

# Addressing the Challenges in 21st-Century Mathematics Teaching

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**Abstract**—In the 21st century, mathematics educators confront significant challenges that must be addressed with care. The prevailing approach to teaching mathematics often presents the subject as a disjointed collection of facts, procedures, and theorems. Consequently, students frequently view mathematics as a collection of memorizable formulas and techniques that have to be memorized at the time of examination and forgotten after it, rather than a meaningful discipline. Changing this misconception is a pressing concern for mathematics educators. This paper examines the primary challenges faced by mathematics teachers and proposes effective strategies for overcoming them. By shedding light on these issues and offering practical solutions, this paper aims to foster a more engaging and meaningful mathematics learning experience.

**Keywords:** Meaningful Mathematics, ICT, GeoGebra, NEP 2020, Ganita Prakash

## I. INTRODUCTION

In the 21st century, learning skills in mathematics are centered on critical thinking, problem-solving, and real-world applications. For example, instead of just memorizing a formula and applying the procedures in a routine exercise, students might explore how they can be used in a real-life situation. This helps them understand the importance of mathematics in real-life contexts, making the learning more meaningful.

Creativity is another key skill for the 21st century. For instance, students might be asked to design a geometric pattern using mathematical concepts, encouraging them to think outside the box. Technology plays a big role too, with tools like graphing calculators and simulation software helping students visualize and analyze data. Communication is essential, as students are required to explain their solutions and reasoning clearly, whether in written form or in discussions with classmates.

Thus, skills like critical thinking, collaboration, creativity, use of technology, and communication are essential in the 21st century because they prepare students for a new world. Mathematics is no longer just about getting the right answer; it's about understanding the process, applying it to new situations, and working effectively with others. This shift is vital as students enter a fast-evolving, technology-driven society that demands flexibility and deep understanding [Trilling and Fadel, 2009].

However, the traditional approach to teaching mathematics has been criticized for its emphasis on procedural knowledge, lack of student engagement, and limited opportunities for creativity. Researchers have identified several issues with traditional classrooms, including overemphasis on memorization, passive learning, ineffective assessment practices, teacher-centered instruction, lack of cultural relevance, limited use of technology, emphasis on individual achievement, neglect of mathematical modeling, and limited attention to mathematical discourse [Gregg, 1995]. These issues contribute to students' difficulties in applying mathematical concepts to real-world problems and developing a deeper understanding of the subject. To better prepare students for the 21st century, educators need to adopt more innovative and student-centered approaches that emphasize conceptual understanding, problem-solving, and critical thinking. In traditional classrooms, mathematics is still often taught in a way that seems disconnected, with students seeing it as a bunch of facts and tricks they need to memorize for exams. This makes it hard for students to truly understand and enjoy the subject. To fix this, math teachers need to keep learning and improving their own knowledge and teaching skills. Teachers need to explain math clearly, so students can understand the ideas and see how they connect. Teachers also need to

make sure that students fully grasp the content and remember it for the long term. The following are the key steps in making math education more effective and meaningful for students.

The main challenges include:

1. Continuously updating content knowledge and pedagogical skills.
2. Delivering content clearly and understandably.
3. Ensuring that students grasp the correct content and develop a deep understanding.
4. Confirming that students' knowledge and comprehension are solid and enduring.

In the following sections, we will explore each of these issues in more depth.

## II. KEEPING UPDATED: MASTERING CONTENT AND NEW TEACHING METHODS

Nobody can question having clear and good content knowledge and its updation regularly. For a mathematics teacher, it is very important to find new approaches and convincing strategies to make mathematics learning enjoyable, comfortable, and understandable. A teacher is an everyday student. For a mathematics teacher, it is very important that he/she has to practice regularly. Most importantly, the basic concept should be understood very clearly, so as to build his/her content knowledge. The document Professional Standards for Teaching Mathematics, published by the National Council of Teachers of Mathematics (NCTM), emphasizes the importance of ongoing professional development, including regular updates to both content knowledge and pedagogical skills [NCTM,1991].

In the present context, the difference between a teacher and a student is indeed just one hour. Because of the technology, it is very difficult for a teacher to create curiosity or interest in the classroom, as the contents are already familiar to the student community through electronic media. So it is a challenge for a teacher to gain the confidence of students in the class with his/her presentation.

Some important concepts/branches are listed here to make the mathematics teacher in a well equipped and strong in content.

- Functions and their visualisation.

- Mensuration and its importance and types.
- Knowledge of the construction of different geometrical figures, like triangles, quadrilaterals, etc.
- Concepts of algebraic expressions and polynomials
- Calculus and its applications.
- Basic probability and its applications.

### 2.1.Knowledge of number system:

Subtraction from a big number to a small number can be explained by borrowing. But some digits in the big number are smaller than the second number, then the real problem starts for a child to understand the process. Here we provide an example where misconceptions about the place value lead to error. For a problem to subtract 115 from 824, a child of grade 5 does the following:

$$\begin{array}{r} 824 - \\ 115 \\ \hline 610 \end{array}$$

The answer was clearly incorrect. When the student was asked to explain how they arrived at it, the response was *that 'since 2 is smaller than 4, we could not "borrow" 1 from it. So I took 1 from the next nearest large number, which is 8'.*

This issue happens because the subtraction is taught as a procedure. From his answer, it is clear that the learner is not clear about the meaning behind the procedure. The teacher with a strong grasp of the number system can explain what is the meaning of borrowing in the case of subtraction. They can help the child understand that 2 in tenth place and 4 in unit place are not separate entities, but it is 20+4. The teacher can help the learner to group them as 10+10+4 = 10+14. This is later considered as "borrowing". This understanding goes beyond merely showing students the mechanics of the process and allows the teacher to explain why the system works, helping students see the logical structure behind operations.

Here we present another simple way of subtraction, which is motivated by the *vinculum method* from Vedic mathematics. We will also discuss why this method works in all cases.

2.2 Vinculum Method of Subtraction:

In the Vinculum method of subtraction, which gives the subtraction without carrying, and also no particular direction is required! Just a little understanding of notation and common sense with presence of mind, a child can joyfully solve this situation, and this work is never taught in classrooms!

$$\begin{array}{r} 7364 - \\ 3415 \\ \hline \end{array}$$

Here at the unit and hundred place, the digit of the second number is bigger than the first number. For this, we use the following steps.

Step 1: Subtract all the digits in the normal way, i.e., from the bigger digit to the smaller digit in any direction.

$$\begin{array}{r} 7364 - \\ 3415 \\ \hline 4151 \end{array}$$

Step 2: Identify the subtraction where we have subtracted in the reverse order. Which means, identify those digits obtained by subtracting the digit in the top number (7364) from the corresponding digit in the bottom number (3415). In this example unit digit and the hundred digit of the answer are obtained by reverse subtraction. Denote them with a notation 'overline' as follows:

$$\begin{array}{r} 7364 - \\ 3415 \\ \hline 4\bar{1}5\bar{1} \end{array}$$

Step 3 : Here  $4\bar{1}$  represents the number  $40 - 1 = 39$ , and  $5\bar{1}$  represents  $50 - 1 = 49$ .

Step 4: Write down the final answer

$$\begin{array}{r} 7364 - \\ 3415 \\ \hline 3949 \end{array}$$

Even though this is also procedural, it is simpler than the usual procedure of subtraction. Once a child becomes very clear about the notation and process, then it became a enjoyable activity for them to solve

the problem. After getting the clear knowledge of the place value system, they can solve the problem traditionally. By that time, the fear of getting the wrong answer is reduced, and they start building confidence in handling such problems with any higher digit numbers. At the same time, they can clearly explain this procedure, as it also depends on the place values in a different way.

III. ENHANCING LEARNING THROUGH MATH LABS

Teachers can use their artistic skills, such as storytelling, singing, and narrating, along with appropriate multimedia to make learning deep, meaningful, and joyful. Activity-oriented contextual concept formation techniques will serve the best to convert disinterest into interest among the students.

The mathematics laboratory can be used as a tool for developing effective mathematics learning, where students find collections of games, puzzles, teaching aids, and other materials. Most importantly mathematics laboratory contributes greatly to the learning of mathematical concepts and skills. It also enhances mathematical awareness and a positive attitude. Details of great mathematical personalities and their works can be exhibited in the math lab in the form of posters, videos, and audio clips.

Teachers should facilitate students to learn important concepts in mathematics, which they can easily apply in their daily life, and so they learn to enjoy mathematics rather than fear it. Mathematics teaching can and should be intellectually stimulating and ever-challenging. It should also be an endless source of satisfaction.

3.1 Visualising Geometry through ICT

Another area for discussion is basic geometry and its visualisation, which includes 2 and 3 dimensions. Students prefer to solve mathematical problems in real-world areas, so educators can use suitable practical problems to create some level of interest among them. There were various research studies regarding this. Osasere and Omoregie [Osasere and Omoregie, 2020] say that the teaching of Mathematics is made interesting to students when teachers can connect mathematical concepts to real-life problems and experiences, as well as establishing connection

between the various forms of Mathematical knowledge.

We have pointed out that a traditional classroom, which is teacher-cetered uses a behaviouristic approach. There are many studies have found that in such classroom mathematics was taught as a collection of unrelated facts and procedures. Also, exam-oriented teaching gave emphasis on rote memorisation. This creates math phobia among the students. Teachers are unable to relate mathematical problems to the real world. These are some of the hindrances to the meaningful learning of mathematics [Noss and Hoyles, 1996]. It has been suggested that digital tools, combined with appropriate pedagogy, have the potential to address some of these issues [Bray and Tangney, 2017]. The technology tools like GeoGebra, Desmos, etc., will help to visualise the mathematical concepts. It also provides opportunities for the learner to experiment with mathematical ideas. This can change the nature of the mathematics classroom from

a transmission-based, teacher-led environment to a student-centred, investigative, and constructivist one [Olive and Makar, 2010].

Some of the states, like Kerala, have included GeoGebra based Labs as a part of their course. Students of classes 11 and 12 explore various mathematical concepts through GeoGebra, similar to laboratory work in other science subjects.

#### Exploring Definite integral through GeoGebra

Here we can show some examples to demonstrate the definite integral as the limit of a sum through the open source software GeoGebra. We will try to find the integral of the function  $f(x) = \sin(2x^2)dx$  between the limits 0.5 and 1. In a traditional classroom, every learner cannot visualise the changes happening when we increase the number of rectangles. Let us use GeoGebra for visualisation.

In each of the following figures, we can compare the actual area (integral) and the area of the upper sum

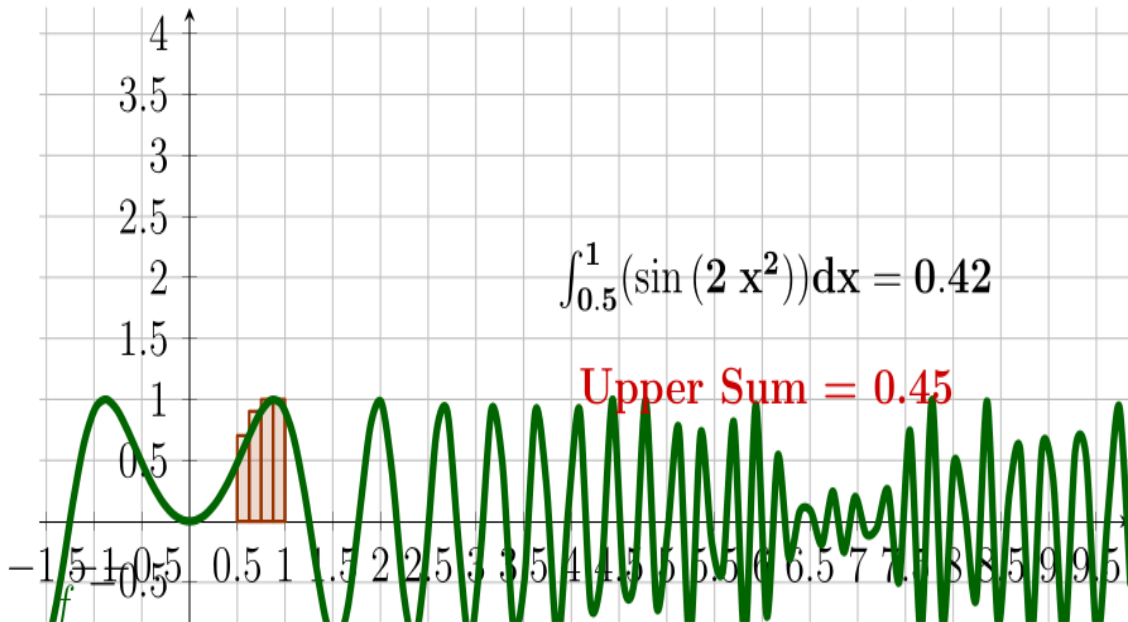


Figure 1 : Area under the curve using 4 Rectangles

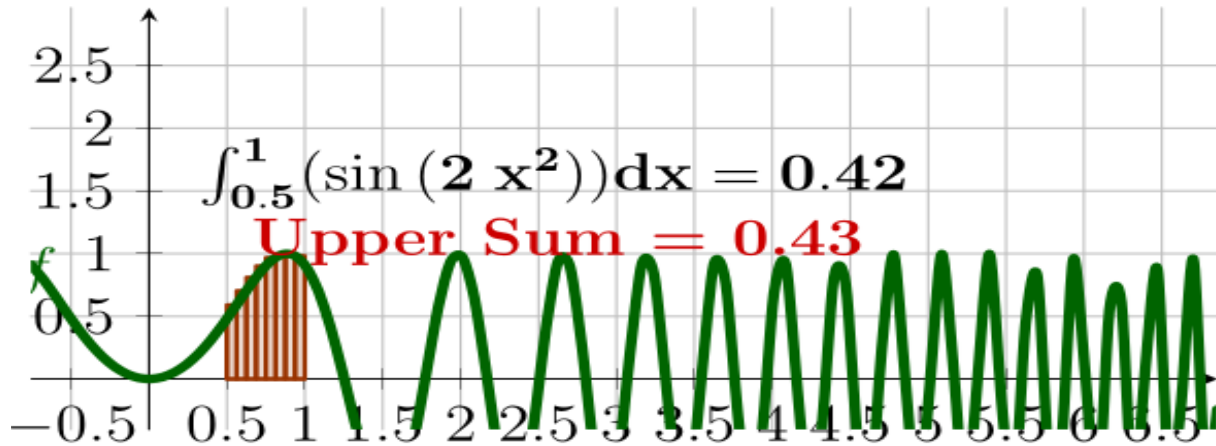


Figure 2: Inserting 8 rectangles

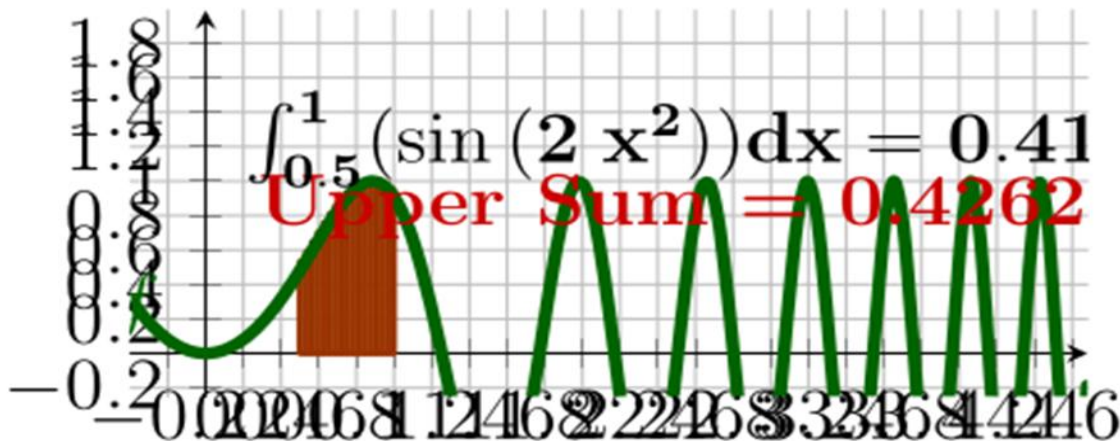


Figure 3: Inserting 16 rectangles

In this GeoGebra applet, students can vary the limits of integration by assigning different values to *a* and *b*. The exact value of the integral is also displayed, allowing students to compare it with the approximate values obtained through the rectangles. Learners can modify the function in the applet and observe how the graphical representation and the approximation change dynamically. Such opportunities for immediate visual feedback and experimentation are difficult to achieve through static blackboard drawings in a traditional classroom. Furthermore, students can

increase the number of rectangles to refine the approximation and zoom in to closely examine the emerging patterns, thereby developing a deeper conceptual understanding of the definite integral as a limit of sums.

#### IV.EFFECTIVE COMMUNICATION STRATEGIES FOR ENHANCED LEARNING

The language of a teacher is one of the most important components of teaching at all levels. As mathematics is considered the language science, the symbols, their

understanding, and usage at appropriate places are of great concern in teaching. If the notations/symbols are understood incorrectly by students, then it is very difficult for them to have proper basic knowledge, which in turn makes it difficult for them to understand higher concepts.

For example, the  $\epsilon - \delta$  definition of continuity of a function is the same as the intuitive definition of continuity of a function, which is observed through the graph as well as presented using terms like  $x$  tends to  $a$  the function  $f(x)$  tends to  $f(a)$ . But to convince this to the students, teachers require effective communication strategies.

Sometimes the teacher may feel that by using sophisticated language techniques, they can communicate in the class very effectively. But it may not facilitate the understanding of the learner. So the teacher has to see the learners' level of understanding and their comfort zone so that they understand the concepts in an easy way.

Also teacher should recollect the basic concepts wherever required and should establish the smooth linkage between the basic concepts and the present concept. This helps the learner in realising the continuity in their learning. In particular, during problem-solving sessions, recollecting the basic ideas and linking them to the given problematic situation is very important. Note that the process by which a student arrives at the answer to a problem becomes as important as the answer itself.

Many times teacher can slowly push the learner outside their comfort zone, where learners will be motivated to learn themselves. For that, they may be using puzzles or games, or brainstorming problems.

#### V. VERIFYING DEEP UNDERSTANDING IN LEARNERS

Understanding of each student's progression in learning mathematics is one of the most difficult things to achieve. Simply by observing their answer papers and their results, we cannot conclude whether they understood the concept correctly or not. Various types of assessment techniques are required to achieve this purpose. Many times, by observing only a few student's response teacher may have a false impression that everyone in the class has understood correctly.

Oral questioning is one of the important techniques through which we can verify students' responses and their accuracy. Teachers can use oral questioning to challenge and elicit students' thoughts, increasing students' curiosity and interest in a topic being taught [Rini et al., 2020]. Oral questioning is not an activity between one student and the teacher. Using this method, we can improve peer learning among students. By hearing their peers' responses in class, children can understand better. Hence, answering oral questions in class not only gives clarity in understanding the mathematical concepts of the learner but also strengthens their mathematical communication skills. Teachers can also use this technique as a reinforcement strategy as well as an assessment tool. Giving reinforcement at the proper time also encourages the child to understand the concepts with enthusiasm.

A teacher can create more challenges and opportunities for the learner. Oral questioning in maths class will develop the reasoning skills of the student. Through reasoning skills, a learner can solve problems without depending on rote memory and routine procedures. Therefore, students should be trained and guided to constantly provide logical explanations and analyze, judge, evaluate, and justify all mathematical activities. In addition, teachers need to provide space and opportunities for mathematical discussions that are not only engaging but also allow each student to be involved [Mahmud and Mohd Drus, 2023]. At the same time, teachers are supposed to ask open-ended questions or diverging questions rather than converging questions. The use of convergent questions causes students' thinking not to diverge and not to develop [Subramaniam et al., 2022]. The correct use of oral questions can help students improve their ability to reason logically and present arguments honestly and convincingly [Mahmud and Mohd Drus, 2023].

With minimal interference and appropriate guidance, teachers can create a constructivist classroom through which children have a better opportunity to learn subjects joyfully.

#### VI. EDUCATIONAL POLICIES SHAPING MATH PEDAGOGY

A policy document is a vision and path for the future. But it will turn out as expected only if all the

supporting systems work well. A change in one policy can have unexpected effects elsewhere in the system. Some unintended effects may take years to emerge. For example, curriculum reform that aims to match international standards will also require other policies, such as teacher education and professional development, to be successful. Transferring one policy without making structural changes can be problematic. Understanding interactions and interdependencies in education policy is challenging, particularly within short political timescales. [Madhu and Biju, 2023] explain how policy documents direct our mathematics education through a period of 75 years in India.

The emphasis on the subject matter to be taught stands in sharp contrast to the emerging policies of the nation [Shulman, 1986]. For example, the policy document [Yashpal, 1993], which later became the foundation stone for the National Curriculum Framework 2005, found that mathematics education in India over emphasis on exam, which in turn create disinterest among the learners. Later National Curriculum Framework 2005 and the NCERT textbooks developed an emphasis on meaningful mathematics and child centered approach. The present New Education Policy 2020 and National Curriculum Framework for School Education 2023 also considers constructivist approach, but give more emphasis on holistic development of the learner. NCF-SE 2023 says that mathematics education should aim to develop capacities of logical thinking, finding patterns, explaining patterns, making, refuting, and proving conjectures, problem solving, computing fluently, and communicating clearly and precisely -through content areas such as arithmetic, algebra, geometry, probability, statistics, trigonometry, and calculus. The grade 6-8 textbook of Mathematics, Ganita Prakash, is prepared according to this philosophy. The books are devoted to one complete chapter on patterns. It also provides opportunities for the learner to experience mathematics through various activities.

## VII. CONCLUSION

In conclusion, we can say that the present mathematics teaching faces many challenges which have to be addressed at various levels, namely by educators, policy makers, teachers, and students. Treating mathematics as a disjoint array of unrelated facts leads to rote memorisation, which is a hazard in making the

learning meaningful. To address the misconception teachers have to continuously update their pedagogical and content knowledge. We have discussed the importance of incorporating technology in an effective manner. In this paper, we have provided some illustrations of how the interactive nature of dynamic softwares help in bridging the gap between abstract concepts by enabling to visualise them.

In this paper, we have tried to present the important challenges faced by teachers and students and tried to propose strategies to help mathematics educators improve learning outcomes.

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