Plane Symmetric Magnetized String Cosmological Model in Lyra Manifold

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***Abstract*—This research investigates the plane-symmetric, magnetized string cosmological model within the framework of the Lyra manifold. We assume the constraint $A = B^n$, where n is constant, between the metric potential and average scale factor, assumed to be $a = e^{\alpha t} t^{\beta}$ describing the hybrid character of the scale factor, in order to obtain a deterministic solution of the model. Accurate solutions of the field equation are found. Additionally, certain major geometrical and physical aspects of this model have been examined.**

***Index Terms*—Cosmic String, Electromagnetic Field, Plane Symmetric, Lyra geometry.**

I. INTRODUCTION

Following the formulation of the gravitational theory by Albert Einstein, numerous attempts have been undertaken to develop a unified field theory. Achieving this goal requires an extension beyond the standard Riemannian description of space-time. A notable contribution was made by Weyl [1], who introduced a generalized framework in which electromagnetic phenomena are represented geometrically. Later, Lyra [2] proposed a revised version of Riemannian geometry, which can also be viewed as a refinement of Weyl’s theory. By incorporating a gauge function into the manifold, Lyra resolved the issue of non-integrability in vector lengths during parallel displacement, and this approach naturally gives rise to a cosmological constant. Further developments by Sen [3] and Dunn [4] led to the formulation of a scalar–tensor gravitational theory within Lyra’s framework, along with field equations analogous to those introduced by Einstein.

According to Halford [5], the present theory yields predictions that agree with the classical solar system tests within observable limits. Soleng [6] further noted that, in Lyra’s geometric framework, a constant displacement field can be interpreted in different ways. It may act as a creation field, equivalent to the one proposed by Fred Hoyle [7-9] in cosmology, or it may represent a particular vacuum field. In the latter case, when combined with the gauge vector term, it can effectively behave like a cosmological constant. Brahma [10] made significant early contributions by constructing specific cosmological models in Lyra's manifold and comparing their behaviour to Einstein's general relativity. Dubey [11] and Brahma [12] investigated various Bianchi-type cosmological models in Lyra's geometry, examining their dynamical properties and late-time behaviour. Katore et al. [13] examined Einstein-Rosen bulk viscous cosmological solutions with zero-mass scalar fields in Lyra geometry, investigating the effects of viscosity on cosmological evolution. Higher-dimensional extensions have been explored by Pawar [14] et al., who investigated higher-dimensional spherically symmetric string cosmological models with zero-mass scalar fields in Lyra geometry. Pradhan et al. [15] studied plane symmetric domain wall in Lyra geometry. Reddy [16] investigated plane-symmetric cosmic strings in the Lyra manifold, discussing the role of the gauge field b in the evolution of the universe. Nimkar and Ugale [17] investigated an anisotropic Bianchi type-VI_0 wet dark fluid cosmological model in Lyra’s manifold. Katore and Kapse [18] studied the dynamical behaviour of coupled magnetized dark energy within Lyra geometry, addressing several key themes in cosmology and theoretical physics.

Cosmic strings are one-dimensional topological defects predicted to form during symmetry-breaking phase transitions in the early Universe and in certain string-theory scenarios. These hypothetical objects, characterised by their lineal energy density, represent a unique intersection of high-energy particle physics, cosmology, and gravitational physics. Mete and Deshmukh [19] obtained a five-dimensional cosmological model with one-dimensional cosmic string coupled with zero mass scalar field in Lyra Manifold. Baruah [20] investigates the problem of cosmic strings taking Bianchi type cosmologies with a self-interacting scalar field. Krori et al. [21] developed a higher-dimensional Bianchi type-I string cosmological model and showed that both matter and strings remain present together during the entire evolution of the universe. Venkateswarlu and Pavan Kumar [22] developed a higher-dimensional string cosmological model within the framework of scale covariant gravitation theory. Reddy [23-24] obtained Bianchi type-IX cosmic string and string cosmological model in scalar-tensor theory of gravitation.

Banerjee et al. [25] analysed an axially symmetric Bianchi type I string dust cosmological model, considering both the presence and absence of a magnetic field. Bali and Upadhyay [26] explored LRS Bianchi type string dust cosmological models influenced by magnetisation. Banerjee and Banerjee [27] examined the stationary distribution of dust along with electromagnetic fields within the framework of general relativity. Pawar et al. [28, 29] investigated plane symmetric string dust cosmological models with bulk viscous fluid and magnetic effects in both general relativity and Lyra manifold. Patil et al. [30, 31] studied thick domain walls incorporating viscous and electromagnetic fields in the contexts of general relativity and Lyra geometry. Bayaskar et al. [32] developed cosmological models involving perfect fluid and massless scalar fields coupled with electromagnetic fields. Bali et al. [33] focused on a magnetized tilted universe with perfect fluid distribution in general relativity. Bagora et al. [34] examined tilted Bianchi type I dust-filled cosmological models under the influence of magnetic fields in general relativity. Bali et al. [35] investigated LRS Bianchi Type II Massive String Cosmological Models with Magnetic Field in Lyra's Geometry. Diamary et al. [36] obtained a five-dimensional Bianchi type-I string cosmological model with electromagnetic field. Parikh et al. [37] studied Lyras's geometry in Bianchi type II string dust cosmological model with an electromagnetic field. Patil et al. [38] investigated a thick domain wall coupled with viscous fluid and electromagnetic field in Lyra's geometry. Ram et al. [39] studied the dynamics of Bianchi type-VI_0 universe with magnetized anisotropic dark energy in Lyra geometry. This paper is organised as follows: Section 1 discusses the motivation behind this study, Section 2 presents the metric and field equations, Section 3 addresses the solutions to the field equations, and Section 4 offers a discussion and concluding remarks.

II. THE METRIC AND FIELD EQUATION

Consider the plane symmetric space-time

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2 dz^2, \qquad (1)$$

where the metric potentials A and B are functions of cosmic time *t* only.

The relativistic field equations in normal gauge in Lyra's manifold are as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\varphi_i \varphi_j - \frac{3}{4} g_{ij}\varphi_k \varphi^k = -T_{ij}, \qquad (2)$$

where φ_i is a displacement vector field, and the other symbols have their usual meaning as in Riemannian geometry. We now assume the vector displacement field γ to be the time-like constant vector.

$$\varphi_i = (0,0,0,\gamma) \qquad (3)$$

where $\gamma = \gamma(t)$ is a function of time alone.

The energy-momentum tensor for a cosmic string with an electromagnetic field (EMF) that has the form

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j + E_{ij} \qquad (4)$$

Here,

ρ = rest energy density of the cloud of string

λ = tension density of string

u^i = (0, 0, 0, 1) cloud four-velocity

x^i = direction of anisotropy, i.e., direction of string

We have

$u^i u_i = -x^i x_j = -1$ And $u^i x_i = 0$

We consider

$$\rho = \rho_p + \lambda \qquad (5)$$

where ρ_p is the rest energy density of particles and x^i to be along the *x*-axis, so that $x^i = (0, 0, A^{-1}, 0)$.

The energy-momentum tensor for the EMF E_{ij} is given by

$$E_{ij} = \frac{1}{4\pi}\left[F_{i\alpha}F_{j\beta}g^{\alpha\beta} - \frac{1}{4}g_{ij}F^{\alpha\beta}F_{\alpha\beta}\right] \qquad (6)$$

where F_{ij} is the electromagnetic field tensor that satisfies Maxwell equations,

$$F_{[ij;k]} = 0 \text{ i.e } \left(F^{ij}\sqrt{-g}\right)_{;j} = 0 \qquad (7)$$

In a commoving coordinate system, the incident magnetic field is taken along the *z*-axis with the help of Maxwell equations (7), the components of F_{ij} except F_{12} are vanish, i.e. F_{12} is the only non-vanishing component along the z-axis,

F_{12} = constant = M (say)

From equation (6), we have

$$E_1^1 = E_2^2 = \frac{M^2}{8\pi A^4} \quad \text{and} \quad E_3^3 = E_4^4 = -\frac{M^2}{8\pi A^4} \qquad (8)$$

Equation (4) yields,

$$T_1^1 = T_2^2 = \frac{M^2}{8\pi A^4},$$

$$T_3^3 = -\lambda - \frac{M^2}{8\pi A^4}$$

and

$$T_4^4 = -\rho - \frac{M^2}{8\pi A^4} \qquad (9)$$

The field equation (2) for the metric (1), with the help of equations (3) - (9), can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\gamma^2 = -\frac{M^2}{8\pi A^4} \qquad (10)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{3}{4}\gamma^2 = \lambda + \frac{M^2}{8\pi A^4} \qquad (11)$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{3}{4}\gamma^2 = \rho + \frac{M^2}{8\pi A^4} \qquad (12)$$

Here, partial differentiation with respect to t is represented by the over-head dot.

The solution of these unknowns is discussed in the next section. Additionally, we discover the following kinematical space-time quantities of physical significance in cosmology:

The definitions of the spatial volume and the average scale factor are as follows:

$$a = \sqrt[3]{AB^2}, \text{ and } V = a^3. \qquad (13)$$

The generalised mean Hubble's parameter H, which describes the universe's volumetric expansion rate, is given by

$$H = \frac{1}{3}\sum_{i=1}^{3} H_i = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \qquad (14)$$

in which $H_1 = \frac{\dot{A}}{A}$, and $H_2 = H_3 = \frac{\dot{B}}{B}$ denotes the directional Hubble's parameters.

We have determined the expansion scalar, mean anisotropy parameter, shear scalar, and deceleration parameter q using (12) and (13), respectively.

$$\Theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} = 3H, \qquad (15)$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \qquad (16)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \Theta^2\right), \qquad (17)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \qquad (18)$$

III. SOLUTIONS AND THE MODEL

Now, the system of equations (10) to (12) are highly non-linear differential field equation with five unknowns, namely *A*, *B*, γ , λ and ρ .

We have considered the linear relationship between the metric potentials A and B as

$$A = B^n \qquad (19)$$

Where *n* is an arbitrary positive constant.

Next, we take a hybrid scale factor [40] as

$$a = e^{\alpha t} t^{\beta} \qquad (20)$$

Where α and β are positive constants.

From equations (13), (19) and (20), we obtained

$$A = t^{\frac{3\beta}{n+2}} e^{\frac{3\alpha t}{n+2}} \text{ and } B = t^{\frac{3\beta n}{n+2}} e^{\frac{3n\alpha t}{n+2}} \qquad (21)$$

From equation (21), equation (1) becomes

$$ds^2 = -dt^2 + t^{\frac{6\beta}{n+2}} e^{\frac{6\alpha t}{n+2}} (dx^2 + dy^2) + t^{\frac{6\beta n}{n+2}} e^{\frac{6n\alpha t}{n+2}} dy^2 \quad (22)$$

It is found that the model built in expression (22) is singularity-free up to infinite duration.

From equations (10) and (12), energy density can be obtained as

$$\rho = \frac{9\left(\alpha^2 t^2 + 2t\beta\alpha + \beta^2 - \frac{1}{3}\beta\right)(n+1)}{(n+2)t^2} \qquad (23)$$

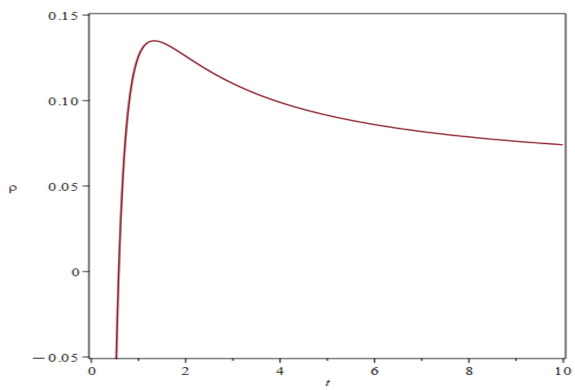

Fig.1: Behaviour of energy density vs. time for $\alpha = 0.1,\ \beta = 0.2,\ n = 0.5$

The energy density in Figure 1, at first grows from negative and approaches to certain state than gradually start to fall. The similar behaviour has been found by Dabre & Makode [41].

Solving equation (10) and (11), the tension density can be obtained as

$$\lambda = -\frac{\left\{ \begin{array}{l} 36\pi\left(\alpha^2 t^2 + 2t\beta\alpha + \beta^2 - \frac{1}{3}\beta\right)(n-1) \\ +M^2 e^{-\frac{12\alpha t}{n+2}} (n+2) t^{\frac{-12\beta+2n+4}{n+2}} \end{array} \right\}}{4(n+2)t^2\pi} \qquad (24)$$

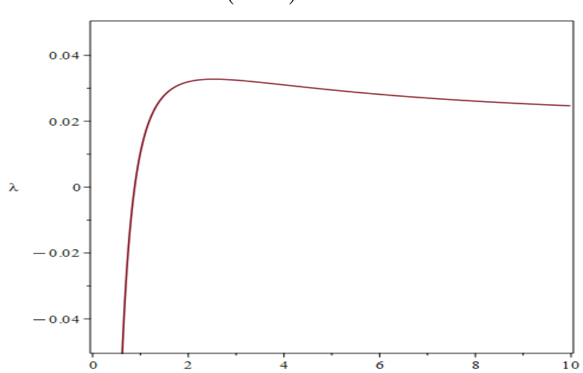

Fig.2: Behaviour of tension density of the string vs. time for $\alpha = 0.1,\ \beta = 0.2,\ n = 0.5$

Figure 2 illustrates that the tension density of the string exhibits initial rapid growth, reaches a maximum and decays gradually towards a steady state. It indicates that initially there is no presence of strings, but over time, strings begin to appear, i.e., indicating a particle-dominated early universe. The results are supported by Letelier's [42].

Equation (10) gives the value of the displacement vector γ as

$$\gamma^2 = -\frac{M^2 e^{-\frac{12\alpha t}{n+2}} (n+2)^2 t^{\frac{-12\beta+2n+4}{n+2}} + 24\pi \left\{ \begin{array}{l} 3\beta^2 (n^2+n+1) \\ +\left[6\alpha(n^2+n+1)t - n^2 - 3n - 2\right]\beta \\ +3t^2\alpha^2 (n^2+n+1) \end{array} \right\}}{6\pi (n+2)^2 t^2}$$

(25)

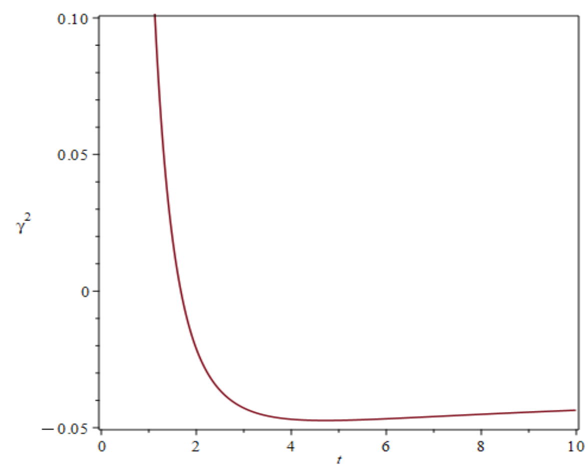

Fig.3: Behaviour of displacement field vs. time for $\alpha = 0.1,\ \beta = 0.2,\ n = 0.5$

According to Figure 3, the displacement field initially decreases sharply from a large positive value. As time increases, it passes below zero and enters the negative region. It then reaches its lowest point before beginning a slow upward movement, gradually approaching a nearly constant negative value.

From equations (13)-(18), the value of the parameters volume V, Hubble expansion factor H, expansion scalar Θ, anisotropy parameter Δ, shear scalar σ, and deceleration parameter q for our model are obtained as

$$v = t^{3\beta} e^{3\alpha t} \qquad (26)$$

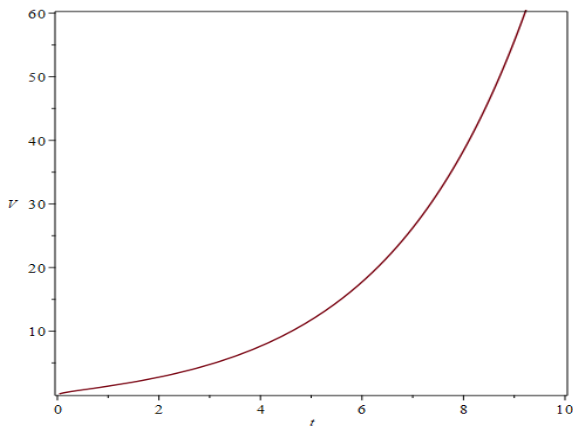

Fig.4: Behaviour of Volume parameter vs. time for $\alpha = 0.1,\ \beta = 0.2$

Figure 4 represents a volume continuously increasing function with an accelerating growth rate, typical of exponential or rapid expansion behavior.

$$H = \frac{\alpha t + \beta}{t} \qquad (27)$$

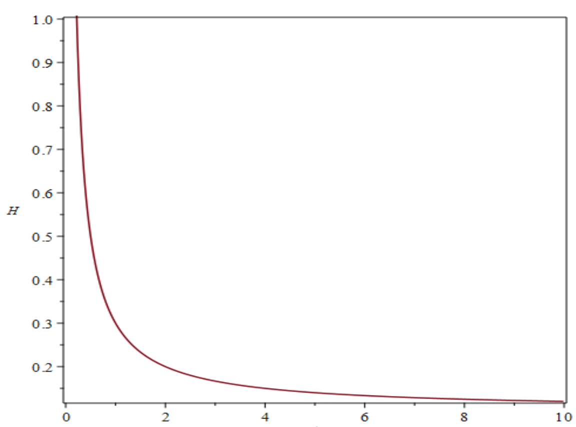

Fig.5: Behaviour of Hubble's parameter vs. time for $\alpha = 0.1,\ \beta = 0.2$

Figure 5 shows that the Hubble parameter decreases over time, indicating that the rate at which the universe expands gradually slows down.

$$\Theta = \frac{3(\alpha t + \beta)}{t} \qquad (28)$$

$$\Delta = \frac{2(n-1)^2}{(n+2)^2} \qquad (29)$$

$$\sigma^2 = \frac{3(n-1)^2(\alpha t + \beta)}{(n+2)^2 t^2} \qquad (30)$$

$$q = -1 + \frac{\beta}{(\alpha t + \beta)^2} \qquad (31)$$

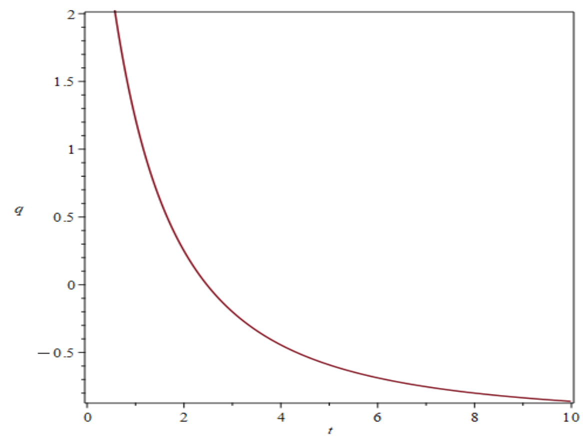

Fig.6 Behaviour of Deceleration parameter vs. time for $\alpha = 0.1,\ \beta = 0.2$

The graph shows how the parameter q changes with time t. At the beginning (small t), the value of q is high and positive, indicating a decelerating phase. As time increases, q decreases very rapidly, and transit from positive to negative, indicating acceleration of the universe.

IV. CONCLUSION

This study explores the viability of a plane-symmetric, magnetized string cosmological model within Lyra geometry. In order to obtain the precise solutions, we have assumed the constraint $A = B^n$, where n is constant, between the metric potential and average scale factor, assumed to be $a = e^{\alpha t} t^{\beta}$ describing the hybrid character of the scale factor. It is discovered that the built model is non-singular for an infinite duration. Additionally, the cosmos exhibits anisotropic acceleration and expansion, which is consistent with a standard cosmological model. Nevertheless, the model shifts from early deceleration to late-time acceleration when the rate of expansion decreases with time.

The model's energy density begins in the negative and continues to grow until it reaches a particular positive value before declining. Dabre and Makode have discovered a similar approach [Ref.No.]. As for energy density, similar patterns have been observed for tension density. Tension density and energy density show similar patterns, although tension density values are lower than particle density, suggesting that the cosmos is in a particle-dominated phase, which is consistent with Letelier's findings [Ref.No.]. Additionally, the displacement field decreased during an early era that shifted from positive to negative and approached a constant negative value.

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REFERENCES

[1] H. Weyl, "Sber Preuss Acad. Wiss. Berlin," p. 465, 1918.
[2] G. Lyra, "Über eine Modifikation der Riemannschen Geometrie," Mathematische Zeitschrift, vol. 54, p. 52, 1951, doi: 10.1007/BF01175135.
[3] D. K. Sen, "A static cosmological model," Zeitschrift für Physik, vol. 149, p. 311, 1957, doi: 10.1007/BF01333146.
[4] D. K. Sen and K. A. Dunn, "A scalar-tensor theory of gravitation in a modified Riemannian manifold," Journal of Mathematical Physics, vol. 12, p. 578, 1971, doi: 10.1063/1.1665623.
[5] W. D. Halford, "Scalar-tensor theory of gravitation in a Lyra manifold," Australian Journal of Physics, vol. 23, p. 863, 1970, doi: 10.1071/PH700863.
[6] H. H. Soleng, "Cosmologies based on Lyra's geometry," General Relativity and Gravitation, vol. 19, p. 1213, 1987, doi: 10.1007/BF00759100.
[7] F. Hoyle, "A new model for the expanding universe," Monthly Notices of the Royal Astronomical Society, vol. 108, p. 372, 1948, doi: 10.1093/mnras/108.5.372.
[8] F. Hoyle and J. V. Narlikar, "A new theory of gravitation," Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, vol. 273, no. 1352, 1963, doi: 10.1098/rspa.1963.0072.
[9] F. Hoyle and J. V. Narlikar, "Cosmological models in Lyra geometry," Proceedings of the Royal Society of London A, vol. 282, p. 191, 1964, doi: 10.1098/rspa.1964.0227.
[10] K. S. Bhamra, "A cosmological model of class one in Lyra's manifold," Australian Journal of Physics, vol. 27, no. 4, pp. 541–547, 1974.
[11] V. C. Dubey, "On Bianchi type cosmological models in Lyra's geometry," World Academy of Science, Engineering and Technology, International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, vol. 10, no. 1, pp. 1–6, 2016.
[12] B. P. Brahma, "A study on Bianchi Type-V cosmological models in Lyra's geometry," 2022.
[13] S. D. Katore, S. P. Hatkar, D. V. Kapse, and S. V. Thakare, "Einstein Rosen bulk viscous cosmological solutions with zero mass scalar field in Lyra geometry," Prespacetime Journal, vol. 3, no. 11, pp. 1098–1107, 2012.
[14] D. D. Pawar, Y. S. Solanke, and S. D. Bayaskar, "Higher dimensional spherically symmetric string cosmological model with zero mass scalar field in Lyra geometry," Journal of Scientific Research, vol. 17, no. 2, pp. 493–502, 2025.
[15] A. Pradhan, I. Aotemshi, and G. P. Singh, "Plane symmetric domain wall in Lyra geometry," Astrophysics and Space Science, vol. 288, no. 3, pp. 315–325, 2003.
[16] D. R. K. Reddy, "Plane symmetric cosmic strings in Lyra manifold," Astrophysics and Space Science, vol. 300, no. 4, pp. 381–386.
[17] H. A. Nimkar and M. R. Ugale, "Anisotropic Bianchi type-VI0 wet dark fluid cosmological models in Lyra's manifold," Indian Journal of Physics, vol. 99, no. 5, pp. 1969–1975, 2025.
[18] S. D. Katore and D. V. Kapse, "Dynamical behavior of coupled magnetized dark energy in Lyra's geometry," Astrophysics, vol. 62, no. 3, pp. 415–433, Sep. 2019, doi: 10.1007/s10511-019-09592-5.
[19] V. G. Mete and V. Deshmukh, "Five-dimensional cosmological model with one dimensional cosmic string coupled with zero mass scalar field in Lyra manifold," Journal of Scientific Research, vol. 15, no. 2, pp. 351–359, 2023.
[20] R. Bhattacharjee and K. K. Baruah, "String cosmologies with a scalar field," 2001, pp. 47–47.
[21] K. D. Krori, T. Chaudhuri, and C. R. Mahanta, "Strings in some Bianchi type cosmologies," General Relativity and Gravitation, vol. 26, no. 3, pp. 265–274, 1994, doi: 10.1007/BF02108006.
[22] R. Venkateswarlu and K. Pavankumar, "Higher dimensional string cosmologies in scale-covariant theory of gravitation," Astrophysics and Space

Science, vol. 298, no. 3, pp. 403–408, 2005, doi: 10.1007/s10509-005-5830-z.

[23] D. R. K. Reddy and R. L. Naidu, "Bianchi type-IX cosmic strings in a scalar-tensor theory of gravitation," Astrophysics and Space Science, vol. 312, no. 1, pp. 99–102, 2007.

[24] D. R. K. Reddy, "A string cosmological model in a scalar-tensor theory of gravitation," Astrophysics and Space Science, vol. 286, no. 3, pp. 359–363, 2003.

[25] A. Banerjee, A. K. Sanayal, and S. Chakraborti, "Cosmological models in Lyra geometry," Pramana Journal of Physics, vol. 34, p. 1, 1990.

[26] R. Bali and R. D. Upadhyaya, "Cosmological models in Lyra manifold," Astrophysics and Space Science, vol. 283, p. 97, 2003.

[27] A. Banerjee and S. Banerjee, "String cosmological solutions," Journal of Physics A: Proceedings of the Physical Society (GB), vol. 2, p. 188, 1968.

[28] D. D. Pawar, S. W. Bhaware, and A. G. Deshmukh, "Cosmological solutions in Lyra geometry," International Journal of Theoretical Physics, vol. 47, pp. 599–605, 2008.

[29] D. D. Pawar and V. R. Patil, "Cosmological models in Lyra manifold," Prespacetime Journal, vol. 4, no. 3, pp. 312–320, 2013.

[30] V. R. Patil, D. D. Pawar, and A. G. Deshmukh, "Higher dimensional cosmological models," Romanian Reports in Physics, vol. 62, no. 4, pp. 722–730, 2010.

[31] V. R. Patil, D. D. Pawar, and G. U. Khapekar, "Thick domain walls coupled with viscous fluid and electromagnetic field in Lyra geometry," International Journal of Theoretical Physics, vol. 51, pp. 2101–2108, 2012.

[32] S. N. Bayaskar, D. D. Pawar, and A. G. Deshmukh, "String cosmological models in Lyra geometry," Romanian Journal of Physics, vol. 54, nos. 7–8, pp. 763–740, 2010.

[33] R. Bali and K. Sharma, "Cosmological investigations in Lyra manifold," Astrophysical Journal, vol. 283, p. 11, 2003.

[34] A. Bagora, G. S. Rathore, and P. Bagora, "String cosmological models," Turkish Journal of Physics, vol. 33, 2009.

[35] R. Bali, M. K. Yadav, and L. K. Gupta, "LRS Bianchi Type II massive string cosmological models with magnetic field in Lyra's geometry," Advances in Mathematical Physics, vol. 2013, Art. no. 892361, 2013.

[36] J. Daimary and R. R. Baruah, "Five-dimensional Bianchi type-I string cosmological model with electromagnetic field," Journal of Mathematical and Computational Science, vol. 11, no. 5, pp. 6599–6613, 2021.

[37] S. Parikh, A. Tyagi, and B. R. Tripathi, "Lyra's geometry in a Bianchi Type II string dust cosmological model with an electromagnetic field," Prespacetime Journal, vol. 9, no. 4, 2018.

[38] V. R. Patil, D. D. Pawar, and G. U. Khapekar, "Thick domain walls coupled with viscous fluid and electromagnetic field in Lyra geometry," International Journal of Theoretical Physics, vol. 51, no. 7, pp. 2101–2108, 2012.

[39] S. Ram, S. Chandel, and M. K. Verma, "Dynamics of Bianchi Type-VI0 universe with magnetized anisotropic dark energy in Lyra geometry," Prespacetime Journal, vol. 8, no. 9, 2017.

[40] J. B. Jimanez et al., "Cosmological implications in modified gravity," Journal of Cosmology and Astroparticle Physics, vol. 03, p. 016, 2009.

[41] A. Dabre and P. Makode, "Viscous plane symmetric string cosmological model in f(R) gravity," Astrophysics, vol. 67, no. 2, pp. 161–177, 2024.

[42] P. S. Letelier, "String cosmologies," Physical Review D, vol. 28, no. 10, pp. 2414–2419, 1983.