F₁F₃}	$F_{F_1F_3} = \frac{M_{F_1F_3}}{M_E} \sim F_{1,28(r-1)}$
F ₂ F ₃	1	Q _{F₂F₃}	M _{F₂F₃} = Q _{F₂F₃}	$F_{F_2F_3} = \frac{M_{F_2F_3}}{M_E} \sim F_{1,28(r-1)}$
F ₁ F ₄	1	Q _{F₁F₄}	M _{F₁F₄} = Q _{F₁F₄}	$F_{F_1F_4} = \frac{M_{F_1F_4}}{M_E} \sim F_{1,28(r-1)}$
F ₂ F ₄	1	Q _{F₂F₄}	M _{F₂F₄} = Q _{F₂F₄}	$F_{F_2F_4} = \frac{M_{F_2F_4}}{M_E} \sim F_{1,28(r-1)}$
F ₃ F ₄	1	Q _{F₃F₄}	M _{F₃F₄} = Q _{F₃F₄}	$F_{F_3F_4} = \frac{M_{F_3F_4}}{M_E} \sim F_{1,28(r-1)}$
F ₁ F ₅	1	Q _{F₁F₅}	M _{F₁F₅} = Q _{F₁F₅}	$F_{F_1F_5} = \frac{M_{F_1F_5}}{M_E} \sim F_{1,28(r-1)}$
F ₂ F ₅	1	Q _{F₂F₅}	M _{F₂F₅} = Q _{F₂F₅}	$F_{F_2F_5} = \frac{M_{F_2F_5}}{M_E} \sim F_{1,28(r-1)}$
F ₃ F ₅	1	Q _{F₃F₅}	M _{F₃F₅} = Q _{F₃F₅}	$F_{F_3F_5} = \frac{M_{F_3F_5}}{M_E} \sim F_{1,28(r-1)}$
F ₄ F ₅	1	Q _{F₄F₅}	M _{F₄F₅} = Q _{F₄F₅}	$F_{F_4F_5} = \frac{M_{F_4F_5}}{M_E} \sim F_{1,28(r-1)}$
3-Factor Interactions	1	Q _{F₁F₂F₃}	M _{F₁F₂F₃} = Q _{F₁F₂F₃}	$F_{F_1F_2F_3} = \frac{M_{F_1F_2F_3}}{M_E} \sim F_{1,28(r-1)}$
F ₁ F ₂ F ₄	1	Q _{F₁F₂F₄}	M _{F₁F₂F₄} = Q _{F₁F₂F₄}	$F_{F_1F_2F_4} = \frac{M_{F_1F_2F_4}}{M_E} \sim F_{1,28(r-1)}$
F ₁ F ₃ F ₄	1	Q _{F₁F₃F₄}	M _{F₁F₃F₄} = Q _{F₁F₃F₄}	$F_{F_1F_3F_4} = \frac{M_{F_1F_3F_4}}{M_E} \sim F_{1,28(r-1)}$
F ₂ F ₃ F ₄	1	Q _{F₂F₃F₄}	M _{F₂F₃F₄} = Q _{F₂F₃F₄}	$F_{F_2F_3F_4} = \frac{M_{F_2F_3F_4}}{M_E} \sim F_{1,28(r-1)}$
F ₁ F ₂ F ₅	1	Q _{F₁F₂F₅}	M _{F₁F₂F₅} = Q _{F₁F₂F₅}	$F_{F_1F_2F_5} = \frac{M_{F_1F_2F_5}}{M_E} \sim F_{1,28(r-1)}$
F ₁ F ₃ F ₅	1	Q _{F₁F₃F₅}	M _{F₁F₃F₅} = Q _{F₁F₃F₅}	$F_{F_1F_3F_5} = \frac{M_{F_1F_3F_5}}{M_E} \sim F_{1,28(r-1)}$
F ₂ F ₄ F ₅	1	Q _{F₂F₄F₅}	M _{F₂F₄F₅} = Q _{F₂F₄F₅}	$F_{F_2F_4F_5} = \frac{M_{F_2F_4F_5}}{M_E} \sim F_{1,28(r-1)}$

$F_3F_4F_5$	1	$Q_{F_3F_4F_5}$	$M_{F_3F_4F_5} = Q_{F_3F_4F_5}$	$F_{F_3F_4F_5} = \frac{M_{F_3F_4F_5}}{M_E} \sim F_{1,28(r-1)}$
4-Factor Interactions $F_1F_2F_3F_4$	1	$Q_{F_1F_2F_3F_4}$	$M_{F_1F_2F_3F_4} = Q_{F_1F_2F_3F_4}$	$F_{F_1F_2F_3F_4} = \frac{M_{F_1F_2F_3F_4}}{M_E} \sim F_{1,28(r-1)}$
$F_1F_2F_4F_5$	1	$Q_{F_1F_2F_3F_5}$	$M_{F_1F_2F_3F_5} = Q_{F_1F_2F_3F_5}$	$F_{F_1F_2F_3F_5} = \frac{M_{F_1F_2F_3F_5}}{M_E} \sim F_{1,28(r-1)}$
$F_1F_3F_4F_5$	1	$Q_{F_1F_3F_4F_5}$	$M_{F_1F_3F_4F_5} = Q_{F_1F_3F_4F_5}$	$F_{F_1F_3F_4F_5} = \frac{M_{F_1F_3F_4F_5}}{M_E} \sim F_{1,28(r-1)}$
$F_2F_3F_4F_5$	1	$Q_{F_2F_3F_4F_5}$	$M_{F_2F_3F_4F_5} = Q_{F_2F_3F_4F_5}$	$F_{F_2F_3F_4F_5} = \frac{M_{F_2F_3F_4F_5}}{M_E} \sim F_{1,28(r-1)}$
5-Factor Interactions $F_1F_2F_3F_4F_5$	1	$Q_{F_1F_2F_3F_4F_5}$	$M_{F_1F_2F_3F_4F_5} = Q_{F_1F_2F_3F_4F_5}$	$F_{F_1F_2F_3F_4F_5} = \frac{M_{F_1F_2F_3F_4F_5}}{M_E} \sim F_{1,28(r-1)}$
Error	$28(r-1)$	Q_E	$M_E = \frac{Q_E}{28(r-1)}$	-
Total	$32r-1$	Q	-	-

Inference: The null hypotheses of insignificance of various factorial effects (main/interaction) is rejected. If $F_0 > F_c$ at α % level of significance. Otherwise it is accepted.

V. CONCLUSION

In this paper, introduced the new method of construction of 2^5 factorial experiments for five factors F_1, F_2, F_3, F_4, F_5 each with two levels 0 and 1. Then the three factor interactions $F_1F_4F_5$ and $F_2F_3F_5$ are partially confounded in four blocks through application. First identified the number of confounded interaction and size of the blocks using the formula of defining contrast rule. In this application the three factor interactions $F_1F_4F_5$ and $F_2F_3F_5$ are partially confounded in four blocks are constructed. In further, the same method of construction can be extended for asymmetrical factorial experiments, fractional factorial experiments, *BIBD* and *PBIBD(m)*.

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