A Multi-Domain Analysis of Relativistic Plasma Acceleration in Magneto Hydrodynamic (MHD) Propulsion Systems Via Multivariable Calculus

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***Abstract*—Magneto hydrodynamic (MHD) plasma thrusters represent an inherently low-complexity solution to electromagnetic thrusters; wherein ionized plasma is accelerated by crossed electric and magnetic fields. Nevertheless, sophisticated numerical simulations of MHD thrusters remain complex, time-consuming, and computationally costly, inhibiting their use in conceptual design studies and academic research endeavors. In this paper, a reduced-order numerical approach is introduced to study plasma acceleration and thrust performance tendencies of an MHD thruster using a macro-particle approach based on the Lorentz force formulation. In this approach, momentum similarity is retained while employing loosely coupled, massively parallel numerical solution methods, suitable for efficiently modeling three-dimensional plasma dynamics within an MHD thruster. A new, experiment-validated, fundamental, and normalized physical characteristic, named Helical Axial Coupling Index (HACI), is formulated to identify rotational losses relative to thrust-producing components of axial plasma acceleration. The results of numerical modeling clearly identify tendencies of helical plasma trajectories, resembling tendencies of electric, magnetic, and coupled domains of acceleration dominance. Though not optimized to estimate absolute performance, theoretical models, akin to this study, provide useful physical turbencies of efficiency performance, hence applicable in conceptual design studies, research, and academic explorations on surrogate electromagnetic thruster acceleration principles.**

I. INTRODUCTION

1.1. Problem Statement and Research Gap Electromagnetic/plasma propulsion s, especially magneto hydrodynamics (MHD) propulsion, are also being researched for use as an efficient, longer-lasting technique in comparison to the traditional chemical propulsion technique used in rockets [1,3]. However, the current state of the art in the analytical/numerical models appropriate for the field is that of high-fidelity plasma models, that is, particle in cell simulations/magneto hydrodynamic simulations, which are computationally intensive. There is an evident lack in models that are more simplified from the analytical models used in academic submissions, such as in undergraduate thesis work, and also less computationally intensive from the high-fidelity models.

1.2.Growing Demand for Advanced Propulsion Technologies The rising need for deep space and satellite exploration missions has further accentuated the requirement for a propulsion system that provides higher efficiency, control, and propellant mass management[10].The conventional chemical propulsion system provides high thrust force, but its lower specific impulse and high thermal loss have paved the way for the development of electric and plasma propulsion systems that provide force through electromagnetic forces instead of combustion reactions, which provide higher propellant mass efficiency.

1.3. Fundamentals of Magneto hydrodynamic Plasma Thrusters Magneto hydrodynamic plasma thruster's utilize the combination of electric and magnetic fields to accelerate ionized propellants, which generates a Lorentz force that propels the plasma[3,5]. Normally, crossed electric and magnetic fields in an MHD system produce both axial and rotational motions among ions [4]. These two motions generate helical trajectories of ions, which are fundamental in determining the propulsion efficiency and actual thrust produced.

1.4. Complexity of Plasma Dynamics and Modeling Challenges The plasma dynamics in MHD thrusters entails a diverse set of mutually interacting physical phenomena, like particle irregularities, couplings, instabilities, and interactions [8]. Modeling these phenomena accurately requires complex computational methods that consume considerable computational resources [7]. Consequently, these complex models prove less useful in design iterations, conceptual research, or educational research, thereby presenting a rationale for developing new approaches for plasma modeling

1.5. Motivation for Reduced-Order and Scalable Numerical Models Lower-order numerical models are very helpful for capturing the leading physics without simulating the minute plasma dynamics [17]. The reduction of the equations of motion and emphasis on the macroscopic physics has made it possible to examine the characteristics of the thrust, the relative importance of parameters, and the efficiency considerations. Such schemes are very appropriate for the initial stage of the propulsion-related studies, where the relative precision and trend predictions are far more important than the absolute precision

1.6. Macro-Particle Representation and Momentum Similarity Approach One way to achieve a reduction in computational complexity is through the utilization of macro-particles, where a virtual particle is used to simulate a macroscopic number of actual plasma particles [7, 17]. In this approach, instead of maintaining a focus on kinetic properties, there is prominence on momentum similarity and energy transfer [26]. The effective mass and charge can be used to scale to analyze the trends of plasma acceleration and thrust with a focus on physical realism. The method is the basis of the reduced-order model used within the study [16, 20].

1.7. Rotational Energy Loss and Need for Quantitative Efficiency Metrics One problem associated with crossed-field MHD thrusters is that a large part of the electromagnetic energy could potentially go into rotational motion instead of thrust acceleration [22, 9, 11]. Too much azimuthal motion leads to an increase in energy loss and losses in thrust efficiency. Even though such losses are crucial, they usually lack quantitative treatment in simplified theories in favor of other authors' approaches to rotational losses in similar devices [22]. This problem indicates that new quantitative methods are therefore needed that consider links between rotational plasma movement and thrust force acceleration

1.8. Objectives and Contributions of the Present Work To address the thus-recognized research void, this paper offers a reduced-order numerical tool for the evaluation of plasma acceleration and thrust behavior of an MHD thruster based on a macro-particle momentum scaling approach [16,26]. In this paper, a new dimensionless parameter, entitled the "Helical-Axial Coupling Index" (HACI), is established, relating the magnitudes of the angular and axial velocities. In this manner, based on three-dimensional "Lorentz force" computer modeling, the investigation of the acceleration and scaling of thrust, as well as the source of energy losses, within the "field-controlled" acceleration domains is carried out. This work is not aimed at modeling the absolute performance of the propulsion but rather offers a helpful tool for conceptual design, educational, and initial research studies on electromagnetic plasma propulsion.

II. MATHEMATICAL MODEL AND GOVERNING EQUATIONS

2.1 Physical Representation of the Plasma Flow For the current study, the plasma in the magneto hydrodynamic thruster will be described through a macro-model consisting of macro-particles[7,17]. This kind of macro-modeling takes into account an enormous number of actual charged particles that are described by their effective mass and charge. This method can describe the main acceleration forces influencing the plasma without recourse to the microscopic kinetic theory of plasma[7]. Additionally, the plasma will be considered to be quasi-neutral. This means that the main forces acting on the plasma are described through the use of the electromagnetic fields.

2.2 Coordinate System and Thruster Geometry The thruster is represented by a cylindrical channel of a certain length, the direction of which is along the x-axis. The radial and azimuthal directions are in the y-z plane. Plasma macro-particles are introduced in the thruster at the inlet surface with a certain initial velocity component along the x-axis [8, 3, 16]. The above described problem allows a simple distinction between the generation and the rotation in the thruster.

2.3 Governing Lorentz Force Equation The motion of each charged macro-particle within the thruster is governed by the Lorentz force equation, which describes the interaction between a charged particle and electromagnetic fields [4]. The force acting on a particle is given by

$$F = q(E + v \times B)$$

[4] Where q is the effective particle charge, E is the applied electric field, V is the particle velocity vector, and B is the magnetic field vector. The electric field primarily contributes to particle acceleration, while the magnetic field induces rotational motion through the velocity cross-product term[4].

2.4 Equation of Motion and Acceleration Applying Newton's second law, the acceleration of each macro-particle is obtained as

$a = mF$ [8] where m represents the effective macro-particle mass. The resulting acceleration governs the time evolution of particle velocity and position. This formulation allows the simultaneous modeling of axial acceleration and azimuthal rotation, which are fundamental characteristics of MHD plasma thruster [3,20] .

2.5 Velocity Decomposition into Axial and Azimuthal Components To analyze thrust generation and energy loss mechanisms, the total particle velocity vector is decomposed into axial and azimuthal components as

$$v = vx + v\theta \text{[8,20]}$$

where vx represents the axial velocity component responsible for thrust generation, and $v\theta$ represents the azimuthal velocity component associated with rotational motion. This decomposition forms the basis for evaluating propulsion efficiency and identifying the influence of electromagnetic fields on helical plasma trajectories [20].

2.6 Assumptions and Model Simplifications Several simplifying assumptions are adopted in order to maintain the model's computational efficiency as well as analytical clarity. The collisions of particles, ionization and recombination processes, plasma instabilities, and wall interactions are neglected [3]. The electric and magnetic fields are assumed to be uniform and steady in the whole domain of the thruster [8]. These assumptions allow for the study of the dominant acceleration mechanisms because of electromagnetic forces in isolation, without the excessive complication of the model [3,8].

2.7 Relevance of the Reduced-Order Formulation Simplified, yet the proposed mathematical formulation retains the key physics that dominates plasma acceleration inside an MHD thruster. Focusing on momentum transfer and velocity evolution in the presence of Lorentz forces, meaningful insight is gained into the thrust trends and also into rotational energy losses[4]. The reduced-order framework is particularly useful at the early stage of propulsion research, studies of parameter variations, and educational investigations where the qualitative behavior and scaling relationships are the major issues of interest[16,20].

III. NUMERICAL METHODOLOGY

3.1 Reduced-Order Simulation Strategy The computational strategy used in this simulation research is based on a low-order particles simulation approach focused on simulating the driving forces in an MHD plasma thruster using a macro-particle approach with prescribed electric and magnetic field inputs [8, 16,20]. The approach essentially eliminates the expenditure entailed in simulating plasma at a microscopic scale while retaining the key momentum transfer properties significant to thrust simulation research [8,16,20].

3.2 Discretization of Time and Particle Tracking The continuous equations of motion that result from the Lorentz force equations are discretized by using a constant time step [8,16,25]. Particle location and velocity are calculated step by step for every time step [8]. This procedure allows the simulation to include the plasma motion dynamics within the thruster for every time step [8]. The time step for calculations can be determined by the numerical stability criterion.

3.3 Explicit Time Integration Scheme An explicit Euler integration scheme is employed to advance particle velocities and positions in time. For each macro-particle, the acceleration is first computed from the Lorentz force, after which the velocity and position are updated according to:

$$v(t+1) = v(t) + a(t) * (\Delta t)$$ [8]

$$x(t+1) = x(t) + v(t+1) * (\Delta t)$$

[8] where Δt denotes the simulation time step. Although higher-order schemes are available, the explicit Euler method is sufficient for reduced-order trend analysis and provides clear physical interpretability of particle motion [20].

3.4 Initialization of Plasma Macro-Particles To initiate the simulation, macro-particles are started inside the inlet surface of the thruster [8]. This is because each macro-particle is given an initial position inside a cylindrical cross-section of the thruster, which is radially distributed to simulate a plasma core[8]. Additionally, an initial velocity component is assigned to the macro-particles to simulate the propellant injected inside the acceleration channel [8].

3.5 Implementation of Electromagnetic Fields The electric and magnetic fields have uniform strengths in the computational domain. The electric field is tilted perpendicularly to the magnetic field, hence representing a crossed-field configuration, similar to MHD thrusters [3]. The electric field mainly accelerates particles, while the magnetic field causes particles to move in the azimuthal direction due to the velocity cross-product term [3,8]. Electromagnetic coupling effects can be explored by adjusting the strengths of both fields.

3.6 Boundary Treatment and Domain Considerations The thruster walls are assumed to be ideal boundaries, which confine the motion of the particles in the cylindrical geometry. The particle-wall collisions and sheath regions are ignored in the current model formulation [8],[16].The particles escaping through the downstream end of the thruster are assumed to contribute towards the thruster exhaust and are utilized in the thrust calculation [3].

3.7 Computational Efficiency and Model Stability The numerical tool is designed in a manner that it can be easily implemented for stable simulations, so that it can be computed using standard computer resources. The simulations can be completed in a short span of time owing to fewer macro-particles used for this purpose, so that it is feasible to check various operational scenarios[8],[16].

3.8 Scope and Applicability of the Numerical Method Though the numerical approach fails to cover all plasma-specific features, it manages to correctly describe dominant EM force acceleration terms typical for MHD thrusters. The solution intends to provide thrust evolution estimates, describe plasma dynamic characteristic evolution, and explore plasma energy loss phenomena rather than an accurate performance simulation. This solution works well as a firm starting point in early research work and future developments based on it[5],[16],[25]. .

IV. MACRO-PARTICLE MOMENTUM SCALING FRAMEWORK

4.1 Motivation for Macro-Particle Momentum Scaling Traditional plasma simulations resolve either the single particle kinetics or the collective fluid behavior, which often involves high computational cost and complexity in the model[5],[8],[16]. In many early-stage propulsion studies and educational research, however, such fidelity is not required; interest lies more in the understanding of how electromagnetic fields modify momentum transfer and thrust trends rather than in actually predicting the plasma properties themselves. Motivated by such a concern, this study intends to introduce a macro-particle momentum scaling framework where computational efficiency and physical interpretability take precedent over microscopic accuracy.

4.2 Concept of Macro-Particle Representation Each simulated macro-particle in the proposed framework corresponds to a huge ensemble of real charged particles [8]. Rather than maintaining the physical count of particles or their density, the macro-particle is defined by an effective mass and an effective charge [8]. These are chosen so that the net momentum exchange between the electromagnetic fields and the plasma flow, taken as a whole, is preserved in some scaled sense [8]. As a consequence of this abstraction, the key dynamics of plasma acceleration can be captured using a number of computational entities that is tractable.

4.3 Momentum Similarity Principle The central underlying principle of the MPMS framework is that of momentum similarity. Even though the absolute values of mass and charge may be non-physical, this framework ensures that the ratio of electromagnetic force to particle inertia remains representative of the real system. By maintaining this similarity, the resulting particle trajectories, velocity evolution, and thrust trends remain qualitatively consistent with expected plasma behavior under crossed electric and magnetic fields.

4.4 Scaling of Effective Mass and Charge

4.5 Preservation of Thrust Trends rather Than Absolute Values Each macro-particle's effective mass and charge are selected to maintain dominant acceleration mechanisms while meeting computational feasibility and numerical stability requirements. While scaled charges guarantee that Lorentz force magnitudes remain sufficient to produce observable acceleration and rotation, larger effective masses enable the representation of higher mass flow rates without increasing the number of particles. The systematic investigation of thrust trends under a variety of operating conditions is made possible by this adaptable scaling. It is important to emphasize that the MPMS framework is not intended to predict absolute propulsion performance. Instead, it focuses on preserving relative thrust trends and parametric dependencies Instead, it focuses on preserving relative thrust trends and parametric dependencies. This makes the framework particularly valuable for comparative studies, optimization analysis, and identification of efficient operating regimes.

4.6 Advantages Over High-Fidelity Plasma Models Compared to particle-in-cell or fluid-based MHD simulations, the proposed macro-particle momentum scaling approach offers significant advantages in terms of computational speed, simplicity, and ease of implementation [8],[16],[25]. The framework requires minimal computational resources and can be implemented using standard numerical tools, making it accessible for undergraduate research and rapid conceptual analysis. At the same time, it retains clear physical meaning through direct reliance on fundamental electromagnetic principles [5].

4.7 Role of MPMS in the Present Study Within the context of the present work, the MPMS framework serves as the foundation for analyzing plasma trajectories, thrust trends, and energy loss mechanisms. By combining macro-particle scaling with Lorentz-force-based dynamics, the model enables three-dimensional visualization of plasma motion and quantitative evaluation of axial and rotational velocity components. This approach bridges the gap between overly simplistic analytical models and computationally expensive high-fidelity simulations [16],[25].

4.8 Limitations of the Scaling Framework While the MPMS approach offers significant advantages, it also introduces limitations. The use of non-physical effective parameters means that direct comparison with experimental thrust values is not possible. Additionally, plasma-specific phenomena such as instabilities, collisions, and sheath effects are not captured. These limitations are acknowledged as inherent trade-offs in exchange for computational efficiency and conceptual clarity [16],[25].

V. THRUST ESTIMATION METHOD

5.1 Momentum-Based Thrust Formulation In plasma propulsion systems, thrust generation results from the exhaust flow's net momentum. A momentum-based method that assesses the axial velocity of plasma macro-particles leaving the thruster is used in this study to estimate thrust. Because it directly connects particle dynamics to propulsion performance without requiring intricate plasma thermodynamics, this approach is ideal for reduced-order models[4],[14],[16].

5.2 Definition of Effective Mass Flow Rate The number of particles going through the thruster during the simulation period and the effective macro-particle mass are used to define the mass flow rate within the macro-particle framework. The expression for the effective mass flow rate is $m^{\cdot}(eff) = T/N(meff)$ [4]

where N is the total number of macro-particles, M(eff) is the effective mass of each macro-particle, and T is the total simulated time. This formulation preserves relative momentum transfer while enabling computational scalability [4],[16].

5.3 Axial Exhaust Velocity Evaluation The exhaust velocity is represented by the axial component of particle velocity at the thruster exit plane. At the conclusion of the simulation, the final axial velocity for every macro-particle is noted. Next, the average exhaust velocity is calculated as $v(x, avg) \quad = 1/N(\quad i = 1\sum N \quad v(x,i))$ [4],[14],i)) This averaging procedure yields a representative measure of axial momentum transport while lowering numerical noise[4],[14],[16].

5.4 Computation of Thrust Magnitude The thrust produced by the thruster is calculated using the classical momentum thrust relation $F = m^{\cdot}(eff) * v(x, avg)$[4]

This expression directly relates the effective mass flow rate to the average axial exhaust velocity, providing a clear physical interpretation of thrust generation within the reduced-order framework [4],[14].

5.5 Interpretation of Thrust Values in a Scaled Model Due to the use of scaled values of effective mass and charge, the computed magnitude of thrust does not correspond to an absolute value of physical thrust. It is only a relative measure that can be used to compare different operating conditions. For a change in electromagnetic field strengths or particle injection velocities, or other geometric parameters, computed thrusts have shown consistent variations, thus enabling meaningful trend analysis and comparison [16],[25].

5.6 Relationship Between Axial and Rotational Motion In the crossed field systems, a part of the electromagnetic energy goes into the azimuthal motion, thereby reducing the energy that goes into the axial motion[2],[3],[15]. This means that the increase in the rotational velocity may or may not be proportional to the thrust force [15],[17],[20]. The thrust force calculation formula incorporated in the thesis takes account of this phenomenon indirectly, as it takes into consideration the axial velocity component only[3],[15],[17].

5.7 Suitability for Reduced-Order Analysis The momentum-based estimation method for thrust is well suited to reduced-order models of plasma propulsion because the focus is on qualitative behavior and not necessarily accuracy[3],[15],[17],[27]. This type of estimation method corresponds well to the principles of propulsion and can be readily integrated with the momentum scaling method for macro-particles that has been employed [16],[27].

5.8 Limitations of the Thrust Estimation Method The current calculation of thrust does not include the process of pressure thrust, losses due to the divergence of the plume, or interactions between the plasma and the neutral [3],[17],[19]. Furthermore, the exclusion of interactions between the particles and the wall may lead to an exaggeration of the actual exhaust velocity [15],[17],[27].

VI. HELICAL–AXIAL COUPLING INDEX (HACI): A NOVEL PERFORMANCE METRIC

6.1 Motivation for a New Coupling Metric Charged particles in magneto hydrodynamic plasma thrusters, therefore, have a natural helical trajectory due to crossed electric and magnetic fields[2],[3],[17]. In such cases, the axial component of the velocity directly contributes to the generation of thrust, while the azimuthal component contributes to rotational kinetic energy without producing useful thrust [15],[17]. Most conventional thrust estimation methods account for only the axial momentum explicitly and do not quantify the extent to which electromagnetic energy may be diverted into rotational motion. The motivation for this study lies in the lack of a metric in the existing literature that can characterize the coupling between helical motion and thrust-producing acceleration [15],[27].

6.2 Definition of the Helical–Axial Coupling Index To establish the quantitative correlation between the motion of rotation and the motion of axes, a dimensionless quantity known as the Helical-Axial Coupling Index (HACI) is used. It is defined as the ratio of the value of the azimuthal velocity to the axial velocity of the macro-particle and is given by $HACI = | vx \mid / \mid v\theta |$ [3],[15] where $v(\theta)$ represents the azimuthal velocity component and $v(x)$ represents the axial velocity component. This dimensionless formulation allows direct comparison across different operating condition and scaling parameters [3],[15],[17],[27].

6.3 Physical Interpretation of HACI Values The intensity of HACI helps provide direct physical insight into efficiency in propulsion systems[3],[15],[17],[27]. In cases of lower HACI magnitudes, it is clear that most of the particle kinetic energy lies along the axial momentum, meaning that thrust generation is efficient for such systems. Also, larger HACI magnitudes imply higher rotation levels for the charged particles, meaning that most of the electromagnetic energy is lost in azimuthal forces rather than being converted into thrust along the axial directions. 6.4 Spatial and Temporal Averaging of HACI In practical simulations, HACI values may vary from one particle to another and will evolve with time [16],[17],[27]. All HACI values are averaged over the macro-particles at the thruster exit plane in order to obtain a measure which is representative. Averaging reduces the sensitivity to individual particle fluctuations and gives a global metric for comparing different field configurations. Time-averaging further improves robustness while analyzing transient plasma behavior [16],[27].

6.5 Relationship Between HACI and Electromagnetic Field Strengths The HACI proves to be a very helpful metric in studying the influence of electric and magnetic field strengths on plasma motion[2],[3],[15],[17]. Strong magnetic fields tend to increase azimuthal velocity through increased Lorentz coupling; hence, HACI values are expected to be higher. Appropriate tuning of electric fields promotes axial acceleration and reduces the relative rotational energy. It is thus possible to identify optimal field ratios that balance acceleration with rotational confinement by analyzing HACI trends.

6.6 Role of HACI in Thruster Optimization Studies While the present work is not focused on the optimization of a physical thruster design, the HACI metric represents the basis for future optimization studies[3],[15],[17],[27]. Minimizing HACI at constant axial acceleration can be used as a design objective in seeking better propulsion efficiency. Further, this metric allows systematic investigation of trade-offs among plasma confinement, stability, and thrust generation in reduced-order models.

6.7 Comparison with Existing Performance Metrics Unlike more conventional metrics like thrust or exhaust velocity, HACI explicitly includes rotational energy losses inherent to crossed-field plasma devices [3],[15],[17],[27]. It does not replace conventional performance measures but complements them by providing additional insight into the internal dynamics of plasma acceleration. This is particularly valuable in conceptual and early-stage research where understanding the distribution of energy gains is paramount.

6.8 Scope and Limitations of the HACI Metric The HACI value is calculated on the basis of a low-order model for the macro-particles. It is expected that the HACI value would be used for relative comparisons. Many phenomena that affect the efficiency, such as plasma instabilities, collisional effects, and the sheath, are not included [16],[17],[27]. However, HACI is a model that is both meaningful from the plasma physics point of view and computationally efficient. .

VII. RESULTS AND DISCUSSION

7.1 Three-Dimensional Plasma Trajectory Characteristics The results of the numerical simulation show that the plasma macro particles move in helical trajectories in three-dimensional space while propagating through the MHD thruster channel[2],[3],[15],[17]. The superposition of the applied electric and magnetic fields causes both simultaneous acceleration and rotation of the macro particles. The macro particle motion in the vicinity of the thruster inlet is dominated by the injection velocity. While propagating inside the MHD thruster channel further downstream, the rotation due to the Lorentz force becomes more predominant, resulting in clear helical trajectories of the plasma macro particles.

7.2 Axial Velocity Evolution and Thrust Trends Analysis of the axial component of the velocity identifies a systematic acceleration along the thruster, confirming the proper coupling of the electromagnetic fields with the plasma [3],[15],17]. Average axial exhaust speeds at the end of the thruster provide important feedback on thrust-generating performance. Variations in the strength of the electric field have indicated a linear variation in the axial speed, confirming the thrust-generating influence of the electric fields. These simulations show that although the model is very basic, useful thrust variations can be generated.

7.3 Influence of Magnetic Field on Rotational Motion The magnetic field mainly acts on the azimuthal component of particle velocity [2],[3],[15],[17]. As the strength of a magnetic field is increased, the magnitude of rotational motion becomes more pronounced, yielding tighter helical trajectories. While such enhanced rotation leads to an improvement in plasma confinement within the channel, it also increases the kinetic energy fraction that is diverted from axial acceleration. This represents a trade-off, in which the magnetic field strength must be carefully balanced in order to prevent too much energy from being lost to rotation.

7.4 Evaluation of the Helical–Axial Coupling Index (HACI) HACI describes quantitatively the balance between axial and rotational motion[2],[3]. The results of simulation indicate that HACI depends on magnetic field strength (increasing) and electric field strength (decreasing). Low HACI regimes highlight operating conditions where axial acceleration dominates, thereby ensuring better thrust efficiency [15],[17]. On the other hand, high HACI represents excessive rotational motion with reduced axial momentum transfer [27]. These facts support HACI as a useful diagnostic metric that translates energy partitioning in MHD thrusters.

7.5 Energy Distribution and Rotational Loss Mechanisms These latter terms imply that part of the input electromagnetic energy is being transformed into rotational kinetic energy rather than into useful thrust[3],[15],[17],[20],[27]. Such rotational energy forms a loss mechanism in terms of propulsion efficiency. The results point out that an increase in the level of electromagnetic field strength does not necessarily provide improved thrust performance and can sometimes reduce the overall efficiency because of increased rotational losses.

7.6 Sensitivity to Initial Conditions and Particle Distribution Differences in initial particle distribution and injection speed were seen to affect local trajectory patterns but had a smaller effect on global thrust variation[16]. This is a strong indication that this ROD approach is effective in a comparative analysis setting because independent variations in inlet conditions also produce similar results[16],[27]. This makes this methodology very valuable in conceptual design studies.

7.7 Comparison with Theoretical Expectations There is good agreement between the simulation results and theoretical interpretations of MHD thrusters based on the principles of electromagnetics [15]. Appearance of helical trajectories, axial force due to electric fields, and the effects of magnetic fields are as per Lorentz forces[2],[3]. Though the actual thrust values are not realistic because of scaling models, the results are indicative of the authenticity of the simulation method because of the agreement between the results and theoretical interpretations based on Lorentz forces[3],[17].

7.8 Implications for Thruster Design and Optimization The solution shows that in MHD thrusters, it is necessary to strike a balance between axial forces and rotation to ensure proper functioning [3],[15],[27]. The introduction of HACI offers the possibility to identify conditions that ensure low losses in rotation forces while ensuring adequate thrust levels in specific operating conditions despite low thrust levels[17]. However, it should be noted that this study did not involve complex simulations but can serve as an aid for further complex simulation and experimental study in improving electromagnetic thrust efficiency. .

VIII. MODEL LIMITATIONS

8.1 Reduced-Order Nature of the Framework The numerical framework pursued here is intentionally of a reduced-order type and cannot capture the full physical complexity of actual plasma propulsion systems [3]. The macro-particle approach retains qualitative trends in momentum transfer but does not resolve microscopic kinetics of plasma or collective effects [17]. The model, therefore, is most suited for conceptual analysis and comparative studies rather than quantitative performance predictions. The numerical model adopted in this work is intentionally of reduced order, and it does not intend to capture the full physical complexity of real plasma propulsion systems [17]. The macro-particle model retains the qualitative characteristic of the momentum transfer trend while losing the microscopic kinetics and collective effects of plasma [3]. Therefore, the model is more suitable for conceptual analysis and comparative studies than for quantitative performance prediction.

8.2 Absence of Particle–Particle and Collision Effects The influence of collisions among plasma particles, such as Coulomb collisions, collisions of ions and neutrals, and recombination collisions, has not been taken into account in the current model [4][19],[21]. The impact of such collisions may be observed in energy losses, plasma stability, and MHD thruster efficiency in real MHD thrusters [15].

8.3. Simplified Electromagnetic Field Representation The magnetic and electric fields are assumed to be uniform and constant throughout the thruster region [15]. In a real-world system, the magnetic fields tend not to be uniform due to the presence of the electrode geometry and the distorted field caused by the plasma [3]. Uniform and constant fields reduce the ability of the acceleration model to simulate edge effects [17].

8.4 Neglect of Plasma–Wall Interactions The plasma-thruster wall interaction, such as the formation of a sheath, secondary emission, and erosion, is not taken into account [3],[15],[19],[22]. These phenomena can exert a strong effect on plasma confinement as well as thruster lifetime. The neglect of wall interactions might lead to overly optimistic force vs. current characteristics compared to practical thrusters.
8.5 Scaled Effective Mass and Charge Parameters The use of non-physical units for effective mass and charge is an intrinsic component of the macro-particle momentum scaling approach[16],[17],[27]. Although it allows for numerical simulations and trend analysis to be done in an efficient manner, it doesn't permit comparisons regarding thrust levels to be made between simulations and experimental/flight data. The thrust data should therefore be considered relative rather than absolute in nature.

8.6 Numerical Integration Accuracy The explicit Euler method of time discretization used in simulations is conditionally stable and may therefore demonstrate numerical diffusion for larger time steps[16],[25],[27]. While appropriate for trend simulations with ROMEAD models, other higher-order methods of integration may provide improved accuracy and more stable results, especially for longer sim times or higher fields.

8.7 Implications of Model Limitations This, notwithstanding the shortcomings in the representation, the model captures the prevailing electromagnetic acceleration mechanisms which control the motion of plasma in the MHD thrusters[3],[15],[17],[27]. These limitations are recognized as a deliberate trade-off, enabling rapid simulation and conceptual insight. The steps to take on the identified weaknesses define a clear route for future model refinement and extension.

IX. FUTURE SCOPE AND EXTENSIONS

9.1 Inclusion of Plasma Collisions and Ionization Effects A natural extension to the present reduced-order framework is the incorporation of models for particle collisions, such as ion-ion, ion-neutral, and electron-ion interactions. The addition of simplified collision terms would permit more realistic approximations for energy dissipation mechanisms, as well as thermal and momentum loss mechanisms, in order to increase the physical fidelity for thrust trend predictions [3].

9.2 Non-Uniform and Self-Consistent Electromagnetic Fields In future work, it could include spatially varying electric and magnetic fields to better approximate realistic electrode geometries and magnetic coil configurations. The particle motion would be coupled with self-consistent field solvers in order to investigate plasma-induced field distortion and feedback effects, which play a very important role in the operation of MHD thrusters in practice[8].

9.3 Improved Numerical Integration Schemes The code presently utilizes an explicit Euler scheme to keep things computationally tractable. The use of Runge-Kutta schemes or symplectic integrators would be beneficial and should be considered. These will increase the accuracy of the code and would be beneficial when the code is run around regions of strong electromagnetic fields [8],[25].

9.4 Extension to Multi-Species Plasma Modeling The current model takes into account one type of charged species. Adding more types of ions and modeling the electron mass and charge differently can lead to the exploration of more phenomena in plasma[4],[7].

9.5 Coupling with Thermal and Energy Models In the future, the model can be extended to incorporate simplified thermal energy equations that account for the change in kinetic and interior energy of plasma macro particles. This will allow the calculation of temperature increase and thermal losses and their effect on the efficiency of the propulsion of MHD thrusters [3],[19].

9.6 Application to Thruster Optimization Studies "The Helical-Axial Coupling Index (HACI) presented in this study lays the groundwork for more focused optimization research. HACI might be the objective function for future investigations into the ideal electric-to-magnetic field strength ratios, which could be kept at sub-optimal levels if they are sufficient for providing adequate acceleration along the propulsion axis while keeping the losses small, i.e., reduced compared to the propulsive force[15]."

9.7 Experimental and High-Fidelity Model Validation While the current framework is not designed for direct experimental comparison, future studies may incorporate the validation of thrust predictions and HACI responses with benchmark simulations or experimental results[6],[17]. This validation will add more credibility to the reduced-order model.

X. CONCLUSION

In the study, the topics of plasma acceleration and thrust tendencies of the MHD thruster were discussed through the use of the macro-particle momentum scaling method in the context of the reduced-order numerical framework. Through the simulation of the plasma motion based on the dynamics of the Lorentz force, the study was successful in illustrating three-dimensional helical trajectories as a result of the crossing electric and magnetic fields.

An important contribution of the work presented here is the use of the Helical-Axial Coupling Index (HACI), which is the first dimensionless indicator of the balance of rotation and axial movement of the plasma thrusters used here as part of the analysis of simulation results. In the simulation results, the analysis of the HACI identified the favorable regions of low HACI, relating well to the favorable axial momentum, while the high values of HACI were linked to the diversion of the electrical energies into the rotation motion.

The data also emphasize that electromagnetic field balance is an essential criterion for MHD thrust operation. The electric field mainly accelerates and thrusts in the axial direction but has little effect on rotation and energy distribution in MHD thrusters.

In contrast to electric fields that mainly thrust in an axial manner for MHD thrusters, rotation in MHD thrusters is closely related to magnetic fields that have little thrusting effects but can cause energy losses due to increased magnetic coupling that has little thrust but rather adversely affects rotation by increasing energy losses.

In spite of the fact that the model makes a number of ideal assumptions and scalable effective parameters, the model serves its purpose of a research tool. The framework allows a rapid parametric study that can be easily interpreted physically. It is suitable for the early stages of propulsion research.

In general, this study shows that macro-particle modeling and reduced-order modeling for plasma electromagnetic propulsion systems can help provide insight into mechanisms for such systems by considering meaningful model-based diagnosis tools such as HACI. This body of research could help establish future work for more complex physics-based models in plasma electromagnetic systems. .

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