# Narayana Prime Cordial Labeling of Some Graphs

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**Abstract- The labeling of graph is an assignment between the numbers and vertices/edges. In this paper, the results on the labeling of graph are studied using Narayana numbers and prime numbers for graphs namely; Crown, Triangular snake, Double Fan, Double Comb graphs.**

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## I. INTRODUCTION AND PRELIMINARIES

Graph labeling has enormous applications within Mathematics, Computer Science and Communication networks refer [5,10]. The authors discussed on prime cordial labeling, Cordial Graphs, duality concepts and applications in [1,2,3,11,12,13,14].
In [4], authors studied Narayana prime cordial labeling of graphs. Also, sufficient quality of research work on Power Exponential Mean and Centroidal Mean labeling, found in [5,6,7,8].

In [9,10,15,16] authors discussed the labeling of well-known classical means, Heron mean and their applications.

The readers are well known about the following notions.
Definition 1.1. Binary vertex labeling(BVL): Let $G(V,E)$ be finite and undirected, then $g$ from V(G) to (0,1) is called BVL of G. Let g(v) is the labeling of vertex v.

For an edge e=uv $g^*$: E(G) to {0,1} is given by $g^*$ (e) = | g(u)-g(v) |, is called the induced edge labeling.

1. The notions of G, $v_g(0)$ and $v_g(1)$; vertices number with labels 0 and 1.
2. The notions of G, $e_{g^*}(0)$ and $e_{g^*}(1)$; edges number with labels 0 and 1.

Definition 1. 2. Cordial labeling(CL): [3] A BVL of G is CL, provided

$$|v_g(0)-v_g(1)| \leq 1 \quad \text{and} \ |e_{g^*}(0)-e_{g^*}(1)| \leq 1.$$

Note: Every cardial graph admits CL.

Definition 1.3. Prime Cordial labeling(PCL):[1,2,11,12,13] A PCL of $G$ is a onto mapping g:V(G) to {1,2,3,….V(G), here V(G) is vertex set and g*: E(G)→{0, 1} is defined by, $g^*$(uv)=1; if gcd[g(u);g(v)]=1, otherwise 0 and | $e_{g^*}$ (0)- $e_{g^*}$ (1) |.

Note: Every prime cordial graph admits PCL.

Definition 1.4. Narayana number(NN): [4] For all set of non-negative integer $N_0$, the numbers, a€$N_0$. The NN can be expressed as, $N(a,k) = \frac{1}{a}\ {}^{a}C_k\ {}^{a}C_{k+1}$ where $0 \leq k < a$. Relevant work on labeling found in ([5]-[10], [14]).

The Narayana-prime-cordial-labeling (NPCL) for well-known graphs are discussed using the following definition.

Definition 1.5. Definition: For all set of non-negative integer $N_0$, the numbers, $a \in N_0$. The NN can be expressed as, $N(a,k) = \frac{1}{a}\,{}^{a}C_k\ {}^{a}C_{k+1}$ where $0 \leq k < a$

## II. MAIN RESULTS

This section provides the main results of the Narayana-prime-cordial-labeling (NPCL) for well known graphs are discussed.

Theorem 2.1. The Crown graph is a Narayana prime cordial graph.

Proof. A crown graph be a graph obtained by joining every vertices of a cycle $C_n$ with $K_1$.

Let U and V be the two vertex set where edges of the vertex set U forms a cycle and each vertex of the set V forms $K_2$ graph with the vertices of U. U is an inner vertex set and V is an outer vertex set of graph $C_n$ .

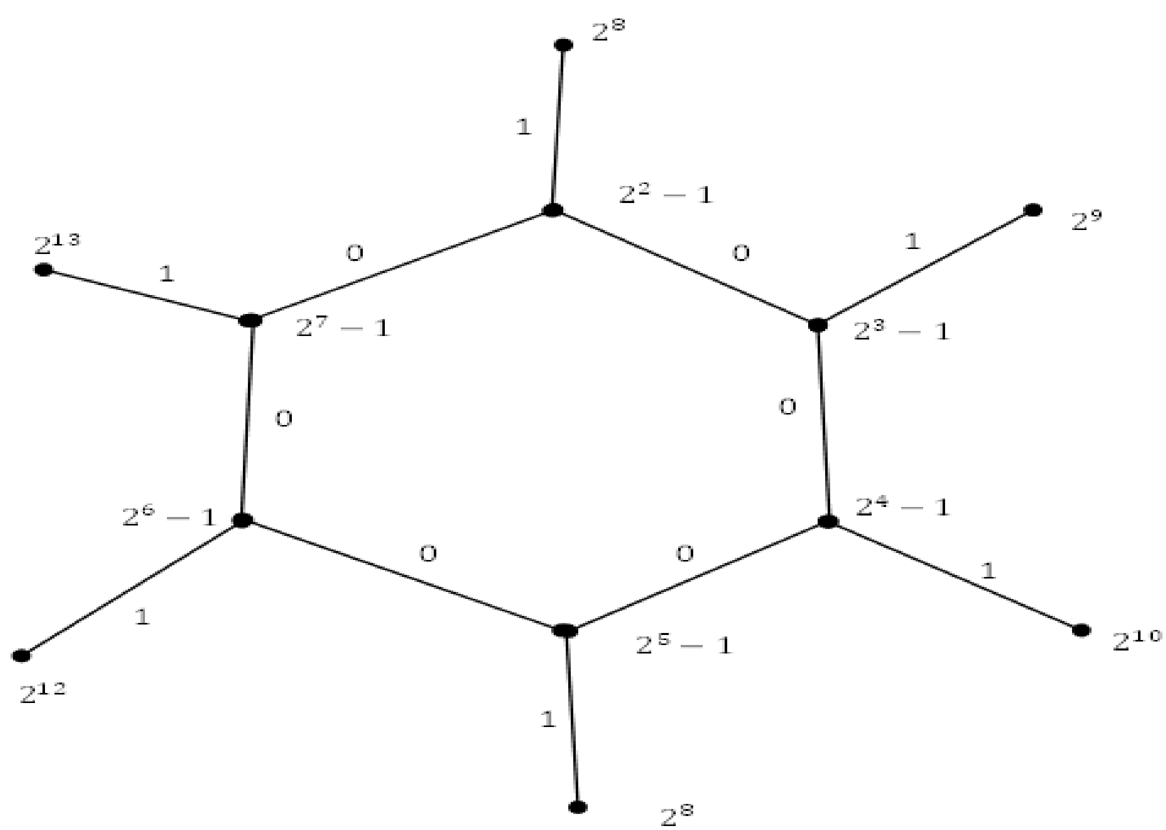
The vertex set can be defined as;
$U + V = \{u_i: 1 \leq i \leq \frac{n}{2}\} \cup \{v_i: \frac{n}{2} \leq i \leq n\}$ where $n$ is the total number of vertices.
Describe one-one function g: V→ $N_0$ such that when n≡0(mod2)

$$g(v_i) = 2^{i+1} - 1; \quad 1 \leq i \leq \frac{n}{2}$$

$$g(u_i) = 2^{i+1}; \quad \frac{n}{2} < i \leq n$$

In this type of labeling pattern,
$e_g^*(0) = \frac{n}{2}$ and $e_g^*(1) = \frac{n}{2}$
So that $\left|e_g^*(0) - e_g^*(1)\right| = \left|\frac{n}{2} - \frac{n}{2}\right| = 0 \leq 1$ is satisfied.
Therefore the crown graph admits Narayana prime cordial labeling.

Illustrations: 2.1 The following figure shows Narayana prime cordial labeling of crown graph.



Theorem 2.2. A triangular snake graph $TL_n$ is a Narayana prime cordial graph.
Proof. Let G be a graph obtained by joining the end vertices of an edge of a path $P_n$ with a new vertex with total number of vertices of a graph is 2n-1

Let U be the set of vertices with (n=1) number of vertices which connects the vertices of a path to make a set of triangles with common vertex. Such that the total number of edges in a graph G is $3n-3$.

Define a one-one function g:V(G) →$N_0$ such that
Case (i) Path $P_n$ when n is odd

$$g(P_i) = 2^{i+1} - 1, \quad i \equiv 1(mod2); \quad 1 \le i \le n$$
$$g(P_i) = 2^{i+1}, \quad i \equiv 0(mod2); \quad 1 \le i \le n$$
$$g(U_i) = 2^{i+1} - 1, \quad i \equiv 1(mod2); \quad n+1 \le j \le 2n-1$$
$$g(U_i) = 2^{i+1} - 1, \quad i \equiv 0(mod2); \quad n+1 \le j \le 2n-1$$

Then, $e_g^*(0) = \left(\frac{n-1}{2}\right) + (n-1) = \frac{3(n-1)}{2}$

$$e_g^*(1) = \left(\frac{n-1}{2}\right) + (n-1) = \frac{3(n-1)}{2}$$

So that $\left|e_g^*(0) - e_g^*(1)\right| = \left|\frac{3(n-1)}{2} - \frac{3(n-1)}{2}\right| = 0 \le 1$
$\left|e_g^*(0) - e_g^*(1)\right| \le 1$ is satisfied.

Therefore the graph G admits Narayana prime cordial labeling.
Case (ii): When path $P_n$ where n is even.

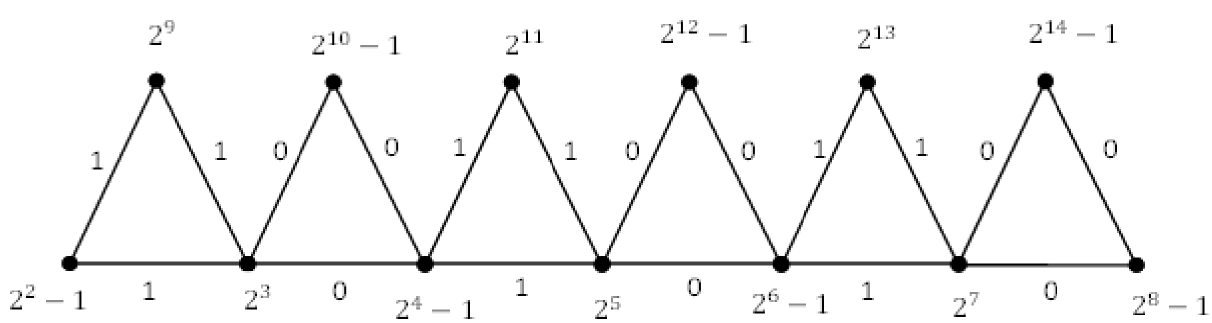
$$g(P_i) = 2^{i+1} - 1, \quad i \equiv 1(mod2); \quad 1 \le i \le n$$
$$g(P_i) = 2^{i+1}, \quad i \equiv 0(mod2); \quad 1 \le i \le n$$
$$g(U_i) = 2^{i+1} - 1, \quad i \equiv 1(mod2); \quad n+1 \le j \le 2n$$
$$g(U_i) = 2^{i+1} - 1, \quad i \equiv 0(mod2); \quad n+1 \le j \le 2n$$

Then $e_g^*(0) = \left(\frac{n}{2}\right) + n = \frac{3n}{2} - 1$

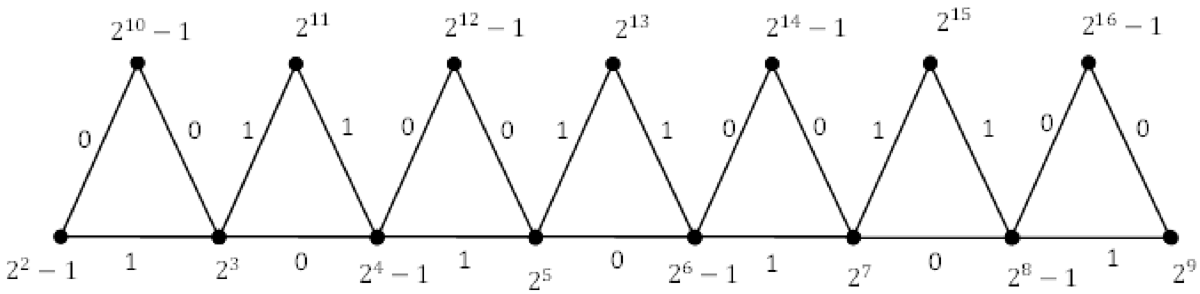
$$e_g^*(1) = \left(\frac{n}{2}\right) + (n-2) = \frac{3(n-1)}{2} - 2$$

So that $\left|e_g^*(0) - e_g^*(1)\right| = \left|\frac{3n}{2} - 1 - \frac{3n}{2} + 2\right| = 1$
Hence $\left|e_g^*(0) - e_g^*(1)\right| \le 1$ is satisfied.
Therefore the triangular snake graph $TL_n$ admits Narayana prime cordial labeling.

Illustrations: 2.2.1 The Narayana prime cordial labeling of Triangular snake $TL_n$ graph for n is odd are shown in the following figure.



Illustrations: 2.2.2 The Narayana prime cordial labeling of Triangular snake $TL_n$ graph for n is even are shown in the following figure.




Theorem 2.3. A Double Fan $DF_n$ graph admits Narayana prime cordial labeling.
Proof. Let G is a graph with two pendent vertices m and k.

Let $P_n$ be the path with n vertices and n=1number of edges m and k are the vertices in which m is the pendent vertex which joins all the vertices of the path $P_n$ from upper side to form n number of edges and k is the pendent vertex which joins all the vertices of path $P_n$ from downside to form n number of edges.

Number of vertices in the graph is n+2 Number of edges in the graph is 3n-1
Case (i): The path $P_n$ with n is odd.

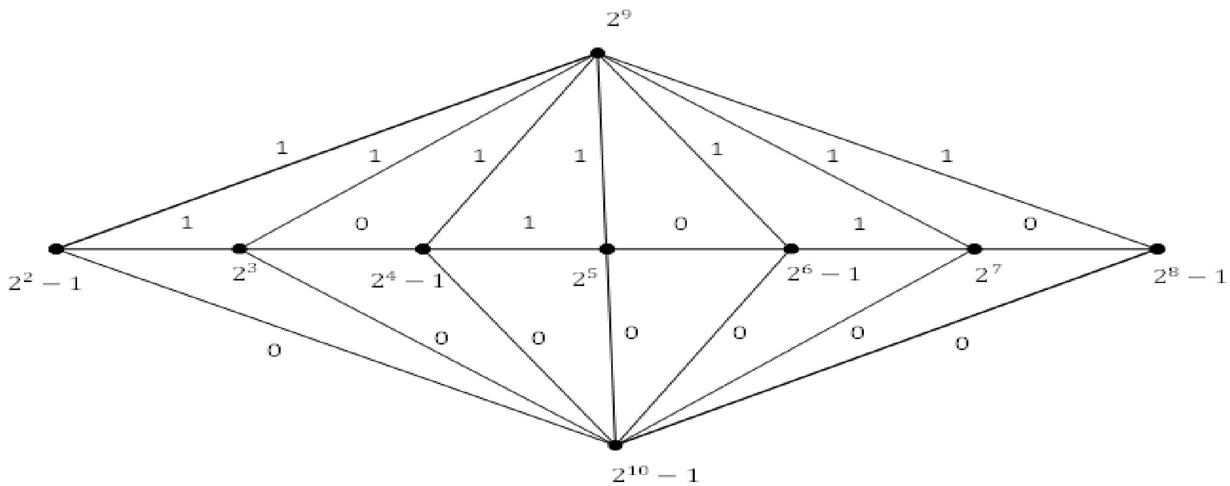
$$g(v_i) = 2^{i+1} - 1; \quad i \equiv 1(mod2); \quad 1 \le i \le n.$$
$$g(v_i) = 2^{i+1}; \quad i \equiv 0(mod2); \quad 1 \le i \le n.$$

$g(m) = 2^{i+1+1} = 2^{n+2}$ and $g(k) = 2^{(n+2)+1} - 1 = 2^{n+3} - 1$.
Then, $e_{g^*}(0) = n + \left(\frac{n-1}{2}\right) = \frac{3n-1}{2}$ and $e_{g^*}(1) = n + \left(\frac{n-1}{2}\right) = \frac{3n-1}{2}$
So that $\left|e_{g^*}(0) - e_{g^*}(1)\right| = \left|\frac{3n-1}{2} - \frac{3n-1}{2}\right| = 0$
Hence $\left|e_g^*(0) - e_g^*(1)\right| \le 1$ is satisfied.
Case (ii): The path $P_n$ with n is odd

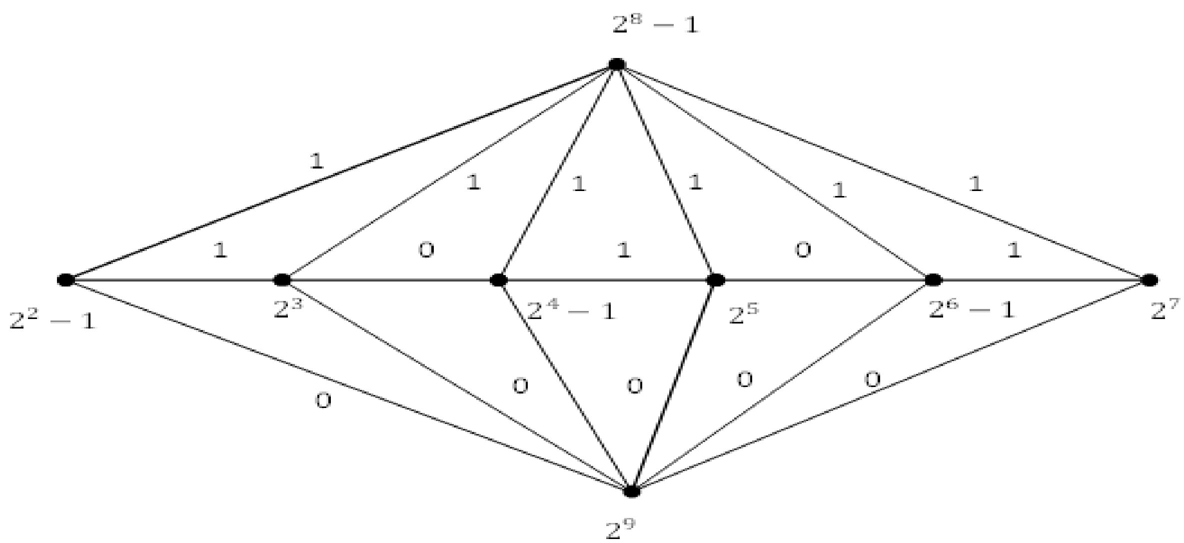
$$g(v_i) = 2^{i+1} - 1; \quad i \equiv 1(mod2); \quad 1 \le i \le n.$$
$$g(v_i) = 2^{i+1}; \quad i \equiv 0(mod2); \quad 1 \le i \le n.$$

$g(m) = 2^{n+2} - 1$ and $g(k) = 2^{n+3}$.
Then, $e_{g^*}(0) = n + \left(\frac{n-2}{2}\right) = \frac{3n-2}{2}$ and $e_{g^*}(1) = n + \frac{n}{2} = \frac{3n}{2}$
So that $\left|e_{g^*}(0) - e_{g^*}(1)\right| = \left|\frac{3n}{2} - 1 - \frac{3n}{2}\right| = 1$
Hence $\left|e_g^*(0) - e_g^*(1)\right| \le 1$ is satisfied.
Hence Double Fan $DF_n$ graph admits Narayana prime cordial labeling.

Illustrations: 2.3.1 Consider the Double Fan with odd vertices in the path. The Narayana prime cordial labeling of $DFL_n$ is as shown below.




Illustrations: 2.3.2 Consider the Double Fan with even vertices in the path. The Narayana prime cordial labeling of $DFL_n$ is as shown below.




Theorem 2.4. A Double Comb $DCm_n$ admits Narayana prime cordial labeling.

Proof. Let G be a double comb graph obtained by $K_2$ on either side of vertices of path $P_n$ with 3n vertices and 3n-1 number of edges.

Define a one-one function by $g:V(G) \to N_0$ such that

Case (i): when path$P_n$ with n is odd.

$$g(v_i) = 2^{i+1}; \quad i \equiv 1(mod2); \quad 1 \le i \le n.$$
$$g(v_i) = 2^{i+1} - 1; \quad i \equiv 0(mod2); \quad 1 \le i \le n.$$
$$g(x_i) = 2^{i+1} - 1; \quad n+1 \le i \le 2n.$$
$$g(z_i) = 2^{i+1}; \quad 2n+1 \le i \le 3n.$$

In this type of labeling,

$e_{g^*}(0) = n + \left(\frac{n-1}{2}\right) = \frac{3n-1}{2}$ and $e_{g^*}(1) = n + \frac{n-1}{2} = \frac{3n-1}{2}$

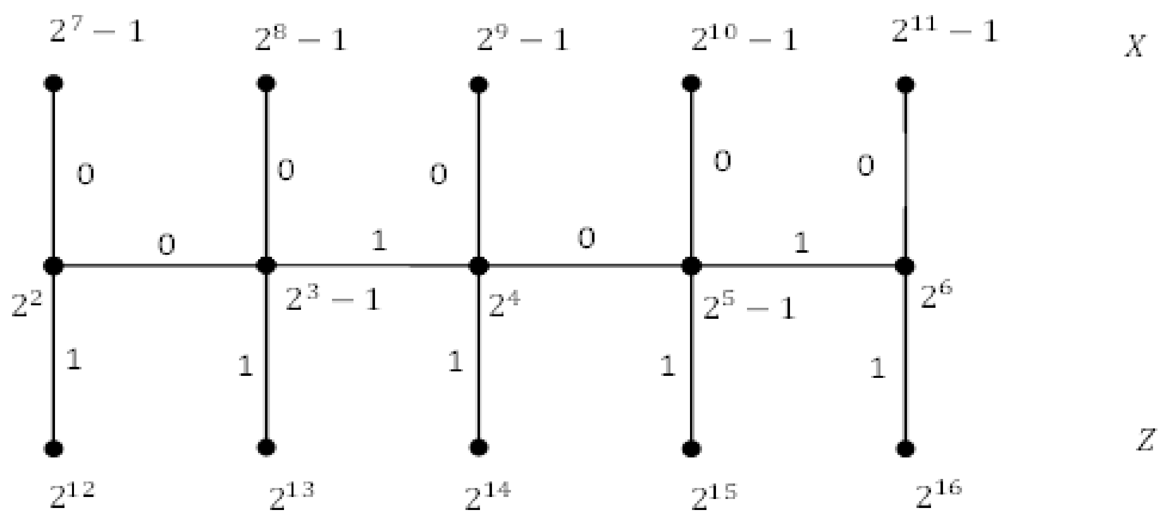
So that $\left|e_{g^*}(0) - e_{g^*}(1)\right| = \left|\frac{3n-1}{2} - \frac{3n-1}{2}\right| = 0$

Hence $\left|e_{g^*}(0) - e_{g^*(1)}\right| \leq 1$ is satisfied.
Case (ii): when path $P_n$ with $n$ is even.

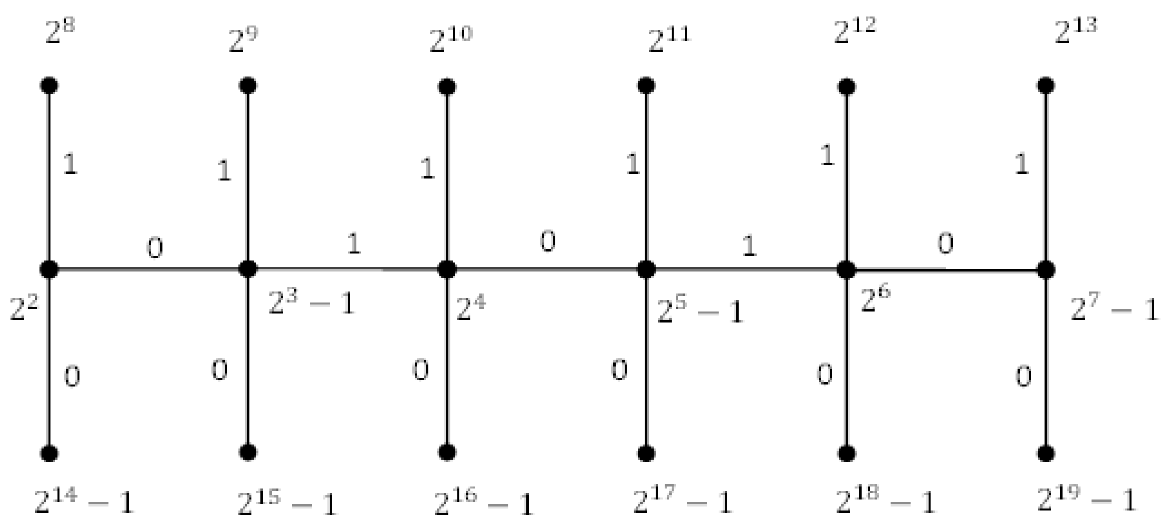
$$g(v_i) = 2^{i+1}; \quad i \equiv 1(mod2); \quad 1 \leq i \leq n.$$
$$g(v_i) = 2^{i+1} - 1; \quad i \equiv 0(mod2); \quad 1 \leq i \leq n.$$
$$g(x_i) = 2^{i+1} - 1; \quad n+1 \leq i \leq 2n.$$
$$g(z_i) = 2^{i+1}; \quad 2n+1 \leq i \leq 3n.$$

Then, $e_{g^*}(0) = n + \left(\frac{n}{2}\right) = \frac{3n}{2}$ and $e_{g^*}(1) = n + \frac{n}{2} - 1 = \frac{3n}{2} - 1$
So that $\left|e_{g^*}(0) - e_{g^*}(1)\right| = \left|\frac{3n}{2} - \frac{3n}{2} + 1\right| = 1$
Hence $\left|e_g^*(0) - e_g^*(1)\right| \leq 1$ is satisfied.
Hence Double Comb $DCm_n$ admits Narayana prime cordial labeling.

Illustrations: 2.4.1 Narayana prime cordial labeling of Double comb with n is odd are as shown below.




Illustrations: 2.4.2 Narayana prime cordial labeling of Double comb with n is even are as shown below.



## III. CONCLUSION

In this paper, the discussion of Narayana-prime-cordial-labeling of the graphs namely, Crown, Triangular snake, Double Fan, Double Comb give remarks and observations to conclude their works.

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