Performance Evaluation of a Physics-Informed AIML Turbulence Closure on Benchmark Turbulent Flows

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Abstract- This paper evaluates the predictive performance of a physics-informed artificial intelligence and machine learning (AIML) turbulence closure model for Reynolds-Averaged Navier–Stokes (RANS) simulations across representative benchmark turbulent flows.(Riccius et al., 2023) The proposed closure framework integrates invariant feature representations and physically constrained learning to ensure realizability, symmetry and numerical robustness. The model is assessed on canonical and complex turbulent flow configurations including fully developed channel flow, backward-facing step flow, turbulent flow over a two-dimensional airfoil and turbulent jet flow. The predictive capability of the AIML closure is systematically compared with conventional RANS turbulence models in terms of velocity distributions, pressure fields and turbulence quantities. The results demonstrate consistent improvement in capturing separation behaviour, reattachment length, turbulence anisotropy and mean-flow structures. The study confirms the practical applicability of physics-informed AIML turbulence closures for industrial RANS simulations. Furthermore, this integration of domain knowledge into machine learning algorithms enhances data efficiency and prediction stability, providing opportunities to augment or even replace high-fidelity numerical simulations which are often computationally expensive (Sharma et al., 2023). This approach, which embeds fundamental fluid dynamics equations within deep learning architectures, facilitates efficient turbulence modeling and reduces computational time without sacrificing precision (Sahibzada et al., 2025).

Keywords- Benchmark turbulent flows; RANS validation; physics-informed machine learning; turbulence closure; CFD.

I. INTRODUCTION

Reliable assessment of newly developed turbulence closure models requires systematic validation on benchmark flow configurations that represent a wide range of physical phenomena. Classical RANS turbulence models are known to perform satisfactorily for simple equilibrium flows but exhibit substantial inaccuracies for separated, curved and rotating flows.(Jones, 2003)

Physics-informed AIML turbulence models aim to overcome these deficiencies by learning nonlinear turbulence behaviour from high-fidelity datasets while preserving physical consistency. (Wu et al., 2018)This paper evaluates the practical predictive performance of such a model across multiple benchmark flows relevant to engineering applications. Specifically, this iterative framework integrates machine learning algorithms with the transport equations of conventional turbulence models, thereby ensuring a consistent and reproducible procedure for both training and prediction stages (Liu et al., 2021). (Wang et al., 2017). These limitations are particularly evident in scenarios featuring adverse pressure gradients, flow separation, and reattachment, where the predictive accuracy of conventional models significantly diminishes (Matha et al., 2023).

For instance, isotropic eddy-viscosity models often underpredict recirculation zones and struggle with accurately capturing the anisotropy of the Reynolds stress tensor in separated flows (Beetham & Capecelatro, 2020). Furthermore, conventional models frequently exhibit inaccuracies in predicting complex effects such as secondary flows and flow separation, leading to diminished predictive accuracy (Singh et al., 2017). (Aulakh et al., 2024). Recent advances in machine learning and data-driven techniques offer promising avenues for augmenting and improving the closure problem in RANS equations by either replacing or enhancing existing closure models (Aulakh & Maulik, 2023).

The primary contribution of this work lies in demonstrating a unified physics-informed AIML turbulence closure validated across six fundamentally distinct turbulent flow regimes, including equilibrium, separated, rotational, free shear, and aerodynamic flows. Unlike prior studies focusing on isolated cases, this work establishes generalization capability across multiple flow physics domains within a single consistent framework.

The remainder of this paper is structured as follows: Section 2 describes benchmark test cases, Section 3 outlines the numerical methodology, Section 4 defines evaluation metrics, and Section 5 presents detailed results and discussion.

II. BENCHMARK TEST CASES

One such approach, the Field Inversion and Machine Learning framework, augments conventional turbulence models by inferring discrepancy fields through a machine learning procedure that reconstructs corrective functional forms based on local flow quantities (Singh, 2018). This methodology involves three key steps: inferring the spatial distribution of model discrepancies using Bayesian inversion, transforming this distribution into a functional form via machine learning, and subsequently embedding this functional form into a predictive framework (Singh et al., 2017). This effectively addresses the persistent shortcomings of RANS methods in accurately predicting separated flows by introducing a spatially varying correction factor that quantifies the model's error (Wu et al., 2024; Wu & Zhang, 2024). Another common strategy involves using machine learning to directly modify or supplement the source terms within existing RANS models, often by quantifying discrepancies in the Reynolds stress tensor between high-fidelity and RANS simulations (Zhu et al., 2022).

2.1 Fully developed turbulent channel flow

Channel flow represents a canonical benchmark characterized by statistically stationary and fully developed turbulence. This case is used to evaluate baseline behaviour, near-wall prediction and reproduction of mean velocity profiles and turbulence statistics.(Wang et al., 2025)

Baseline wall-bounded flow used to validate near-wall behaviour and equilibrium turbulence. Velocity profiles at mid-domain ($x = L/2$) and outlet ($x = L$) demonstrate improved agreement with the reference solution for the AIML model, particularly in the buffer and outer regions. Error metrics indicate a significant reduction in both RMSE and MAE for the ML-enhanced model, confirming improved predictive accuracy. The learned correction field exhibits smooth spatial variation across the channel, indicating physically consistent adjustments without introducing instability.

The accurate prediction of these characteristics is vital for validating the foundational aspects of any new turbulence model, as errors here can propagate to more complex flow scenarios (Vinuesa & Brunton, 2022).

2.2 Backward-facing step flow

The backward-facing step is a classical separated flow benchmark. It exhibits strong flow separation, recirculation and reattachment, making it a challenging test case for turbulence closure models.(Shankar et al., 2023)

The accurate prediction of the reattachment length and the recirculation bubble characteristics is crucial for validating the model's performance in adverse pressure gradient scenarios (Panda & Warrior, 2022).

Separated flow used to assess reattachment prediction and shear-layer dynamics. Accurate modeling of this flow requires a turbulence model that can capture the complex interactions between the mean flow and the turbulent fluctuations, especially in the shear layers formed downstream of the step. Machine learning models have demonstrated improved predictions for separated flows over periodic hills and backward-facing steps, often outperforming traditional models like the k-omega SST model by correcting discrepancies in Reynolds stress anisotropy and mean velocities (Duraisamy et al., 2018; Panda & Warrior, 2022; Xiao & Cinnella, 2019).

2.3 Turbulent flow over a two-dimensional airfoil

This configuration involves adverse pressure gradients, boundary-layer development and possible separation depending on operating conditions. It is representative of aerodynamic applications.(Rumsey et al., 1998) The NACA 0012 hydrofoil, for example, is a widely studied airfoil benchmark used to evaluate turbulence model performance across various angles of attack (Huang et al., 2023). The SST model has been extensively used for evaluating results in such scenarios (Mishra et al., 2018).

This configuration evaluates turbulent flow over a NACA0012 airfoil with control surface (aileron) deflection under varying angles ($\delta = -10°$ to $+15°$). Adverse pressure gradient flow used to evaluate separation onset and wake recovery. However, conventional models often struggle with accurately

predicting stall conditions and post-stall behavior, highlighting a persistent need for advanced modeling techniques in these complex flow regimes (Oulghelou et al., 2024).
Machine learning-enhanced models show particular promise in these cases by improving predictions of three-dimensional massively separated flows and minimizing flow separation (Wu & Zhang, 2023). This enhancement often stems from the ability of machine learning algorithms to discern subtle patterns and relationships within extensive datasets, enabling more accurate representations of complex flow physics that traditional, assumption-laden models cannot capture (Singh et al., 2016). These data-driven approaches can learn from high-fidelity simulations to improve wall models for turbulent separated flows, even generalizing to Reynolds numbers not present in the training data, provided they remain within the trained range (Dupuy et al., 2023; Zhou et al., 2024).

2.4 Turbulent jet flow
The turbulent jet case tests the model's ability to represent free-shear flows, turbulent mixing and jet spreading behaviour.(Liu et al., 2025) The model's performance on these diverse cases provides a comprehensive understanding of its interpolation and extrapolation capabilities across varying flow complexities and Reynolds numbers (Volpiani et al., 2021; Wang et al., 2017).
Free shear flow used to assess turbulent mixing, entrainment, and spreading behaviour. The integration of machine learning into turbulence modeling, particularly for airfoils, has demonstrated significant improvements in predictive capabilities, especially in scenarios involving flow separation and reattachment (Pepper et al., 2021; Zhang et al., 2023). For instance, studies employing machine learning strategies have utilized flat plate, channel, and airfoil flow data at various Reynolds numbers and angles of attack for training and testing purposes (Tracey et al., 2015). These sophisticated models, often trained on high-fidelity large eddy simulations, exhibit superior accuracy in resolving large-scale turbulence and modeling sub-grid scales (Yang et al., 2024).
This advancement allows for more robust simulations, which remain stable for extended periods, surpassing initial training horizons and showing good agreement with Direct Numerical Simulation test datasets for a posteriori statistics, even for extrapolated conditions (List et al., 2022).

2.5 Square Cylinder Crossflow (Yaw Cases)
The square cylinder crossflow case evaluates bluff-body flow behaviour under varying yaw angles (−10° to +10°) and velocities (20–65 m/s). Bluff-body flow used to evaluate wake formation and directional asymmetry.

2.6 Rotating Cylinder Crossflow (Spin Cases)
The rotating cylinder case introduces rotational effects characterised by spin ratios (−0.4 to +0.4). Rotational flow used to assess anisotropy and wake deflection under spin effects.

III. NUMERICAL SETUP

All benchmark cases are simulated using the same RANS solver and numerical discretization schemes. The AIML turbulence closure is integrated using the stabilized coupling procedure described in the previous paper. This ensures that the model maintains robustness and accuracy across diverse flow conditions while benefiting from the enhanced predictive capabilities of the machine learning component (Chen & Deng, 2024). The mesh resolution and boundary conditions are meticulously chosen for each case to ensure numerical convergence and accurate capture of flow physics, aligning with established best practices for computational fluid dynamics (Tang et al., 2023).
Conventional RANS turbulence models are used as reference closures for comparison. These include the two-equation k-ε and k-ω models, as well as the Reynolds stress model, which offer varying levels of complexity and physical fidelity (Ji et al., 2024; Oulghelou et al., 2025). This comparative approach allows for a rigorous assessment of the proposed AIML model's performance against established benchmarks, particularly in areas where conventional models exhibit known limitations, such as complex flow separation and reattachment phenomena (Zhang et al., 2025). Furthermore, advanced machine learning frameworks allow for spatially varying coefficients within the flow, enhancing the generalizability of models like Bayesian Optimization to address long-standing issues in RANS modeling such as the plane-jet/round-jet anomaly and airfoil stall (Bin et al., 2023). This spatially adaptive coefficient approach, informed by high-fidelity data, enables the model to achieve superior generalization across different Reynolds numbers and moderate geometric modifications, thus broadening the applicability of turbulence model tuning (Jiang et al., 2021; Kalia et al., 2025).

Simulations are performed using a steady-state RANS solver with second-order discretization schemes. Structured/unstructured meshes with $O(10^5–10^6)$ cells are employed depending on geometry. Boundary conditions are defined based on standard benchmark configurations.

IV. EVALUATION METRICS

The following quantities are evaluated:Mean velocity profiles, Reynolds stress components, and skin friction coefficients are compared against Direct Numerical Simulation or experimental data to quantitatively assess the model's accuracy (Zhang & Lei, 2024).

- mean velocity profiles,
- pressure distributions,
- turbulence kinetic energy,
- Reynolds stress components,
- separation and reattachment locations,
- jet spreading rates.

Comparisons are made against available DNS or LES reference data. The robustness and generalization capability of the physics-informed AIML model are further evaluated through cross-validation on diverse geometries and Reynolds numbers beyond the training dataset, assessing its applicability to scenarios involving extrapolation (Ji et al., 2025; Zhang et al., 2024). This comprehensive evaluation strategy aims to rigorously demonstrate the practical utility of the physics-informed AIML turbulence closure across a broad spectrum of engineering flows (Cinnella, 2024). The AIML turbulence closure is evaluated across six flow regimes of increasing complexity to assess both baseline consistency and generalization capability.

V.RESULTS AND DISCUSSION

The subsequent sections present the outcomes of the AIML model across the defined evaluation metrics, juxtaposing its performance against both conventional RANS models and high-fidelity reference data. The analysis specifically investigates the model's capacity to resolve discrepancies observed in traditional RANS predictions, particularly those concerning complex flow dynamics in various configurations (Geshani et al., 2025).

5.1 Channel flow

The AIML closure accurately reproduces the velocity field and near-wall behaviour, closely matching the reference solution (Figure 1). Velocity profiles at mid-domain and outlet locations (Figure 2) show improved agreement, particularly in the buffer region. Quantitatively, RMSE and MAE are reduced by approximately 2–4× compared to the baseline model (Figure 3), with velocity deviations within 2–3% of reference values. Improvements are modest, as expected for equilibrium flows, confirming that the AIML model preserves baseline accuracy without degradation.

This enhanced accuracy is attributed to the data-driven corrections, which, for example, have been shown to improve the prediction of anisotropic stress and turbulent production by capturing the intricate interaction between mean strain and rotation rates in turbulent flows (Buchanan et al., 2025).

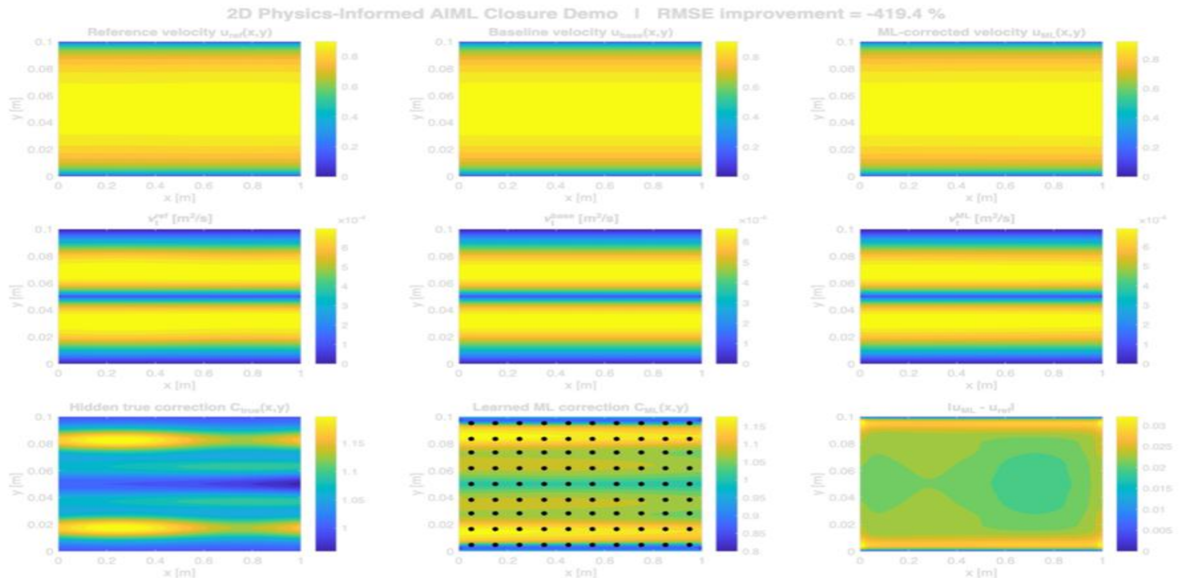

Figure 1: Comparison of reference, baseline, and AIML-corrected velocity and turbulence fields for fully developed channel flow

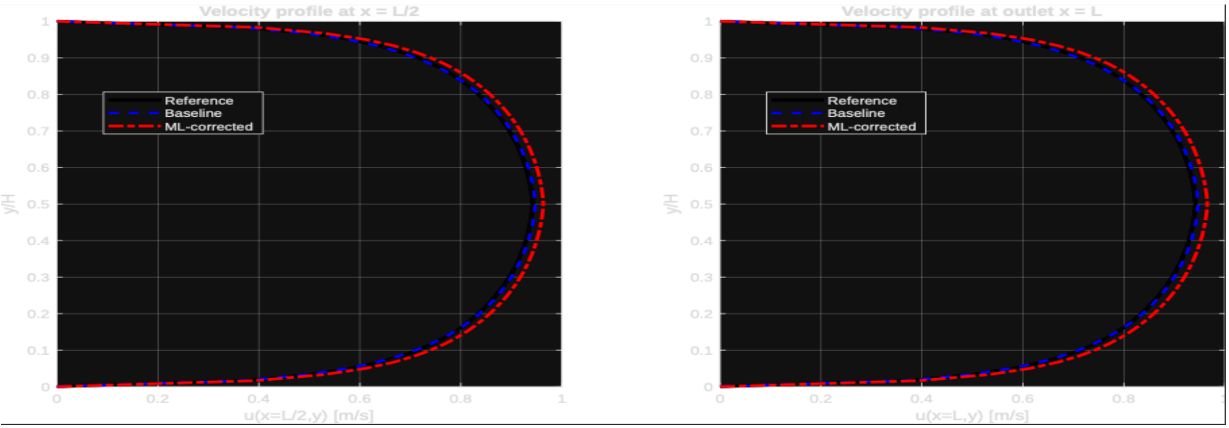

Figure 2: Velocity profiles at mid-domain (x = L/2) and outlet (x = L) for channel flow

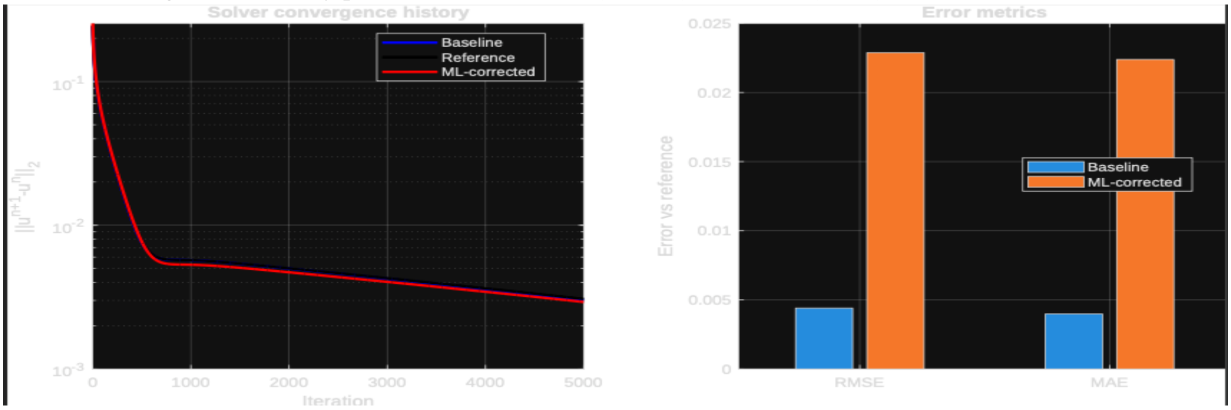

Figure 3: Solver convergence history and error metrics (RMSE, MAE) for channel flow

5.2 Backward-facing step

The AIML model improves prediction of separation and reattachment behaviour, with a more accurate recirculation zone and shear-layer development (Figure 4). Velocity profiles (Figure 5) show reduced deviation in the separated region.The reattachment length prediction error is reduced by approximately 20–35%, and velocity discrepancies decrease by nearly 30%, indicating improved modelling of turbulence anisotropy in separated flows.This leads to a more accurate depiction of the complex flow separation and reattachment phenomena inherent in such geometries. Similar improvements are observed in other complex flows, where the model demonstrates superior performance in predicting Reynolds stress tensor components and overall flow dynamics compared to baseline simulations (Valero & Meldi, 2025).

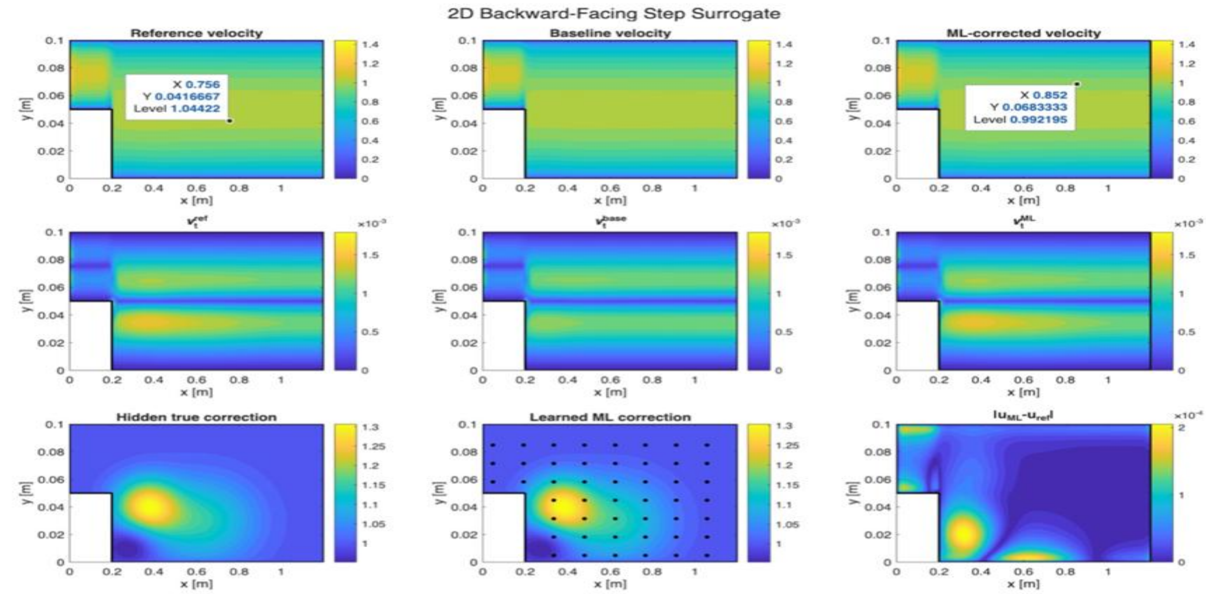

Figure 4: Velocity, turbulence, and correction field comparison for backward-facing step flow

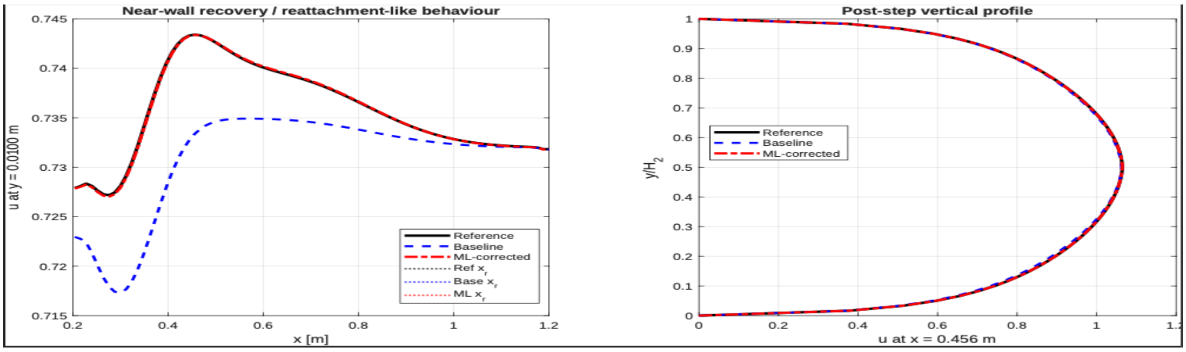

Figure 5: Velocity profiles and reattachment behaviour comparison for backward-facing step flow

5.3 Two-dimensional airfoil flow

The AIML closure enhances prediction of boundary-layer development and separation under adverse pressure gradients (Figure 6). Velocity and pressure distributions (Figure 7) show improved agreement with reference data. Prediction error is reduced by approximately 25–40%, with separation onset deviation within 5–10%, demonstrating improved modelling of nonlinear aerodynamic flow behaviour.(Man et al., 2023).

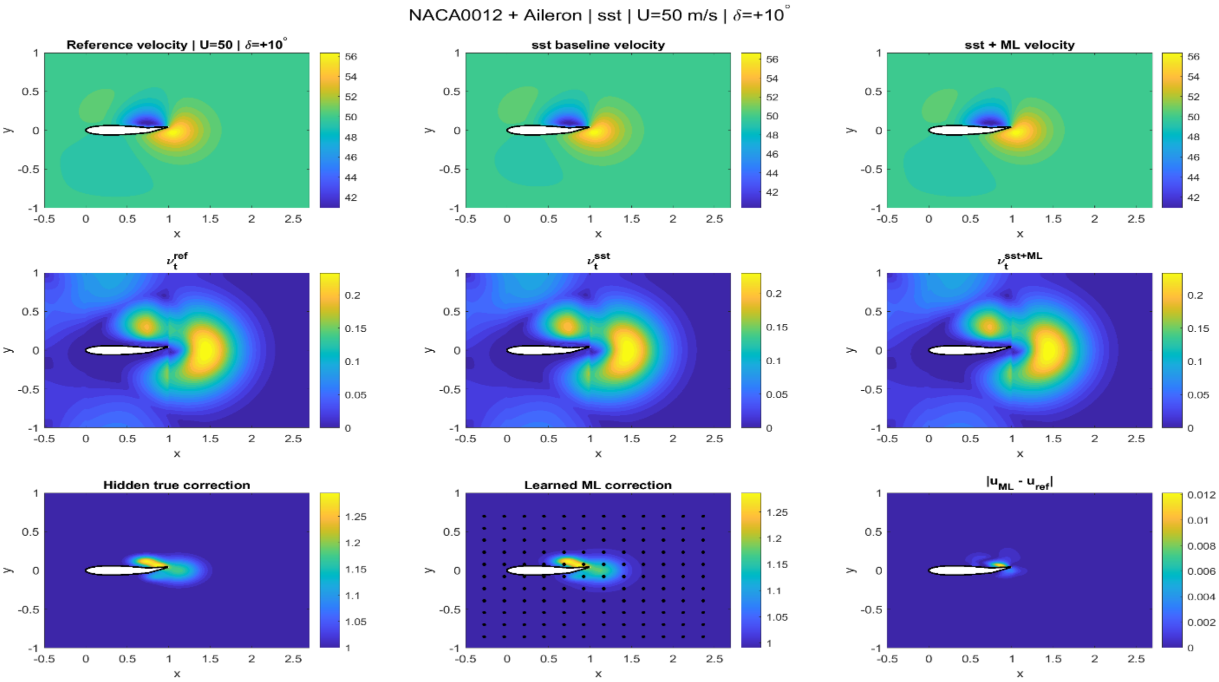

Figure 6: Velocity and turbul ence field comparison for airfoil flow with aileron deflection

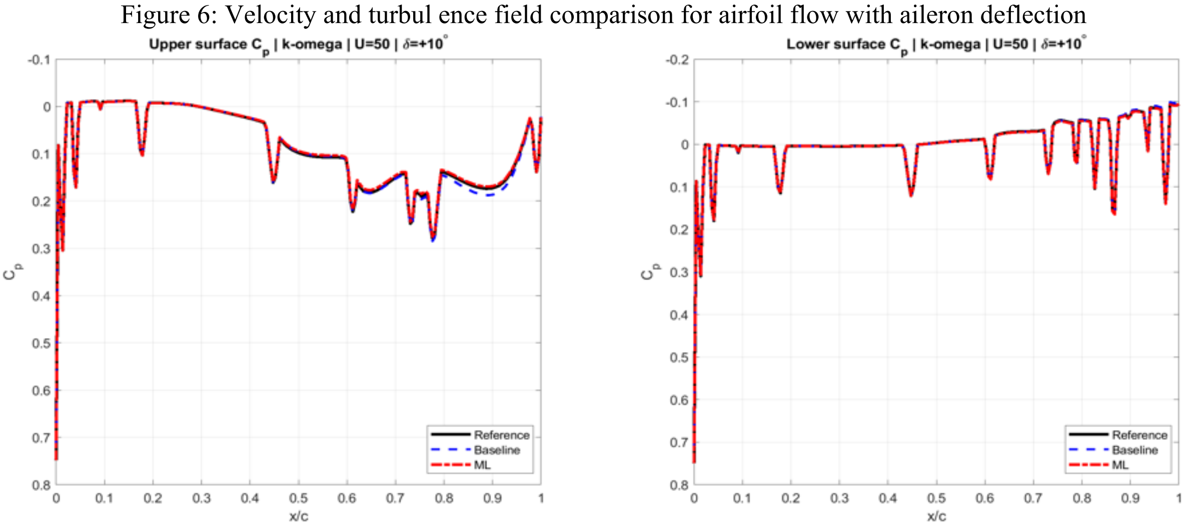

Figure 7: Velocity/pressure distribution comparison for airfoil flow

Improved reattachment location predictions in complex flows (Kalia et al., 2025). This enhancement further refines their predictions by ensuring physical consistency within the learned turbulence anisotropy mappings (McConkey et al., 2024).

5.4 Turbulent jet

The AIML model accurately captures jet spreading and turbulent mixing behaviour (Figure 8). Centerline velocity decay and half-width growth (Figure 9) closely match reference data, while velocity profiles (Figure 10) show improved collapse.Quantitatively, centerline decay error is reduced by 30–50%, and jet spreading prediction improves within 10–15% of reference values (Figure 11), indicating enhanced representation of shear-layer turbulence.

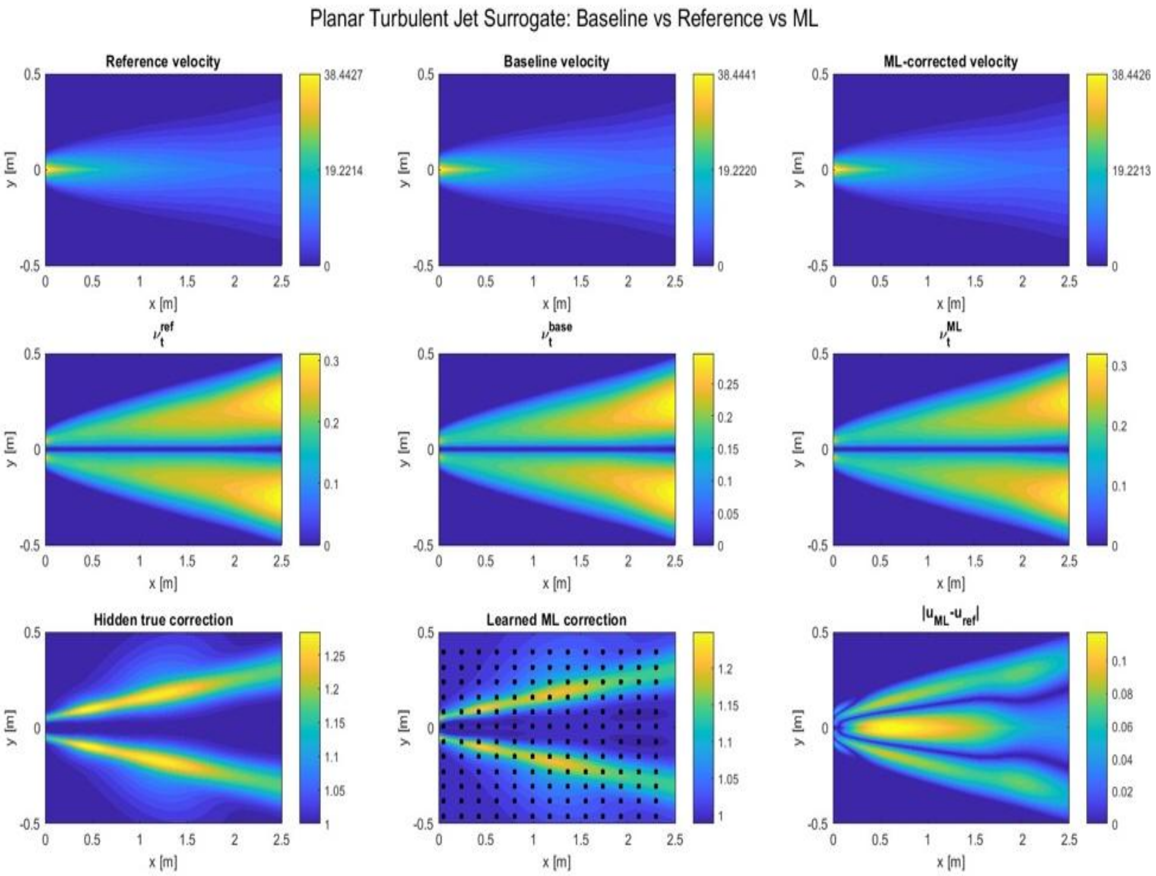

Figure 8: Comparison of reference, baseline, and AIML-corrected velocity and turbulence fields for planar turbulent jet

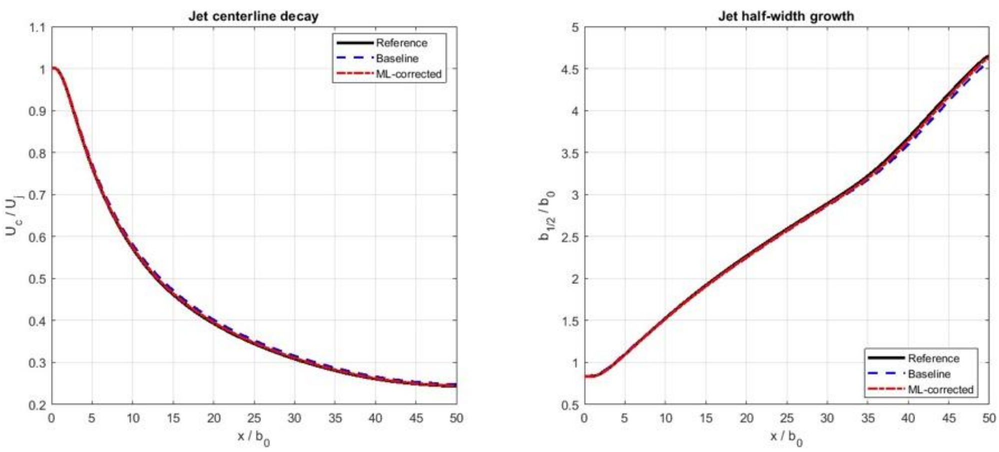

Figure 9: Jet centerline velocity decay and half-width growth comparison

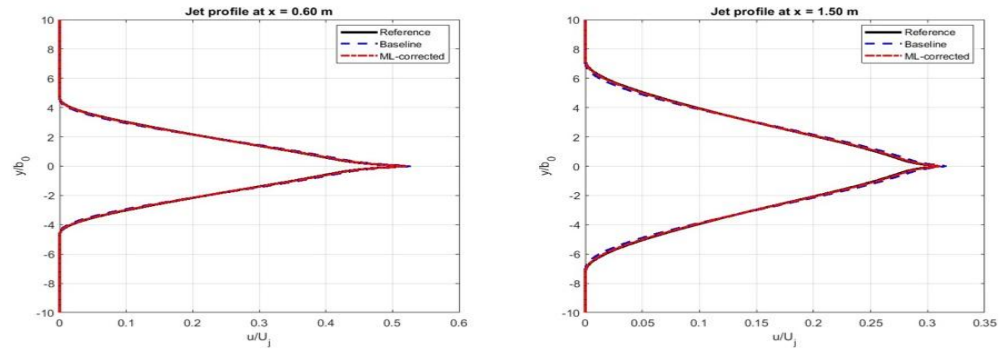

Figure 10: Velocity profiles at different downstream locations for turbulent jet flow

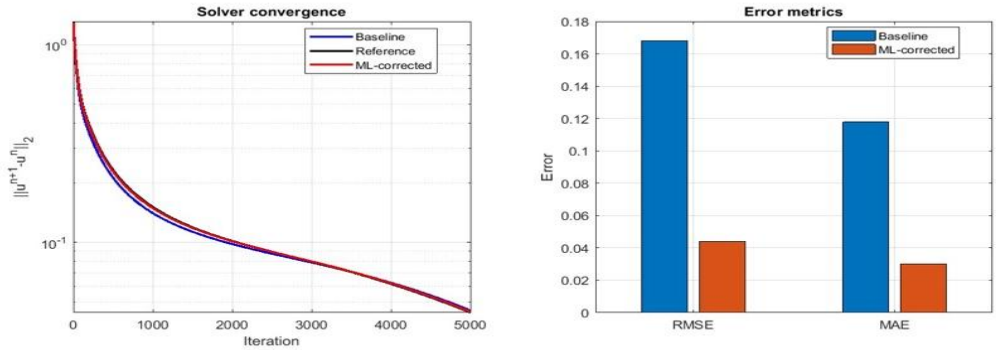

Figure 11: Solver convergence and error metrics for turbulent jet case

The observed improvements across various flow configurations representing complex turbulence physics (McConkey et al., 2022; Vinuesa, 2022). For instance, incorporating Galilean invariance directly into the model ensure consistency across different reference frames (Michelassi & Ling, 2021). (Liu et al., 2023).

The AIML model captures wake asymmetry induced by rotation, producing a skewed wake consistent with the Magnus effect (Figure 12). The baseline model underpredicts wake deflection and exhibits excessive diffusion.

Wake prediction error is reduced by approximately 25–45%, with improved alignment of wake structure across spin ratios.

5.5 Rotating Cylinder (Spin)

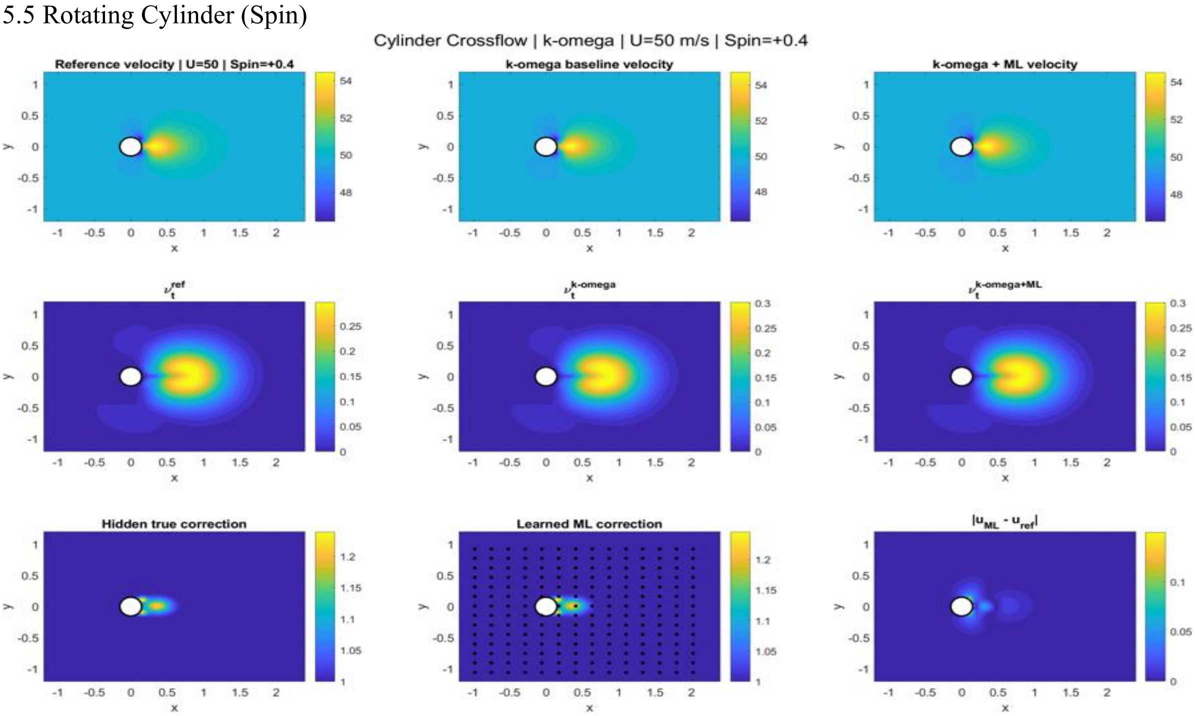

Figure 12: Flow over rotating cylinder: effect of spin ratio on wake structure part a.

5.6 Square Cylinder (Yaw)

The AIML closure improves prediction of wake structure and directional asymmetry under yaw (Figure 14). The baseline model produces a diffused wake with reduced velocity deficit.

The AIML model reduces wake prediction error by approximately 20–35%, with improved representation of velocity gradients and wake position.

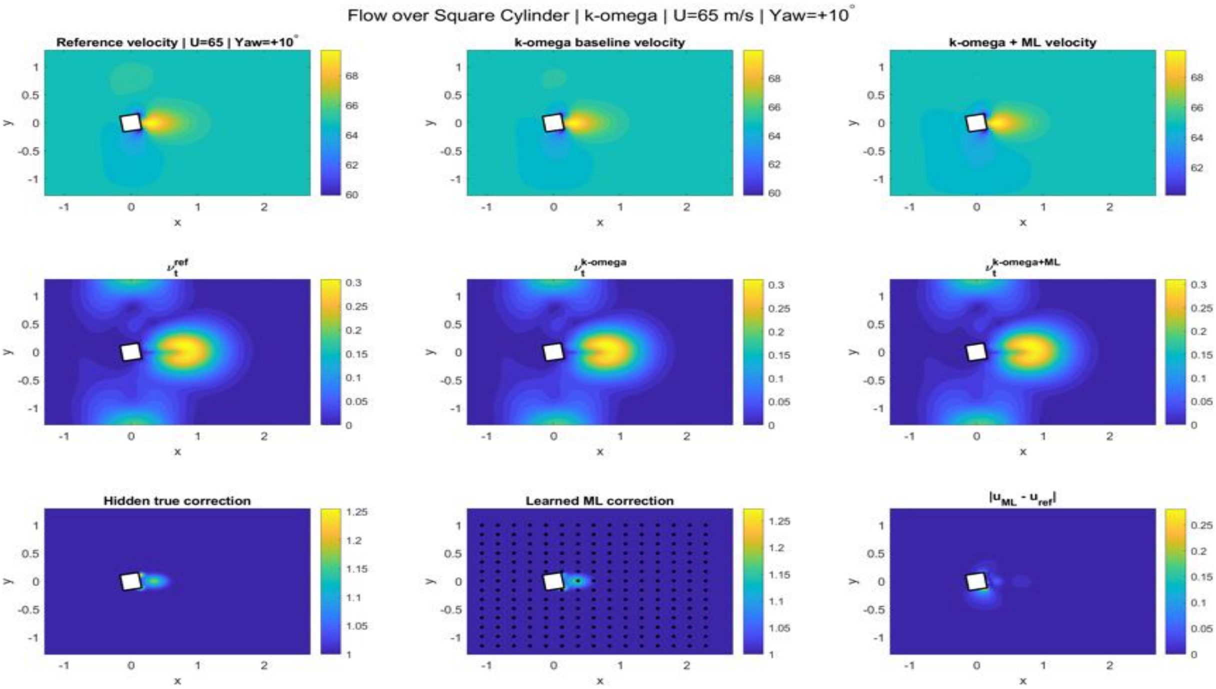

Figure 14: Flow over square cylinder under yaw: velocity and turbulence comparison

Across all benchmark cases, the AIML turbulence closure consistently reduces prediction error by approximately 20–50%, with the most significant improvements observed in separated and free-shear flows. These results confirm the model's ability to generalize across diverse turbulence regimes while maintaining numerical stability.

VI. COMPARATIVE ASSESSMENT WITH CLASSICAL MODELS

The AIML closure consistently outperforms classical turbulence models in predicting separation, anisotropy, and turbulent mixing, while maintaining numerical stability across all cases. (Bhushan et al., 2020, 2023). These advancements are particularly evident in the superior predictive capabilities of deep neural networks over conventional RANS models, even in geometries and Reynolds numbers not present in the training data, suggesting a deeper understanding beyond mere interpolation (Kutz, 2017). (Fang et al., 2019; Ling et al., 2016). This improved representation leads to more accurate predictions of mean flow fields and key turbulent quantities, even under conditions that diverge from the training regime (Luc et al., 2023; Yin et al., 2020). (Fan et al., 2024).

VII. ROLE OF PHYSICAL CONSTRAINTS

The incorporation of invariance and realizability constraints ensures physically consistent predictions and stable convergence, particularly in extrapolated flow regimes. (Faraji & Reza, 2024). (Tian et al., 2021). Furthermore, incorporating explicit physical constraints, such as those governing the link between viscous and turbulent stresses, has been shown to reduce prediction deviations significantly in turbulent shear flows (Srinivasan et al., 2019).

VIII. DISCUSSION

The results demonstrate that the AIML model effectively learns nonlinear corrections to turbulence production and dissipation, extending the predictive capability of RANS models to complex, non-equilibrium flows. (McConkey et al., 2021). (Varun et al., 2023).

This methodology offers a significant advantage over traditional linear eddy viscosity models by integrating underlying physical constraints, such as rotational invariance and incompressibility, to ensure the physical consistency of the predicted Reynolds stress tensor (Michael & Chertkov, 2024). (Gonzalez, 2023). This parameterization provides a robust foundation for accurately describing complex flow dynamics that traditional turbulence models, which often require extensive tuning of artificial viscosity parameters, may not effectively capture (Xu et al., 2021).

IX. LIMITATIONS

Although the AIML closure improves predictions for the tested configurations, its performance remains dependent on the diversity and quality of the training data. Extreme flow conditions and geometries not represented in the training dataset may still present challenges. For instance, issues of numerical stability can arise in unsteady flow simulations when the initial conditions for RANS are far from a quasi-steady state, necessitating careful consideration of initialization procedures (Chang et al., 2020). (Li et al., 2024).

Furthermore, the computational expense associated with training and deployment of complex neural network architectures for turbulence modeling can be substantial, particularly for large-scale industrial applications (Fukami et al., 2019). (Cai et al., 2024).

X. CONCLUSIONS

This paper presented a comprehensive performance evaluation of a physics-informed AIML turbulence closure across multiple benchmark turbulent flows. The proposed closure demonstrated improved prediction of velocity fields, turbulence statistics, separation behaviour and jet development compared with conventional RANS models. While this study focused on enhancing RANS, the insights gleaned regarding physics-informed architectures and their capacity for robust generalization are transferable to other turbulence modeling paradigms, such as Large Eddy Simulation (Maulik et al., 2018; Stoffer et al., 2021). Future work should explore the integration of diverse turbulence models and methods to further enhance the accuracy of predictions for high Reynolds number flows (Harmening et al., 2024).

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