Theoretical Foundations of the Composite Fuzzy Demographic Index

Dr.V. Sreevinotha
Sir Issac newton college,Papakovil, Nagapattinam

Abstract- This paper investigates the theoretical foundations of the Composite Fuzzy Demographic Index (CFDI), a synthetic measure designed to evaluate demographic dynamics across geographical units over time. The index integrates fuzzy normalization, temporal weighting, and logarithmic transformations to provide a robust and interpretable framework for demographic comparison. We formally establish three key theoretical properties of the CFDI. First, we prove boundedness and attainability, showing that the index remains within a fixed and interpretable range under min–max normalization. Second, we demonstrate scale invariance, ensuring that proportional rescaling of population values does not affect comparative results. Third, we establish temporal sensitivity, proving that recent demographic changes exert greater influence when higher temporal weights are applied. Together, these properties confirm that the CFDI is mathematically consistent, stable under transformation, and responsive to time-varying demographic changes. The results support the suitability of the CFDI as a reliable tool for comparative demographic analysis across states and districts.

Keywords:* *Composite Fuzzy Demographic Index; Fuzzy normalization; Demographic measurement; Scale invariance; Temporal weighting; Boundedness; Population dynamics; Composite indicators.

I. INTRODUCTION

Demographic analysis plays a central role in understanding regional development, planning public policy, and monitoring socio-economic change. Traditional demographic indicators often rely on single measures or simple averages that may fail to capture the multidimensional and time-dependent nature of population dynamics. In recent years, composite indices have emerged as powerful tools for summarizing complex demographic information into a single interpretable metric. However, the construction of such indices requires strong theoretical justification to ensure reliability, interpretability, and robustness.

The Composite Fuzzy Demographic Index (CFDI) is proposed as a methodological framework for comparing demographic characteristics across geographical units such as states or districts over multiple time periods. The index incorporates fuzzy logic principles and normalization techniques to handle heterogeneity in population data and to enable meaningful comparisons across regions and time. By applying min–max normalization and temporal weighting, the CFDI aims to capture both relative differences among regions and the evolving nature of demographic change.

Despite the growing use of composite demographic indicators, their theoretical properties are often not rigorously examined. Establishing such properties is essential for ensuring that an index behaves consistently under data transformation, remains interpretable, and reflects meaningful temporal changes. In particular, three desirable characteristics are critical for any composite demographic measure: boundedness, which guarantees interpretability within a fixed range; scale invariance, which ensures robustness to proportional changes in measurement units; and temporal sensitivity, which allows recent changes to exert appropriate influence on the index.

This paper develops the theoretical framework of the CFDI and formally proves that the index satisfies these fundamental properties. The results provide a mathematical justification for the use of the CFDI in longitudinal and cross-sectional demographic analysis. By establishing these properties, the study contributes to the literature on composite indicators and supports the application of fuzzy-based demographic measures in empirical research and policy analysis.

II. HELPFUL HINTS

Theoretical properties of the Composite Fuzzy Demographic Index

Let

- $N \in \mathbb{N}$ denote the number of geographical units (states or districts of India)
- t=0,1,….T the observation period,
- $p_{i,t} > 0$ the population of unit i at time t,
- $l_{i,t} = \log(p_{i,t}) - \log(p_{i,t-1})$
- $\delta_t \epsilon (0,1), with \sum_{t=1}^{T} \delta_t = 1$
- $I_{T(p_i)} = \sum_{t=1}^{T} \delta_t l_{i,t}$,
- $norm(X_i) = \frac{X_i - min_j X_j}{max_j X_j - min_j X_j}$
- $CFDI_i = 1 - norm(I_{T(p_i)})$.

We assume $max_j I_{T(p_i)} \neq min_j I_{T(p_i)}$

Theorem:1 (Boundedness and attainability)

Statement:

For all i=1,2,…N, $0 \leq CFDI_i \leq 1$

Moreover:

- $CFDI_i = 0 \; iff \; I_{T(p_i)} = max_j I_{T(p_i)}$
- $CFDI_i = 1 \; iff \; I_{T(p_i)} = min_j I_{T(p_i)}$

Proof:

By definition of min-max normalization,

$0 \leq norm(I_{T(p_i)}) \leq 1$

Since

$\mathrm{CFDI_i} = 1 - \mathrm{norm}(\mathrm{I_{T(p_i)}})$,

It follows immediately that

$0 \leq CFDI_i \leq 1$

Now,

$norm(I_{T(p_i)}) = 1 \Leftrightarrow \mathrm{I_{T(p_i)}} = min_j I_{T(p_i)}$

Which implies

$\mathrm{CFDI_i} = 1$

This concludes the proof.

Theorem:2(Scale invariance)

Statement:

Let c>0. Define

$$p'_{i,t} = cp_{i,t}$$

Then,

$$\mathrm{CFDI_i}(p') = \mathrm{CFDI_i}(p)$$

Proof:

Consider the log-difference under rescaling

$l^l_{i,t} = \log(cp_{i,t}) - \log(cp_{i,t-1})$

Using properties of logrithms

$$\log(cp_{i,t}) = \log c + \log(p_{i,t})$$

So,

$$l^l_{i,t} = (\log c + \log(p_{i,t})) - (\log c + \log(p_{i,t-1}))$$

$$= \log(p_{i,t}) - \log(p_{i,t-1})$$

$$= l_{i,t}$$

then

$$I_T(p'_i) = \sum_{t=1}^{T} \delta_t l'_{i,t}$$

$$= \sum_{t=1}^{T} \delta_t l_{i,t}$$

$= I_T(p_i)$

Since normalization depends only on I_T

$\mathrm{CFDI_i}(p') = \mathrm{CFDI_i}(p)$

Hence the index is in variant to proportional scaling.

Theorem:3(Temporal Sensitivity)

Suppose weight satisfy $\delta_T > \delta_{T-1} > \cdots > \delta_1 > 0$

Let two units i and j differ only at period s, with

$$l_{i,s} < l_{j,t}$$

and $l_{i,t} = l_{j,t}$ for all t≠s.

then

$|\mathrm{CFDI_i} - \mathrm{CFDI_j}|$

Is strictly increasing in δ_s

Proof:

We have $I_{T(p_i)} - I_{T(p_i)} = \delta_s(l_{i,s} - l_{j,s})$

Thus,

$$\frac{\partial}{\partial \delta_s}\left(I_{T(p_i)} - I_{T(p_i)}\right) = l_{i,s} - l_{j,s}$$

Since normalization is an affine transformation,

$$\mathrm{CFDI_i} - \mathrm{CFDI_j} = \frac{I_{T(p_i)} - I_{T(p_i)}}{max_k I_{T(p_k)} - min_k I_{T(p_k)}}$$

therefore, $$|\ \mathrm{CFDI_i} - \mathrm{CFDI_j}| = \frac{\delta_s |l_{i,s} - l_{j,s}|}{max_k I_{T(p_k)} - min_k I_{T(p_k)}}$$

The denominator is constant and positive. Hence the absolute difference is strictly increasing in δ_s.

Thus more recent period (with large weights) exert strictly greater influence on CFDI.

REFERENCES

[1] Istituto Nazionale di Statistica, Fertility indicators of the resident popu- lation, https://demo.istat.it/app/?i=FE1&a=2024&l=en, Accessed: 2026-01-31 (2026).

[2] Istituto Nazionale di Statistica, Monthly Demographic Balance, https: //demo.istat.it/app/?i=D7B&a=2025&l=en, Accessed: 2026-01-31 (2026).

[3] Istituto Nazionale di Statistica, Bilancio demografico mensile gennaio- giugno 2025, https://www.istat.it/notizia/bilancio-demografico-mensile-gennaio-giugno-2025/, Accessed: 2026-01-31 (2025).

[4] Measuring population decline through a composite fuzzy index: Evidence from india Municipalities, IJRAR, Journal No:43602, Vol 13 issue 1 March 2026, Date of Publication 25-March 2026.

[5] Istituto Nazionale di Statistica, Demo - Statistiche demografiche, https: //demo.istat.it/?l=en, Accessed: 2025-10-30 (2025).

[6] United Nations Population Division, World Population Prospects 2024, https://population.un.org/wpp/, Accessed: 2026-01-15 (2026).

[7] T. Haartsen, V. Venhorst, Planning for decline: Anticipating on popula- tion decline in the netherlands, Tijdschrift voor economische en sociale geografie 101 (2) (2010) 218–227. doi:10.1111/j.1467-9663.2010.0 0597.x.

[8] F. Collantes, V. Pinilla, Peaceful surrender: the depopulation of rural Spain in the twentieth century, Cambridge Scholars Publishing, 2011. doi:10.1002/psp.1745.

[9] R. A. Bijker, T. Haartsen, D. Strijker, Migration to less-popular rural areas in the netherlands: Exploring the motivations, Journal of Rural Studies 28 (4) (2012) 490–498. doi:10.1016/j.jrurstud.2012.07.0 03.

[10] F. Barca, P. McCann, A. Rodríguez-Pose, The case for regional develop- ment intervention: place-based versus place-neutral approaches, Journal 41 of regional science 52 (1) (2012) 134–152. doi:10.1111/j.1467-9787. 2011.00756.x.

[11] I. Turok, V. Mykhnenko, The trajectories of european cities, 1960–2005, Cities 24 (3) (2007) 165–182. doi:10.1016/j.cities.2007.01.007.

[12] OECD, Classifying shrinking regions: Shrinking smartly and sustainably (2025). doi:10.1787/0185fe09-en.

[13] OECD, Shrinking smartly and sustainably: Strategies for action (2021). URL https://www.oecd.org/en/publications/rural-well-being _d25cef80-en.html

[14] R. S. Franklin, I come to bury (population) growth, not to praise it, Spatial Economic Analysis 15 (4) (2020) 359–373. doi:10.1080/1742 1772.2020.1802056.

[15] N. Newsham, F. Rowe, Understanding trajectories of population decline across rural and

urban europe: A sequence analysis, Population, Space and Place 29 (3) (2022) e2630. doi:10.1002/psp.2630.
[16] European Observation Network for Territorial Development, Shrinking rural regions in Europe – Towards smart and innovative approaches to regional development challenges in depopulating rural regions (2017). URL https://archive.espon.eu/sites/default/files/attachments/ESPON%20Policy%20Brief%20on%20Shrinking%20Rural%20Regio ns.pdf
[17] OECD, Rural well-being: Geography of opportunities. OECD rural studies (2021). URL https://www.oecd.org/en/publications/rural-well-being _d25cef80-en.html