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Optimizing Fuzzy Assignment: A Novel Sub-Interval Average Ranking ($Siar^{\sim O}$) Approach Using Octagonal Fuzzy Numbers

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***Abstract*—Fuzzy Assignment (FA) problems are important in optimality technique, especially when there's a lot of uncertainty and imprecision involved. In this study, we introduce an improved fuzzy assignment model that uses a Sub-Interval Ranking Approach (*$SIAr^{\sim O}$*) along with Octagonal Fuzzy Numbers (*OFN*) to boost both solution accuracy and computational efficiency. Our proposed ranking method transforms the Octagonal fuzzy numbers into a clear, crisp form through defuzzification. By optimizing the assignment schedule cost δ_{ij} based on the defuzzified fuzzy assignment costs using the Enumeration method, we provide numerical examples that showcase how effective and practical this model can be in real-world situations, leading to optimal solutions.**

***Index Terms*—Fuzzy Assignment Problem, Octagonal Fuzzy Number, Enumeration method, Sub - Interval Ranking Technique, Optimal Solution Octagonal**

I. INTRODUCTION:

Fuzzy numbers, first coined by Zadeh in 1965, enable decision-makers to model uncertainty in optimization problems. Rather than assigning a fixed cost value, fuzzy numbers offer a range of possible values with varying degrees of membership. Among the different types of fuzzy numbers, Octagonal Fuzzy Numbers (OFN) provide a more accurate representation of uncertainty due to their structured shape and added parameter flexibility. To tackle Fuzzy Assignment Problems (FAPs) with Octagonal Fuzzy Numbers, it's essential to convert these fuzzy values into crisp numbers through defuzzification techniques, we use Sub-Interval Ranking Approach ($SIAr^{\sim O}$) to gained attention for its accuracy and efficiency in maintaining the characteristics of fuzzy data. These method helps to transform fuzzy cost matrices into deterministic values, which can then be processed using the Enumeration method algorithm to find the optimal assignment cost δ_{ij}. This study aims to enhance ranking approach in computational efficiency, and robustness in optimization, making it a highly effective approach for managing uncertainty in fuzzy optimization techniques.

II. BASIC PRELIMINARIES

2.1 Definition: Fuzzy Set

A fuzzy set Ä in a nonempty set X which is categorized by the membership function $\mu_{\text{Ä}} : X \rightarrow [0,1]$ and $\mu_{\text{Ä}(X)}$ is the degree of membership of element X in fuzzy set for each $x \in X$

2.2 Definition: Fuzzy Number

A fuzzy number is a fuzzy set Ä defined on a set of real number R in which the memberships consist of following conditions such as it should be Normal, Convex, and Upper Semi - Continuous in $X \in R$

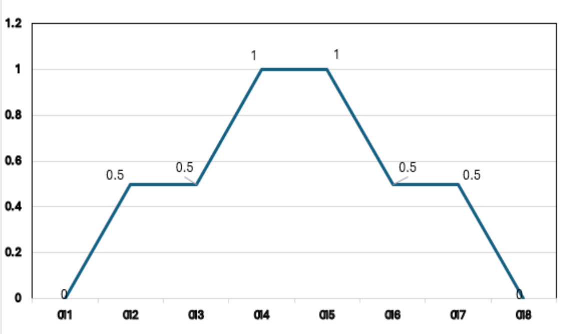
2.3 Definition: Octagonal Fuzzy Number

An Octagonal Fuzzy number denoted by

$\text{Ä}_{Ɋ} = \{Ɋ_1, Ɋ_2, Ɋ_3, Ɋ_4, Ɋ_5, Ɋ_6, Ɋ_7, Ɋ_8\}$ where $Ɋ_i \in R$ and its membership function $\mu_{\text{Ä}(X)}$ given below

$$\mu_{\tilde{A}(X)} = \begin{cases} 0, & x < Ɋ_1 \\ \frac{1}{2}\frac{(x - Ɋ_1)}{(Ɋ_2 - Ɋ_1)} & Ɋ_1 \le x \le Ɋ_2 \\ 0.5 & Ɋ_2 \le x \le Ɋ_3 \\ \frac{1}{2} + \frac{1}{2}\frac{(x - Ɋ_3)}{(Ɋ_4 - Ɋ_3)} & Ɋ_3 \le x \le Ɋ_4 \\ 1 & Ɋ_4 \le x \le Ɋ_5 \\ \frac{1}{2} + \frac{1}{2}\frac{(Ɋ_6 - x)}{(Ɋ_6 - Ɋ_5)} & Ɋ_5 \le x \le Ɋ_6 \\ 0.5 & Ɋ_6 \le x \le Ɋ_7 \\ \frac{1}{2}\frac{(Ɋ_8 - x)}{(Ɋ_8 - Ɋ_7)} & Ɋ_7 \le x \le Ɋ_8 \\ 0 & x \ge Ɋ_8 \end{cases}$$

Fig 2.3 Graphical representation of Octagonal Fuzzy Number



2.4 General Mathematical Model of Assignment Problem

The mathematical formulation of the assignment problem can be expressed as follows:

Minimum Z = $\sum_{i=1}^{n}\sum_{j=1}^{n} \delta_{ij}\, x_{ij}$; *i =1,2,3, … n and j =1,2,3, … m*

Subject to Constraints $\sum_{i=1}^{n} x_{ij} = 1\ \forall$ i *=1,2,3, … n. such that* $x_{ij} \in (0,1)$

$\sum_{i=1}^{m} x_{ij} = 1\ \forall$ j *=1,2,3, … m;*

$$x_{ij} = \begin{cases} 1; & if\ i^{th}\ person\ is\ assigned\ j^{th}\ job \\ 0; & Elsewhere \end{cases}$$

Table 2.4

	1	2	3	……	j	…..	m
1	δ_{11}	δ_{12}	δ_{13}	….	δ_{1j}	….	δ_{1m}
2	δ_{21}	δ_{22}	δ_{23}	….	δ_{2j}	….	δ_{3m}
….	….	….	….	….	….	….	….
. i	δ_{i1}	δ_{i2}	δ_{i3}	….	δ_{ij}		δ_{im}
….	….	….	….	….	….	….	….
n	δ_{n1}	δ_{n2}	δ_{n3}	….	δ_{nj}		δ_{nm}

III. RANKING PROCEDURE

The *x-axis* features discrete real points Ɋi, where i ranges from 1 to n. There are *xn* intervals between these discrete points. Each sub-interval has its own upper and lower limits essentially the x least upper bound and greatest lower bound for each interval, let's call them Ɋj and Ɋk for all j ≤ k, where 1 ≤ j ≤ k ≤ n. We can find the average of these limits as (Ɋj +Ɋk)/2. The Average of average of all these sub - intervals gives us our ranking function.

3.1. Defuzzification of Fuzzy Number:

Defuzzification is all about taking a fuzzified output and turning it into a crisp value that relates to a fuzzy set. In this study, we explored the method for defuzzifying a Octagonal fuzzy number, as illustrated in the flowchart

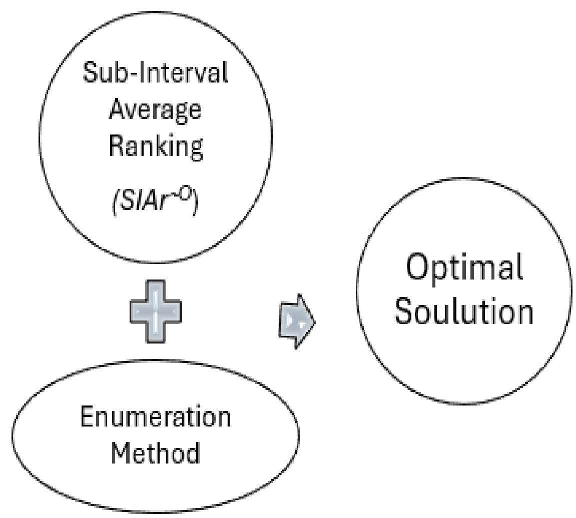


Fig 3.1 Defuzzification - Flow Chart

3.2. Sub -Interval Average Method of Ranking function:

Let $A^8 = \{Ɋ_1, Ɋ_2, Ɋ_3, Ɋ_4, Ɋ_5, Ɋ_6, Ɋ_7, Ɋ_8\}$ be an Octagonal fuzzy number and its ranking technique on sub interval average method on the fuzzy number of A^8 split into several sub intervals by using rule of ranking procedure[3]

Interval Fuzzy number (A^8) =

$$\begin{cases} (Ɋ_1, Ɋ_2), (Ɋ_2, Ɋ_3), (Ɋ_3, Ɋ_4), (Ɋ_4, Ɋ_5) \\ (Ɋ_5, Ɋ_6), (Ɋ_6, Ɋ_7), (Ɋ_7, Ɋ_8), (Ɋ_1, Ɋ_8) \\ (Ɋ_1, Ɋ_1), (Ɋ_2, Ɋ_2), (Ɋ_3, Ɋ_3), (Ɋ_4, Ɋ_4) \\ (Ɋ_5, Ɋ_5), (Ɋ_6, Ɋ_6), (Ɋ_7, Ɋ_7), Ɋ_8, Ɋ_8) \end{cases}$$

Average of upper and lower limits (A^8) for each sub interval

$$\left\{ \frac{\mathfrak{a}_1+\mathfrak{a}_2}{2} + \frac{\mathfrak{a}_2+\mathfrak{a}_3}{2} + \frac{\mathfrak{a}_3+\mathfrak{a}_4}{2} + \frac{\mathfrak{a}_4+\mathfrak{a}_5}{2} + \frac{\mathfrak{a}_5+\mathfrak{a}_6}{2} + \frac{\mathfrak{a}_6+\mathfrak{a}_7}{2} + \frac{\mathfrak{a}_7+\mathfrak{a}_8}{2} + \frac{\mathfrak{a}_8+\mathfrak{a}_1}{2} + \frac{\mathfrak{a}_1+\mathfrak{a}_1}{2} + \frac{\mathfrak{a}_2+\mathfrak{a}_2}{2} + \frac{\mathfrak{a}_3+\mathfrak{a}_3}{2} + \frac{\mathfrak{a}_4+\mathfrak{a}_4}{2} + \frac{\mathfrak{a}_5+\mathfrak{a}_5}{2} + \frac{\mathfrak{a}_6+\mathfrak{a}_6}{2} + \frac{\mathfrak{a}_7+\mathfrak{a}_7}{2} + \frac{\mathfrak{a}_8+\mathfrak{a}_8}{2} \right\}$$

$$r(A^i) = \frac{(i+1)}{2x_i}\sum_{i=1}^{n}\mathfrak{a}_i = \frac{(i+1)}{2i(i+1)/2}\sum_{i=1}^{n}\mathfrak{a}_i \qquad (1)$$

when i =8 then

$$r(A^8) = \frac{(8+1)}{2x_8}\sum_{i=1}^{8}\mathfrak{a}_i = \frac{(8+1)}{2(8)(8+1)/2}\sum_{i=1}^{8}\mathfrak{a}_i$$

$$r(A^8) = \frac{9}{2x_8}\sum_{i=1}^{8}\mathfrak{a}_i = \frac{9}{\frac{2(8)(9)}{2}}\sum_{i=1}^{8}\mathfrak{a}_i = \frac{9}{72}\sum_{i=1}^{8}\mathfrak{a}_i \qquad (2)$$

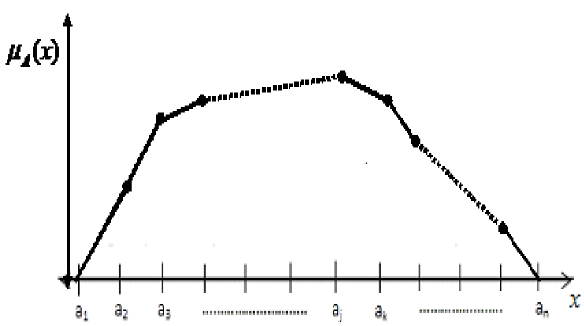


Fig 3.2 Fuzzy number $A^n = \{a_1, a_2, a_3, \ldots\ldots a_{n-1}, a_n\}$

Hence, Sub -Interval Average Ranking technique formula of $r(A^8)$ Octagonal fuzzy number converting into crisp form is each cell δ_{ij} given below

$$\delta_{ij} = r(A^8) = \frac{1}{8}\sum_{i=1}^{8}\mathfrak{a}_i = \frac{1}{8}\left(\mathfrak{a}_1 + \mathfrak{a}_2+\mathfrak{a}_3+\mathfrak{a}_4+\mathfrak{a}_5+\mathfrak{a}_6\mathfrak{a}_7 + \mathfrak{a}_8\right) \qquad (3)$$

IV. ENUMERATION METHODOLOGY IN ASSIGNMENT PROBLEM:

When it comes to tackling assignment problems, the Enumeration method is all about systematically laying out every possible way to assign resources, like people to various tasks. From there, you simply pick the assignment that offers the lowest cost or the highest profit, quickest time, or shortest distance. This approach, often referred to as the complete enumeration method, means you're generating a comprehensive list of all potential pairings between your resources and tasks.

4.1. Algorithm on Enumeration Method:

First, defuzzify the Octagonal fuzzy number into general Assignment problem and then using procedural steps of Enumeration algorithm on solving the numerical problem

Step1: First, we need to indentify the *tasks (or jobs)* and the *people (workers)* who will be part of the assignment problem are equal

Step 2: Next, we should figure out the *cost (or time, distance, etc.)* tied to each potential assignment of a task to a person.

Step 3: We can visualize this problem using a cost matrix, where each *cell (i, j)* shows the cost of assigning *task i to worker j.*

Step 4: Let's generate all possible assignments: Since each task can only go to one individual, the total number of assignments is n!, where n represents the number of tasks (which is the same as the number of individuals). (e.g. If order of matrix n = 4 ther 4! =24 assignemnt allotment)

Step 5: Now, we evaluate each assignment: For every possible assignment, we'll calculate the total cost by adding up the costs of all individual assignments allotment.

Step 6: Finally, we identify the optimal solution: The assignment that has the lowest total cost is our best option.

4.2. Numerical Example

Consider a octagonal fuzzy number on balanced fuzzy assignment in which all profit co.efficient are given below

Table 4.2 Octagonal fuzzy Assignement Problem

	Level 1	Level 2	Level 3
Task 1	1,3,5,7,9,11,12,13	9,10,14,13,15,16,11,12	2,4,5,7,9,10,12,13
Task 2	5,6,8,10,12,13,14,15	2,3,4,5,6,7,8,9	3,6,7,8,9,10,12,13
Task 3	8,9,10,11,12,13,14,15	2,4,5,6,7,9,10,11	5,6,7,10,12,13,14,15

Solution:

Defuzzificating the above table [4.2] matrix of order 3, Using the ranking procedure of Sub -interval average method ($SIAr^{\sim O}$) on using the formula [3.2] of (3) to each cell δ_{ij} assigning *task ɸ ᵢ to worker βj*, we get cost Matrix of Assignment problem

Table 4.2.1

	Φ1	*Φ2*	*Φ3*
β1	7.63	12.50	7.75
β2	10.38	5.50	8.50
β3	11.50	6.75	10.25

Now, we generate all possible assignment since each task can only assigned to one workers

Here, Order of matrix n = 3 then 3! = 6 different ways of assignment allotment are possible.

Here we go for following Enumeration method of iteration 1 proceeding on using table [4.2]

Table 4.2.2

	Φ1	*Φ2*	*Φ3*
β1	[7.63]	12.50	7.75
β2	10.38	[5.50]	8.50
β3	11.50	6.75	[10.25]

Iteration	Assignment Allotment	Cost value
1	Φ1β1→ɸ2β2→ɸ3β3	7.63 + 5.50 + 10.25 = 23.38

Here,Iteration 2 proceeding on using table [4.2.2]

Table 4.2.3

	Φ1	*Φ2*	*Φ3*
β1	[7.63]	12.50	7.75
β2	10.38	5.50	[8.50]
β3	11.50	[6.75]	10.25

Iteration	Assignment Allotment	Cost value
2	Φ1β1→ɸ2β3→ɸ3β2	7.63 + 6.75 + 8.50 =22.88

Iiteration 3 preceeding on using table [4.2.3]

Table 4.2.4

	Φ1	*Φ2*	*Φ3*
β1	7.63	[12.50]	7.75
β2	[10.38]	5.50	8.50
β3	11.50	6.75	[10.25]

Iteration	Assignment Allotment	Cost value
3	Φ1β2→ɸ2β1→ɸ3β3	10.38 +12.50+10.25 =33.73

Iiteration 4 preceeding on using table [4.2.4]

Table 4.2.5

	Φ1	*Φ2*	*Φ3*
β1	7.63	12.50	[7.75]
β2	[10.38]	5.50	8.50
β3	11.50	[6.75]	10.25

Iteration	Assignment Allotment	Cost value
4	Φ1β2→ɸ2β3→ɸ3β1	10.38 + 6.75 +7.75 = 24.88

Iiteration 5 preceeding on using table [4.2.5]

Table 4.2.6

	Φ1	*Φ2*	*Φ3*
β1	7.63	[12.50]	7.75
β2	10.38	5.50	[8.50]
β3	[11.50]	6.75	10.25

Iteration	Assignment Allotment	Cost value
5	Φ1β3→ɸ2β1→ɸ3β2	11.50 +12.50+ 8.50 = 32.50

Iiteration 6 preceeding on using table [4.2.6]

Table 4.2.7

	Φ1	*Φ2*	*Φ3*
β1	7.63	12.50	[7.75]
β2	10.38	[5.50]	8.50
β3	[11.50]	6.75	10.25

Iteration	Assignment Allotment	Cost value
6	Φ1β3→ф2β2→ф3β1	11.50 + 5.50 + 7.75 = 24.75

Finally, we obtain n = 3! Ways such as 6 numbers of possible assignment allotment along with their cost value, now we choose the *Minimum* of overall assignment allotment cost value which defines the optimal solution for the study taken as the numerical example

In this above numerical example study on octagonal fuzzy number, we identify that table [4.2.2] of Iteration II gives the minimum cost value = 22.88 come under the assignment allotment Φ1 β1 → ф2 β3 → ф3 β2

V. CONCLUSION:

This study presents a new and improved way to calculate the defuzzification of various polygons, which can really help when it comes to ranking criteria or alternatives in fuzzy multi-criteria decision-making problems. With the Sub -interval average ranking method we've proposed, we can defuzzify triangular, trapezoidal, pentagonal, hexagonal, and even heptagonal fuzzy numbers. It can also be applied to octagonal fuzzy numbers. The calculations are straightforward, making it accessible. Ultimately, this study paves the way for generalized fuzzy number systems that can tackle any fuzzy multi-criteria decision-making problem, regardless of how many vertices are involved.

VI. FUTURE WORK:

Future research could take the concept of Pentagonal Fuzzy Number (*pFn)* defuzzification and apply it to other fuzzy types, such as Pentagonal, Trapezoidal and Triangular numbers to achieve even greater accuracy. By incorporating advanced hybrid ranking techniques that leverage AI and machine learning, we can significantly boost decision-making processes. Additionally, diving into optimization methods like Genetic Algorithms and Deep Learning will enhance computational efficiency. Real-world applications in areas like logistics, healthcare, and supply chain management can serve to validate these proposed models. Through statistical and experimental analysis, we can evaluate their accuracy and stability when tackling large-scale problems. These enhancements will ultimately make fuzzy assignment models more robust and adaptable in real-world situations.

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