A Decision-Making Approach for Medical Diagnosis Using Generalized Fuzzy Soft Set Matrices

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***Abstract*—This paper presents a decision-making approach for medical diagnosis using generalized fuzzy soft set matrices. In real-world medical scenarios, symptoms are often uncertain and overlapping, making accurate diagnosis difficult. The proposed method combines fuzzy set theory and soft set theory to effectively handle such uncertainties. Generalized fuzzy soft matrices are constructed based on symptom evaluations, and a systematic algorithm is applied to compute the weights of patients. A case study on COVID-19 diagnosis is presented to demonstrate the applicability of the method. The results show that the approach successfully identifies the most affected patient. This method is simple, flexible, and can be extended to various decision-making problems in healthcare and other domains.**

***Index Terms*—Fuzzy Soft Sets, Generalized Fuzzy Matrices, Medical Diagnosis, Decision Making, Uncertainty, COVID-19**
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I. INTRODUCTION

The concept of soft sets was first introduced by Molodtsov (1999) as a novel mathematical tool for dealing with uncertain and imprecise problems. Soft set theory provides a flexible framework for handling objects that are not clearly defined. To enhance its applicability, various operations on soft sets have been developed, leading to improved results and the formulation of decision-making methods such as the uni-int (union–intersection) decision-making approach.

To simplify computations, soft matrix theory was later introduced, along with max–min decision-making techniques. These methods have proven to be practical and effective in solving real-world problems involving uncertainty. Classical mathematical approaches often fail to address such problems due to the presence of vagueness and incomplete information, particularly in fields like economics, engineering, and environmental science.

To overcome these limitations, several mathematical theories have been proposed, including probability theory, fuzzy set theory, and interval mathematics. Among these, fuzzy set theory plays a significant role in representing uncertainty through degrees of membership.

Further developments in this area include the work of Yang et al. (2009), who introduced the combination of interval-valued fuzzy sets and soft sets. Çağman and Enginoğlu (2010) developed soft matrix theory and successfully applied it to decision-making problems. Basu et al. (2012) proposed a balanced solution for fuzzy soft set-based decision-making in medical science. Additionally, Dinda et al. (2010) extended the concept through generalized intuitionistic fuzzy soft sets and demonstrated their application in decision-making.

II. PRELIMINARIES

Definition 2.1(Soft set)

Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U and let $A \subseteq E$. A pair (F_A, E) is called a soft set over U, where F_A is a mapping given by,

$F_A : A \to P(U)$.In other words, a soft set over U is a parameterized family of subsets of U.

Definition 2.2 (Fuzzy Soft Set)

Let U be an initial universe set and E be a set of parameters (which are fuzzy words or sentences involving fuzzy words). Let P(U) denotes the set of all fuzzy sets of U. Let $A \subset E$. A pair (F_A, E) is called a Fuzzy Soft Set (FSS) over U, where F_A is a mapping given by, $F_A : E \to P(U)$ such that $F_A(e) = \tilde{\varphi}$ if $e \notin A$ where $\tilde{\varphi}$ is a null fuzzy set.

Definition 2.3 (Interval-Valued Fuzzy Soft Set)

Let U be an initial universe, E be a set of parameters and $A \subseteq E$. Then a pair $(\overline{F}_A, E)$ is called an interval-valued fuzzy soft set over P(U) where $\overline{F}_A$ is a mapping given by $\overline{F}_A : A \to P(U)$.

Definition 2.4 (Relation Form of a Soft Set)

Let (F_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e) : e \in A, u \in F_A(e)\}$ which is called a relation form of (F_A, E). Now the characteristic function of R_A is written by, $\chi R_A : U \times E \to \{0, 1\}$, $\chi R_A = 1, (u, e) \in / R_A$ & $\chi R_A = 0, (u, e) \notin / R_A$

Definition 2.5 (Fuzzy Soft Set over Soft Universe)

Let $U = \{ u_1, u_2,u_3,..., u_m \}$ be the universal set of elements and $E = \{ e_1, e_2,e_3..., e_n \}$ be the set of parameters. The pair (U, E) will be called soft universe. Let $F: E \to I^U$ and λ be a fuzzy subset of E i.e., $\lambda: E \to I = [0,1]$, I^U is the collection of all fuzzy subsets of U . Let $F_\lambda: E \to I \times I^U$ be a function define as $F_\lambda(e) = (F(e), \lambda(e))$, where $F(e) \in I^U$ .Then F_λ is called the fuzzy soft set over the soft universe (U,E) . Here for each parameters i.e. $F_\lambda(e_i) = (F(e_i),\lambda(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\lambda(e_i)$

III. A GENERALIZED FUZZY MATRICES BASED DECISION MAKING PROBLEM

Let more than one decision-maker wish to take a decision or jointly select an object from m objects having n features, i.e., parameters E.

Suppose that each decision-maker has the freedom to choose and evaluate the parameters associated with the objects. They may select the same set of parameters or different subsets of parameters from E, depending on their individual preferences and viewpoints.

It is assumed that the evaluation of parameters by the decision-makers is expressed in terms of generalized fuzzy values. These evaluations may be represented in linguistic form, as generalized fuzzy soft sets, or in the form of generalized fuzzy soft matrices.

The objective of the decision-makers is to jointly identify the most suitable object among the m objects as effectively as possible.

Definition 3.1 (Weight of an Object)

Let $[c_{ij}, \gamma_j] = A' \cap' B' \in$ where A' and B' are two generalized fuzzy soft matrices. Then the set $W(u_i) = \{ u_i \in U / \sum c_{ij} \}$ is called weight for each $u_i \in U$.

Procedure of solving the problem:

A practical problem can be solved using various mathematical techniques. The decision-maker may choose the most suitable and simple method from the available alternatives. Once the approach is selected, the problem can be addressed using the principles of generalized fuzzy soft matrices by following the algorithm given below.

Algorithm:

Step 1:

Construct the generalized fuzzy soft matrices based on the individual choice of parameters of the decision-makers.

Step 2:

Compute the union or intersection of the generalized fuzzy soft matrices.

Step 3:

Calculate the weight of each object W(ui) by summing the membership values of all entries in the i-th row of the resulting matrix.

Step 4:

Select the object with the highest weight as the optimal choice.

Case Study: Medical Diagnosis Using Generalized Fuzzy Soft Matrices

In medical science, a patient suffering from a disease may exhibit multiple symptoms. It is also observed that some symptoms are common to more than one disease, which leads to difficulty in accurate diagnosis. In such situations, especially when a new disease spreads widely in a region, doctors may face challenges in identifying the disease based on overlapping symptoms. Therefore, diagnosis is often carried out by analyzing the common symptoms observed in patients.

Let $U = \{CP_1,CP_2,CP_3,CP_4\}$ be the set of patient and $S=\{s_1,s_2,s_3,s_4,s_5,s_6,s_8,s_9\}$ parameters of symptoms of COVID 19 where

- S1=High fever (104 F, 40°C)
- S2=Muscle or body aches(tiredness)

- S3= New loss of taste or smell
- S4= Headache
- S5=Cough
- S6=Congestion or runny nose
- S7=Fatigue
- S8=Sore throat

Suppose two decision-makers, denoted by DM_1 and DM_2, evaluate the patients based on the same set of parameters.

Let $\lambda: E \rightarrow [0, 1]$ be a function representing the degree of importance or weight assigned to each parameter. The decision-makers evaluate the symptoms and assign appropriate fuzzy values based on their observations. These evaluations are then represented in the form of generalized fuzzy soft matrices.

Step 1 : Parameter Weights Assigned by Decision Makers

parameter	DM_1	DM_2
$\lambda(s_1)$	0.7	0.8
$\lambda(s_2)$	0.6	0.5
$\lambda(s_3)$	0.8	0.7
$\lambda(s_4)$	0.4	0.3
$\lambda(s_5)$	0.3	0.2
$\lambda(s_6)$	0.6	0.3
$\lambda(s_7)$	0.5	0.4
$\lambda(s_8)$	0.2	0.6

As shown in Figure 1 illustrates the variation in membership values assigned by the decision-makers $DM_1$and DM_2 for different COVID-19 symptoms. It can be observed that both decision-makers show similar trends for certain symptoms, while variations exist in others, reflecting subjective evaluation.

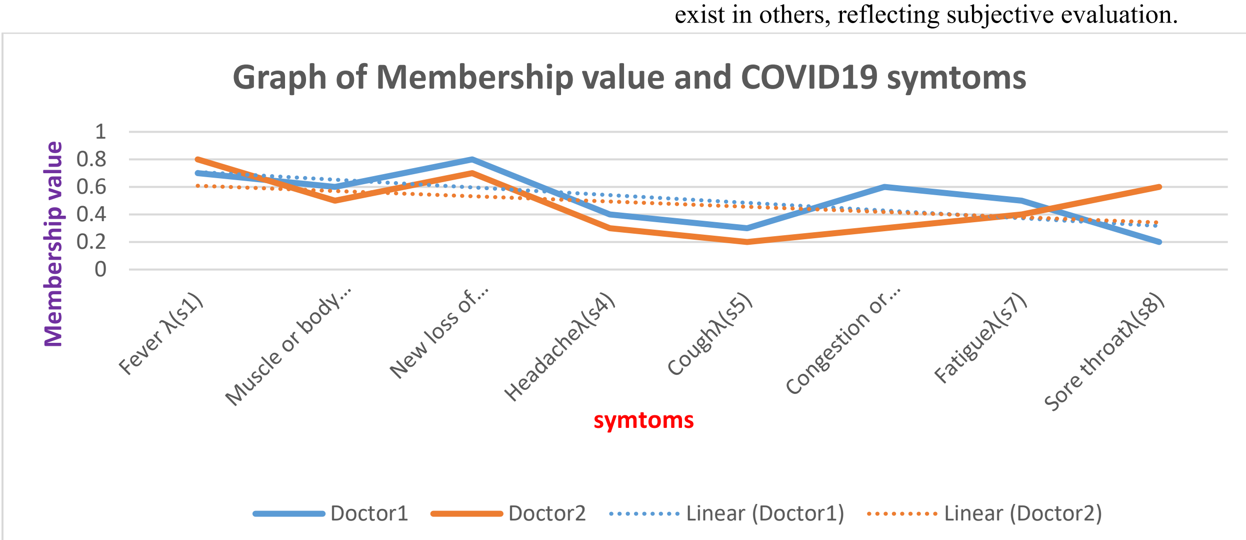

Figure 1: Graph of Membership Values for COVID-19 Symptoms

Generalized Fuzzy soft set Matrices

Matrix A′ (from DM_1)

$$
\begin{array}{cc}
 & \begin{array}{cccc} CP_1 & CP_2 & CP_3 & CP_4 \end{array} \\
A' = & \begin{pmatrix}
(0.6,0.7) & (0.5,0.7) & (0.3,0.7) & (0.2,0.7) \\
(0.4,0.6) & (0.2,0.6) & (0.2,0.6) & (0.4,0.6) \\
(0.7,0.8) & (0.3,0.8) & (0.4,0.8) & (0.1,0.8) \\
(0.3,0.4) & (0.2,0.4) & (0.4,0.4) & (0.2,0.4) \\
(0.2,0.3) & (0.1,0.3) & (0.3,0.3) & (0.2,0.3) \\
(0.6,0.6) & (0.2,0.6) & (0.4,0.6) & (0.3,0.6) \\
(0.7,0.5) & (0.4,0.5) & (0.4,0.5) & (0.2,0.5) \\
(0.6,0.2) & (0.3,0.2) & (0.1,0.2) & (0.3,0.2)
\end{pmatrix}
\end{array}
$$

Matrix B′ (from DM_2)

$$
\begin{array}{cc}
 & \begin{array}{cccc} CP_1 & CP_2 & CP_3 & CP_4 \end{array} \\
B' = & \begin{pmatrix}
(0.6,0.8) & (0.5,0.8) & (0.3,0.8) & (0.2,0.8) \\
(0.4,0.5) & (0.2,0.5) & (0.2,0.5) & (0.4,0.5) \\
(0.7,0.7) & (0.3,0.7) & (0.4,0.7) & (0.1,0.7) \\
(0.3,0.3) & (0.2,0.3) & (0.4,0.3) & (0.2,0.3) \\
(0.2,0.2) & (0.1,0.2) & (0.3,0.2) & (0.2,0.2) \\
(0.6,0.3) & (0.1,0.3) & (0.3,0.3) & (0.3,0.3) \\
(0.6,0.4) & (0.4,0.4) & (0.3,0.4) & (0.2,0.4) \\
(0.6,0.6) & (0.3,0.6) & (0.1,0.6) & (0.4,0.6)
\end{pmatrix}
\end{array}
$$

Step 2: Intersection of Matrices A′∩B′

$$A'\cap' B' = \begin{array}{c} \begin{array}{cccc} CP_1 & CP_2 & CP_3 & CP_4 \end{array} \\ \begin{pmatrix} (0.7,0.8) & (0.5,0.8) & (0.3,0.8) & (0.2,0.8) \\ (0.4,0.6) & (0.3,0.6) & (0.2,0.6) & (0.4,0.6) \\ (0.7,0.8) & (0.4,0.8) & (0.4,0.8) & (0.1,0.8) \\ (0.5,0.4) & (0.2,0.4) & (0.5,0.4) & (0.2,0.4) \\ (0.6,0.3) & (0.2,0.3) & (0.3,0.3) & (0.2,0.3) \\ (0.6,0.6) & (0.2,0.6) & (0.4,0.6) & (0.3,0.6) \\ (0.7,0.5) & (0.4,0.5) & (0.4,0.5) & (0.2,0.5) \\ (0.8,0.6) & (0.3,0.6) & (0.1,0.6) & (0.4,0.6) \end{pmatrix} \end{array}$$

Step 3: Weight Calculation

The weight of each patient W(ui) is obtained by summing the membership values of all elements in the i-th row of the resultant generalized fuzzy soft matrix.

$W(CP_1) =$ 0.7+ 0.4+0.7+0.5+0.6+0.6+0.7+0.8 =5.0

$W(CP_2) =$ 0.5+0.3+0.4+0.2+0.2+0.2+0.4+0.3 =2.5

$W(CP_3) =$ 0.3+0.2+0.4+0.5+0.3+0.4+0.4+0.1 =2.6

$W(CP_4) =$ 0.2+0.4+0.1+0.2+0.2+0.3+0.2+0.4 =2.0

From Figure 2, it is observed that patient CP_1 has the highest weight (41%), indicating the highest severity among the patients. Patients CP_2 and CP_3 show moderate values (21%), while CP_4 has the lowest weight (17%). This confirms that CP_1 is the most affected patient.

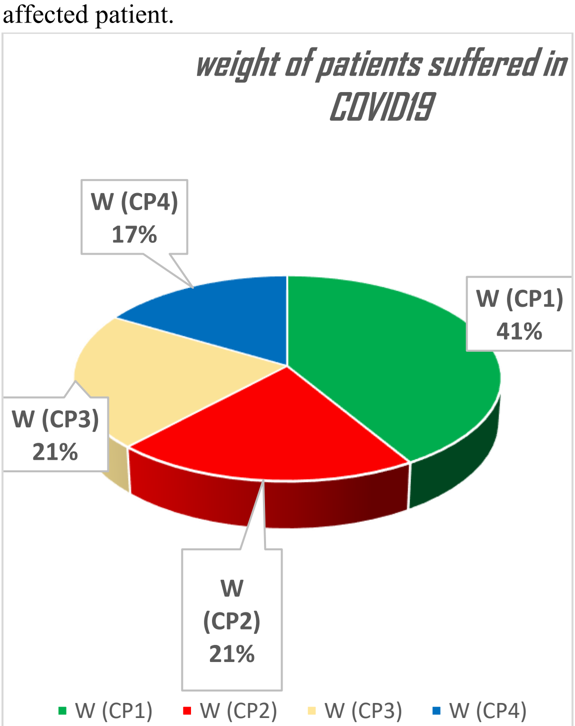

Figure 2: Weight distribution of patients affected by COVID-19

Step4: Selection of the Optimal Patient

The patient with the highest weight is identified as the optimal choice. The maximum weight, 5.0, is obtained for patient CP_1, indicating that CP_1 is the most affected COVID-19 patient.

IV. CONCLUSION

This case study demonstrates the simplicity and effectiveness of the proposed approach. The concept of fuzzy soft matrices has been successfully applied in the field of medical diagnosis to handle uncertainty and imprecise information. In this work, generalized fuzzy soft matrices have been introduced, and their properties have been discussed. A weighting method based on generalized fuzzy soft sets has also been defined to support decision-making.

The application of the proposed method to COVID-19 diagnosis shows that the patient with the highest weight can be identified effectively. The results indicate that patient CP_1 has the maximum weight, highlighting the severity of symptoms.

This approach can be used to improve awareness and assist in identifying affected patients based on symptom analysis. Furthermore, the method can be extended to other real-world problems involving uncertainty. Future work may focus on integrating real-time data and advanced fuzzy models to enhance the accuracy of medical decision-making.

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