

# Beal's Conjecture, Ptolemy theorem

Murugesha P

Guest faculty in Chamarajnagara University

**Abstract: The below theorems explain about the link between number theory, determinant and Ptolemy theorem.**

problem by the United States of America's Dallas, Texas number theory enthusiast and billionaire banker, Mr. Daniel Andrew Beal. Beal's conjecture states that: The equation  $A^x+B^y=C^z$  ----(1)

## I. INTRODUCTION

Beal's Conjecture is a conjecture in number theory formulated in 1993 while investigating generalizations of Fermat's Last theorem set forth in 1997 as a Prize

If A, B, C, x, y, z are integers with all exponents (x,y,z) greater than 2 then the bases A, B, C must share a common prime factor. In other words if A, B, C are relatively prime then the equation (1) has no solution.

## II. THEOREM

Statement: The equation  $x^{l+2}+y^{m+2}=z^{n+2}$  implies  $t^2 y^m z^n + r^2 z^n x^l = s^2 y^m x^{l+1}$ .

Proof:- Assume,  $az-by=x$ ,  $cx-dy=z$ ,  $ex-fz=y$  then we go to write this equation in the form of matrix as

$$\begin{pmatrix} 1 & b & -a \\ -c & d & 1 \\ e & 1 & f \\ & A & \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ X \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ B \end{pmatrix}$$

then  $AX=B$  -----(2). now, we get determinant of A is equal to zero.

Implies  $df+bcf+ac=be+aed+1$ ----(3)

Consider  $(rz^{n+1}-sy^{m+1})x^{l+1}+(sx^{l+1}-tz^{n+1})y^{m+1}=(rx^{l+1}-ty^{m+1})z^{n+1}$  Here, take  $rz^{n+1}-sy^{m+1}=x$ ,  $sx^{l+1}-tz^{n+1}=y$  and  $rx^{l+1}-ty^{m+1}=z$  by our assumption, replace a, b, e, f, c, d as  $rz^n$ ,  $sy^m$ ,  $sx^l$ ,  $tz^n$ ,  $rx^l$ ,  $ty^m$  respectively then by equation(3) becomes

$$t^2 y^m z^n + r^2 z^n x^l = s^2 y^m x^{l+1}$$
----(5)

Theorem: Ptolemy theorem  $(AB+CD=EF)$  implies  $(az-by)x+(bx-cz)y=(ax-cy)z$

Proof:-Let  $B=x$ ,  $D=y$ ,  $F=z$  then clearly there exist two real numbers a and b such that  $az-by=A$  since b is existed, there exist a real number c such that

$bx-cz=C$  since left hand side of equation is satisfied so it must satisfy right hand side also (equality property).

### III. CONCLUSION

It conclude that determinant is used to convert one form of beal conjecture into other form and Ptolemy theorem implies a determinant.

### REFERNCES

- [1] Zhang Yue's Fermat's last theorem for  $n=3$ .<https://www.mathematicaljournal.com>.
- [2] linear differential equation wikipedia. <https://en.wikipedia.org>.