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A Deterministic Inventory Model with Credit Incentives: Optimal Retail Pricing, Ordering Cycles, And Profit Maximization

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***Abstract*—This paper develops a deterministic inventory model under a two-level trade credit financing system, where the supplier offers a credit period to the retailer, and the retailer, in turn, extends a credit period to customers. The model assumes a constant demand rate, no shortages, and negligible lead time, reflecting a realistic retail environment. The main objective is to determine the optimal retail price, ordering cycle, and order quantity that maximizes the retailer's average profit per unit time.**

Two major cases are analyzed based on the relationship between the supplier's credit period and the retailer's credit period. Mathematical formulations are developed to incorporate ordering cost, purchasing cost, holding cost, interest earned, and interest charged. The optimal solutions are obtained by solving the profit functions under different scenarios.

Numerical examples are provided to illustrate the applicability of the model and to validate the theoretical results. A detailed sensitivity analysis is also conducted to examine the effects of key parameters such as ordering cost, interest rates, and credit period on the optimal decisions. The results indicate that financial factors, especially interest earned and credit period, significantly influence profitability.

The study concludes that retailers can enhance profit by utilizing longer credit periods, earning higher interest, and adopting smaller and more frequent replenishment policies under favorable trade credit conditions. The model offers practical insights for decision-making in inventory management under modern credit-based business environments and can be extended to incorporate uncertain demand or deterioration in future research.

***Index Terms*—Two-level trade credit, EOQ, Retail pricing, Optimal ordering cycle, Ordering policy.**

I. INTRODUCTION:

Payment conditions play a crucial role in inventory control situations. Many researchers have used credit policies (or delays in payment) as a coordination mechanism in their models. In

Modern business transactions, it is increasingly common for suppliers to offer a permissible delay in payment or a price discount to buyers when the order quantity meets or exceeds a specific predetermined level.

Traditional inventory models assume that the retailer pays for the goods immediately upon receiving them from the supplier. However, in real practice, suppliers often grant a fixed credit period or a price concession for settling the amount owed for delivered items. During the trade credit period, the retailer can sell the goods, earn revenue, and accumulate interest. If the retailer fails to clear the payment within the credit period, a higher interest rate is usually imposed.

Whitin (1957) noted that for retail businesses, especially those dealing with fashion goods, internal control challenges become more complex because inventory and sales are not independent. An increase in inventory can lead to increased sales. Wolfe (1968) provided empirical evidence of this relationship, showing that sales of fashion items such as women's dresses or sportswear are proportional to the amount of stock displayed.

Traditional inventory models have been built under assumptions such as constant or time-dependent demand. Several models have shown that demand may depend on inventory levels. Levin et al. (1972) pointed out that large displays of consumer goods in supermarkets encourage customers to purchase more.

Baker and Urban (1988) [2] considered a demand pattern influenced by stock levels, which may decrease along with inventory throughout the cycle.

Aggarwal and Jaggi (1995) [1] analyzed the impact of deteriorating items when payment delays are permitted. Jamal et al. (1997) [8] generalized inventory models to include shortages and deterioration. Hwang and Shin (1997) [6] developed optimal pricing and ordering strategies for retailers under trade credit. Teng (2002) demonstrated that well-established retailers may benefit by placing smaller but more frequent orders to take advantage of available trade credit.

Earlier, only the supplier offered a credit period to the retailer; the retailer did not extend credit to customers. Huang (2003) [3] introduced an inventory model in which the supplier provides the retailer a credit period X, and the retailer gives customers a shorter credit period Y, where Y<X. This concept is known as two-level trade credit financing. Huang (2006, 2007) [4-5] further expanded the model by incorporating limited storage capacity and finite production rate. Teng and Chang (2009) enhanced Huang's model by removing the restrictive assumption Y<X. Jaggi et al. (2008) [7] developed a credit-linked demand inventory model under two-level trade credit. Liao (2008) [9] proposed an EOQ model with non-instantaneous replenishment and exponentially deteriorating items under this two-level trade credit environment.

Shah et al. (2013a) incorporated order-quantity-dependent credit for deteriorating items under stock-based demand. Shah et al. (2013b) examined a two-player supply chain with price-sensitive trapezoidal demand under a net credit scheme for deteriorating inventory. Shah et al. (2015) [10] proposed a model for optimal pricing and ordering policies under two-level trade credit and price-sensitive trended demand.

This article investigates the case of constant demand under two-level trade financing, in which the supplier extends a credit period X to the retailer and the retailer gives customers a credit period Y. The key objective is to maximize the retailer's total profit per unit time through optimal pricing and ordering decisions.

Two situations are analyzed:

Case 1: X≥Y and Case 2: X≤Y. The study provides a decision-making guideline for retailers to maximize profit per unit time.

A model is developed for the practical scenario in which both the supplier and the retailer offer credit periods. The optimal solution is obtained through different scenarios. The theoretical results indicate how the cycle time and order quantity change under two-level trade credit. A numerical example used to demonstrate the proposed model.

II. NOTATIONS

B: replenishment cost per order

c: purchasing cost per unit

s: selling price per unit, where s > c

I_C: interest charge per $ for unsold items per year by the supplier

I_e; interest earned per $ per year

Q: order quantity per cycle

X: credit period offered by the supplier to the retailer

Y: credit period offered by the retailer to the customer

T: length of the inventory cycle

I(t): inventory level at any instant of time t where $0 \leq t \leq t_1$

r (s, T): retailer's total profit per unit time.

2.1. Assumptions

- A single-item inventory system is considered with an infinite planning horizon.
- Shortages are not permitted,
- The lead time is assumed to be zero or negligible.
- The demand rate is constant and denoted by R.
- The retailer offers a credit period Y to customers, which results in revenue inflow during the interval$(Y, T+Y)$.
- If $X \leq T+Y$, the retailer settles the account at time $X$and pays interest on the unsold items over the interval $(Y, T+Y)$at an interest rate I_C.
- Conversely, if $X > T+Y$, the account is settled at time Xwithout incurring any interest charges for that cycle. Furthermore, the retailer earns interest on revenue received from sales over the period (Y, X)at the rate I_e.

III. MATHEMATICAL FORMULATIONS

Assume that the inventory level at any time t is described by the following differential equation.

$$\frac{dI(t)}{dt} = -R \qquad 0 \leq t \leq T \qquad (1)$$

Boundary condition I(T) = 0. Then the solution of the equation (1) is

I(T) = R (T-t) *(2)*

When I(0) = Q, then the purchase quantity per cycle by retailer's is

$Q = RT$ *(3)*

So, sales revenue is

$ST = \int_0^r Rdt = sRT$ *(4)*

The ordering cost is

$OC = A_i$ *(5)*

The cost of purchasing the ordered quantity of units is

$PC = cQ = c\,RT$ *(6)*

The holding cost will be

$HC = h\int_0^T I(T)dt = hR\frac{T^2}{2}$ *(7)*

Two cases are considered for interest charges and interest earned based on the durations X and Y.

3.1. Case 1: X ≥ Y

3.1.1. Sub case 1: X ≥ T + Y

Since, $X \geq T + Y$, the retailer has no unsold inventory in the system; therefore, no interest is charged during the cycle, i.e., $IC_1 = 0$. The retailer receives revenue from the start of the cycle and settles the account at time Y. Hence, the retailer can earn interest at the rate I_e(per dollar per year) on this revenue from Yuntil X. Thus, the interest earned during the cycle is given by

$IE_1 = sI_e[\int_0^T \int_0^t Rdudt + (X - T - Y)\int_0^T Rdt\,] = sI_e[R\frac{T^2}{2} + (\,X - T - Y\,)RT$ *(8)*

3.1.2. Sub case 2 X ≤ T + Y

In this sub-case, the retailer lacks sufficient funds to settle the account at time X, since payment from the customers will only be received at $T + Y$. Consequently, the retailer must pay interest on the unsold items over the interval $[X, T + Y]$at an interest rate I_c per dollar per year. Therefore, the interest charges incurred in each cycle are given by

$IC_2 = cI_c\,[\int_X^{T+Y} I(t - Y)dt\,] = cI_c\int_{X-Y}^T I(t)dt = cI_c\,[\,R\frac{T^2}{2} + (X - Y)\left(RT - \frac{(X-Y)^2}{2}\right)]$ *(9)*

In this case, the retailer earns interest on the revenue collected during the interval [Y, X], i.e.,

$IE_1 = sI_e\,[\int_0^{X-Y}\int_0^t Rdudt\,] = sI_e\,[\,R(X - Y)]$ *(10)*

Therefore, the retailer's average profit per unit time is

$r = r_1, 0 \leq T \leq X - Y = r_2, T \geq X - Y$ *(11)*

Where;

$r_1 = \frac{1}{T}[SR - PC - OC - HC - IC_1 + IE_1]$ *(12)*

$r_2 = \frac{1}{T}[SR - PC - OC - HC - IC_2 + IE_2]$ *(13)*

3.2. Case 2: X ≤ Y

In this case, the retailer does not earn any interest; that is, $IE_3 = 0$. Now, the interest charged for all the items is given by

$IC_3 = cI_c\,[\,(Y - X)Q + \int_Y^{T+Y} I(t - Y)dt\,] = cI_c\,[\,(Y - X)Q + \int_0^T I(t)dt] = cI_c\,[\,(Y - X)RT + \frac{RT^2}{2}\,]$ *(14)*

So, average profit per unit time is

$r_3 = \frac{1}{T}[SR - PC - OC - HC - IC_3 + IE_3]$ *(15)*

The primary objective of this model is to maximize the average profit per unit time, expressed as, $r_i(s, T)$, $i = 1,2,3$, with respect to the retail price and cycle time. The objective functions used to obtain the optimal solution are provided in equations (12), (13), and (15).

IV. SOLUTION PROCEDURE

The optimal solution of the proposed system can be determined by examining the optimal solution under each individual case. Therefore, our objective is to derive the optimal solution of the inventory model by analyzing these different cases. The procedure is outlined in the following steps.

Differentiating, ri = 1, 2, 3 with respect to T and equating it to zero.

Step 1

Assigning values to all inventory parameters.

Step 2

For X ≥ Y, we solve simultaneously, $\frac{\partial r_1}{\partial T} = 0$ *and* $\frac{\partial r_2}{\partial T} = 0$

If X ≥ T + Y then we compute r_1 from equation (12) else r_2 from equation (13). After having, the retailers purchase quantity that can be obtained using the equation (3).

Step 3

For X < Y, we solve $\frac{\partial r_3}{\partial T} = \mathbf{0}$ and find the average profit per unit time r_3 from equation (15) and purchase quantity using the equation (3).

4.1. For Case 1 X ≥ Y:

$$\left(\frac{1}{T^2}\left(\left(sRT - cRT - A - \frac{RT^2}{2} + sI_e\left(\frac{RT^2}{2} + (X - Y - T)RT\right)\right)\right) + \frac{1}{T}\left(\left(sR - cR - RT + sI_e((X - Y - T)R)\right)\right)\right) = 0 \quad (16)$$

$$\left(\frac{1}{T^2}\left(\left(sRT - cRT - A - \frac{RT^2}{2} + cI_e\left(\frac{RT^2}{2} + (X - Y)\left(RT - \frac{X}{2} + \frac{Y}{2}\right)\right)\right) + \frac{sI_eR}{2}(M - N)\right) - \frac{1}{T}\left(\left(sR - cR - RT + cI_c((RT + (X - Y)R))\right)\right)\right) = 0 \quad (17)$$

4.2. For Case 2: X ≤ Y

$$\left(\begin{array}{c}\frac{1}{T^2}\left(\left(sRT - cRT - A - \frac{RT^2}{2} - cI_c\left(\frac{RT^2}{2} + (Y - X)RT\right)\right)\right) \\ + \frac{1}{T}\left(\left(sR - cR - RT + cI_c((Y - X)R)\right)\right)\end{array}\right) = 0$$

(18)

V. NUMERICAL EXAMPLES:

To illustrate the model, we consider an inventory system with the following parameter values. Two numerical examples are examined to demonstrate the cases $X \geq Y$and$X < Y$, respectively.

Example 1:

Given R=2,500, A=75, c=8, s=12, I_c=0.10, I_e=0.12, X=0.5, Y=0.3

For case 1,

We get, total profit is 4515 with the order quantity 625 in the cycle time 0.25.

Example 2:

Given R=2,500, A=75, c=8, s=12, Ic=0.10, Ie=0.12, X=0.5, Y=0.7

For case 2,

We get, total profit is 3788 with the order quantity 560 in the cycle time 0.22.

For both examples, the optimal total profit functions are concave in time T, as in Figures 1 and 2, guaranteeing a unique optimal solution.

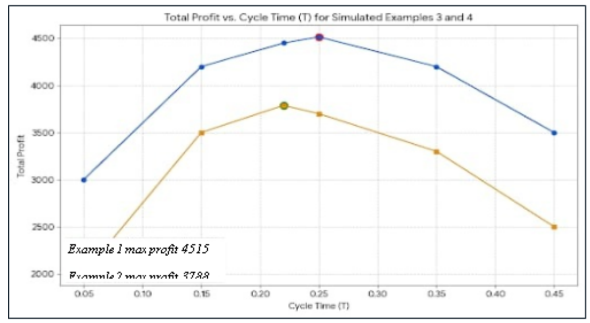

Figure: -1 Total Profit Vs. Cycle Time (T) in above simulation:

VI. SENSITIVITY ANALYSIS

This section analyzes the sensitivity of the optimal profit, order quantity, and cycle time with respect to key system parameters. The analysis is carried out using Example 1 (X ≥ Y) as the base case, since it yields the maximum profit.

Table: 1 Sensitivity Analysis of the Optimal Profit:

Parameter	Value
Demand rate (R)	2500 units/year
Ordering cost (A)	RS.75
Purchasing cost (c)	RS.8/unit
Selling price (s)	RS.12/unit
Interest charged (IC)	0.1
Interest earned (Ie)	0.12
Supplier credit (X)	0.5
Retailer credit (Y)	0.3

Base Results:

Optimal cycle time, T^*=0.25, Optimal order quantity, Q^*=625, Maximum profit, r^*=RS.4515.

6.1. Sensitivity with Respect to Ordering Cost (A):

Ordering cost is varied by ±10% and ±20%.

Table: 2 Sensitivity keeping Ordering Cost in view:

Change in A	A (RS.)	Q* (units)	T*	Profit (RS.)
−20%	60	590	0.236	4680
−10%	67.5	605	0.242	4598
Base	75	625	0.25	4515
10%	82.5	648	0.259	4420
20%	90	670	0.268	4330

When the ordering cost goes up, the profit goes down. If the ordering cost is higher, the firm places bigger orders and orders less often, so the cycle time becomes longer.

6.2. Sensitivity with Respect to Interest Earned (Ie):
Interest earned rate is varied while keeping other parameters constant.

Table: 3 Sensitivity keeping Interest Earned (Ie) in view:

Ie	Q* (units)	T*	Profit (RS.)
0.08	610	0.244	4250
0.1	618	0.247	4380
0.12 (Base)	625	0.25	4515
0.14	632	0.253	4670
0.16	640	0.256	4825

Profit changes a lot when the interest earned changes. A higher interest earned rate greatly increases the retailer's profit. When the interest earned rate increases, the best order quantity increases a little.

6.3. Sensitivity Table:

Table: 4 Sensitivity keeping Interest Charged (IC) in view:

IC	Q* (units)	T*	Profit (RS.)
0.06	640	0.256	4720
0.08	632	0.253	4615
0.10 (Base)	625	0.25	4515
0.12	615	0.246	4370
0.14	600	0.24	4200

Profit goes down quickly when the interest charged increases. To earn more profit, retailers
prefer suppliers who charge lower interest rates.

6.4. Sensitivity with Respect to Supplier Credit Period (X)

Table: 4 Sensitivity keeping Credit Period X in view:

X	Q* (units)	T*	Profit (RS.)
0.3	590	0.236	4180
0.4	605	0.242	4350
0.5 (Base)	625	0.25	4515
0.6	645	0.258	4690
0.7	670	0.268	4880

Profit increases a lot when the supplier gives a longer credit period. Retailers earn more when they are allowed more time to pay (a larger credit window).

6.5. Sensitivity Summary

Table:4. Conclusion of sensitivity analysis:

Parameter	Sensitivity Level	Effect on Profit
Ordering cost (A)	Moderate	Negative
Interest earned (Ie)	Very High	Positive
Interest charged (Ic)	Very High	Negative
Credit period (X)	High	Positive

Retailers should try to get longer credit periods and higher interest earned rates. Profit is affected more by financial factors than by changes in demand. When trade credit benefits are good, placing smaller orders more often is the best choice.

VII. CONCLUSION:

In this study, we develop an inventory model that helps a retailer choose the best price and order quantity to earn the highest profit per unit time. A numerical example is used to explain how the decisions work in practice. The results show that the retailer can gain benefits from the credit period by ordering smaller quantities more often and by giving customers part of the credit period to increase demand. This approach also helps the retailer pay suppliers earlier and lowers the chance that customers will not pay. In the future, this work can be extended by studying inventory models with uncertain demand, uncertain deterioration, or both.

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