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Fuzzy EOQ Model for Two-Parameter Weibull Deteriorating Items with Time-Dependent Cubic Demand under No Shortage Condition

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***Abstract*— In this study we present a fuzzy model for two-parameter Weibull deteriorating items with cubic time dependent demand under a condition of no shortages. The demand function represents market products with variation due to seasonal changes. Example of product such mobile phones and fashion apparel. Due to uncertainties in measuring critical parameters such as demand, deterioration rate, deterioration cost and holding costs parameters are modelled as fuzzy and represented using a triangular fuzzy number. The objective of this model is to minimize the total inventory cost by using optimal order quantity and the cycle length. The resulting nonlinear fuzzy differential equation is defuzzified by decomposing into upper and lower bound using $\alpha-$cut methodand unified using the centroid method. A numerical example is presented and sensitivity analysis presented by varying the cost, demand and Weibull parameters to show their effects on the optimal cycle length, total inventory cost and optimal ordering quantity.**

***Index Terms*—Fuzzy Inventory, Weibull Deterioration, sensitivity analysis, Cubic demand function.**

I. INTRODUCTION

Deterioration plays an important role in inventory control. In general, deterioration refers to decay, damage, spoilage and loss of values of an item that result in its usefulness Wee (1993). Deterioration have been modelled by different authors on different patterns. Ghare and Schrader (1963) in their earlier work developed an EOQ model for deteriorating items for considering deterioration rate as constant. Roy (2008) presented an inventory model for deteriorating items with time proportional demand rate with shortages. Covert and Philip (1973) studied inventory model my modelling deterioration using the Weibull distribution. Giri et al (2003) extended this by developing a ramp-typed demand and Weibull deterioration. Wu (2002) presented a generalize EOQ model with Weibull deteriorating items with shortages partially backlogged. Demand also plays a dominant role on how inventory is been controlled and has been considered using different pattern over the years. The first EOQ assumed that the demand over time as constant and instantaneous. Dave and Patel (1981) extended this by modelled demand as a linear function of time. Donaldson (1977) presented an inventory replenishment policy with linear demand with shortages. Also, Bahari-Kashani (1989) developed a heuristic inventory model for determining the replenishment schedule for deteriorating items with linear demand rate. Mohanty et al (2024) developed a model with two phase demand (constant and quadratic) function of time. Mishra (2016) developed an EOQ model with quadratic demand rate and shortages partially backlogged, Mohan and Venkateswarlu (2013) considered and inventory management model for deteriorating products with quadratic function demand rate with shortages and varying holding cost. Rangaranjan and Karthikeyan (2017) extended the demand function by considering demand a cubic function of time in their model for constant deteriorating items with shortages.

Santhi and Karthekeyan (2017) developed a Two-Parameter Weibull deteriorating items by considering demand as a cubic function of time with shortages partially and fully backlogged.

In real-life scenario parameters such as demand, deterioration and various cost parameters are often fuzzy and imprecise in nature thereby making it insufficient in modelling inventory. Zadeh (1965) introduced the concept of fuzzy set on inventory modelling. Ever since several researchers have applied this knowledge in modelling inventory with imprecise nature. Dutta and Kumar (2013) formulated inventory model for deteriorating items with shortages by allowing demand, deterioration and cost parameters as fuzzy using trapezoidal numbers. In extension of Zadeh's principles Mishra et al (2015), presented an optimal fuzzified economic order quantity model by using triangular membership function. Fuzziness was modelled in deteriorating items inventory model with linear demand with shortages by considering deterioration and holding cost parameters as hexagonal fuzzy numbers Samanta et al (2017). Mandal and Islam (2016) developed an EOQ model for deteriorating items with constant demand with demand, holding and shortage cost as fuzzy numbers. Patro et al (2017) in their study of time dependent Weibull deteriorating item model with shortages partially backlogged modelled demand parameters as triangular fuzzy numbers.

Murthy and Karthigeyan (2020) developed deteriorating items inventory model with quadratic demand by considering holding cost, deterioration and shortage cost as trapezoidal fuzzy numbers. Despite successes recorded in modelling Deteriorating items model with various demand pattern and the growing application of fuzzy set in modelling inventory, none have considered the effect of fuzziness in inventory model with cubic demand to the best of our knowledge. In this paper, we developed a Fuzzy Inventory model for Two-Parameter Weibull deterioration items with cubic demand function of time under no shortage condition. Both demand, deterioration and cost parameters are modelled as triangular fuzzy numbers.

II. ASSUMPTIONS AND NOTATIONS OF THE MODEL

2.1. Assumptions

(1) The demand rate is time dependent cubic function with fuzzy parameters, i.e.,$\widetilde{D}(t) = \tilde{a}_1 + \tilde{a}_2 t + \tilde{a}_3 t^2 + \tilde{a}_4 t^3 \tilde{a}_1 \geq 0, \quad \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \neq 0$.

(2) The deterioration rate follows two-parameter Weibull distribution with fuzzy scale $\tilde{\alpha}$and shape parameter $\tilde{\beta}$, i.e., $\tilde{\theta}(t) = \tilde{\alpha}\tilde{\beta}t^{\tilde{\beta}-1}$.

(3) Shortages are not allowed.

(4) Lead time is zero.

(5) Replenishment rate is infinite.

(6) Fuzzy parameters are modelled as a triangular number

2.1. Notations

Notation	Meaning
$I(t)$	Inventory level at time t
Q_0	Optimal ordered quantity
T	Inventory cycle length
$\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4$	Fuzzy demand parameters
$\widetilde{D}(t)$	Fuzzy time dependent demand
$\tilde{\alpha}, \tilde{\beta}$	Fuzzy parameters of Weibull deterioration
$I_L(t)$	α-cut inventory lower bound
$I_U(t)$	α-cut inventory upper bound
Q_{L0}	α-cut initial quantity ordering lower bound
Q_{U0}	α-cut initial quantity ordering upper bound
TC	Total inventory cost
$\widetilde{HC}$	Fuzzy Inventory holding cost
OC	Ordering cost
PC	Purchasing cost
c_0	Ordering cost per unit
$\tilde{h}$	Fuzzy holding cost per unit
T^*	Optimal inventory cycle length
$\widetilde{DC}$	Fuzzy deterioration cost

$\tilde{d}_c$ Fuzzy deterioration cost per unit

p_c Purchasing cost per unit

III. MODEL FORMULATION

The fuzzy inventory model with instantaneous level $I(t)$ is governed by the differential equation

$$\frac{dI(t)}{dt} + \tilde{\alpha}\tilde{\beta}t^{\tilde{\beta}-1}I(t) = -(\tilde{a}_1 + \tilde{a}_2 t + \tilde{a}_3 t^2 + \tilde{a}_4 t^3) \quad (1)$$

With boundary conditions $I(0) = Q_0$ and $I(T) = 0$

Applying the α-cut approach, equation is converted into lower and upper bound and the fuzzy differential equations becomes:

$$\frac{dI_L(t)}{dt} + \alpha_U \beta_U t^{\beta_U - 1} I_L(t) = -(a_{1U} + a_{2U}t + a_{3U}t^2 + a_{4U}t^3) \quad (2)$$

With boundary $I_L(0) = Q_0$ and $I_L(T) = 0$

And

$$\frac{dI_U(t)}{dt} + \alpha_L \beta_L t^{\beta_L - 1} I_U(t) = -(a_{1L} + a_{2L}t + a_{3L}t^2 + a_{4L}t^3) \quad (3)$$

With boundary $I_U(0) = Q_0$ and $I_U(T) = 0$

The solution of equation (2) and (3) is

$$I_L(t) = a_{1U}(T-t) + \frac{a_{2U}}{2}(T^2 - t^2) + \frac{a_{3U}}{3}(T^3 - t^3) + \frac{a_{4U}}{4}(T^4 - t^4) + \alpha_L\left[\frac{a_{1U}}{\beta_U+1}\left(T^{\beta_U+1} - t^{\beta_U+1}\right) + \frac{a_{2U}}{\beta_U+2}\left(T^{\beta_U+2} - t^{\beta_U+2}\right) + \frac{a_{3U}}{\beta_U+3}\left(T^{\beta_U+3} - t^{\beta_U+3}\right) + \frac{a_{4U}}{\beta_U+4}\left(T^{\beta_U+4} - t^{\beta_U+4}\right)\right] - \alpha_U t^{\beta_U}\left[a_{1U}(T-t) + \frac{a_{2U}}{2}(T^2 - t^2) + \frac{a_{3U}}{3}(T^3 - t^3) + \frac{a_{4U}}{4}(T^4 - t^4)\right] - \alpha_U^2 t^{\beta^U}\left[\frac{a_{1U}}{\beta_U+1}(T^{\beta_U+1} - t^{\beta_U+1}) + \frac{a_{2U}}{\beta_U+2}(T^{\beta_U+2} - t^{\beta_U+2}) + \frac{a_{3U}}{\beta_U+3}(T^{\beta_U+3} - t^{\beta_U+3}) + \frac{a_{4U}}{\beta_U+4}(T^{\beta_U+4} - t^{\beta_U+4})\right] \quad (4)$$

$$I_U(t) = a_{1L}(T-t) + \frac{a_{2L}}{2}(T^2 - t^2) + \frac{a_{3L}}{3}(T^3 - t^3) + \frac{a_{4L}}{4}(T^4 - t^4) + \alpha_L\left[\frac{a_{1L}}{\beta_L+1}\left(T^{\beta_L+1} - t^{\beta_L+1}\right) + \frac{a_{2L}}{\beta_L+2}\left(T^{\beta_L+2} - t^{\beta_L+2}\right) + \frac{a_{3L}}{\beta_L+3}\left(T^{\beta_L+3} - t^{\beta_L+3}\right) + \frac{a_{4L}}{\beta_L+4}\left(T^{\beta_L+4} - t^{\beta_L+4}\right)\right] - \alpha_L t^{\beta_L}\left[a_{1L}(T-t) + \frac{a_{2L}}{2}(T^2 - t^2) + \frac{a_{3L}}{3}(T^3 - t^3) + \frac{a_{4L}}{4}(T^4 - t^4)\right] - \alpha_L^2 t^{\beta^L}\left[\frac{a_{1L}}{\beta_L+1}(T^{\beta_L+1} - t^{\beta_L+1}) + \frac{a_{2L}}{\beta_L+2}(T^{\beta_L+2} - t^{\beta_L+2}) + \frac{a_{3L}}{\beta_L+3}(T^{\beta_L+3} - t^{\beta_L+3}) + \frac{a_{4L}}{\beta_L+4}(T^{\beta_L+4} - t^{\beta_L+4})\right] \quad (5)$$

$$I(t) = \frac{a_{1U}(T-t)}{2} + \frac{a_{2U}}{4}(T^2 - t^2) + \frac{a_{3U}}{6}(T^3 - t^3) + \frac{a_{4U}}{8}(T^4 - t^4) + \frac{\alpha_L}{2}\left[\frac{a_{1U}}{\beta_U+1}\left(T^{\beta_U+1} - t^{\beta_U+1}\right) + \frac{a_{2U}}{\beta_U+2}\left(T^{\beta_U+2} - t^{\beta_U+2}\right) + \frac{a_{3U}}{\beta_U+3}\left(T^{\beta_U+3} - t^{\beta_U+3}\right) + \frac{a_{4U}}{\beta_U+4}\left(T^{\beta_U+4} - t^{\beta_U+4}\right)\right] - \alpha_U t^{\beta_U}\left[a_{1U}(T-t) + \frac{a_{2U}}{2}(T^2 - t^2) + \frac{a_{3U}}{3}(T^3 - t^3) + \frac{a_{4U}}{4}(T^4 - t^4)\right] - \frac{\alpha_U^2 t^{\beta^U}}{2}\left[\frac{a_{1U}}{\beta_U+1}(T^{\beta_U+1} - t^{\beta_U+1}) + \frac{a_{2U}}{\beta_U+2}(T^{\beta_U+2} - t^{\beta_U+2}) + \frac{a_{3U}}{\beta_U+3}(T^{\beta_U+3} - t^{\beta_U+3}) + \frac{a_{4U}}{\beta_U+4}(T^{\beta_U+4} - t^{\beta_U+4})\right] + \frac{a_{1L}(T-t)}{2} + \frac{a_{2L}}{4}(T^2 - t^2) + \frac{a_{3L}}{6}(T^3 - t^3) + \frac{a_{4L}}{8}(T^4 - t^4) + \frac{\alpha_L}{2}\left[\frac{a_{1L}}{\beta_L+1}\left(T^{\beta_L+1} - t^{\beta_L+1}\right) + \frac{a_{2L}}{\beta_L+2}\left(T^{\beta_L+2} - t^{\beta_L+2}\right) + \frac{a_{3L}}{\beta_L+3}\left(T^{\beta_L+3} - t^{\beta_L+3}\right) + \frac{a_{4L}}{\beta_L+4}\left(T^{\beta_L+4} - t^{\beta_L+4}\right)\right] - \alpha_L t^{\beta_L}\left[a_{1L}(T-t) + \frac{a_{2L}}{2}(T^2 - t^2) + \frac{a_{3L}}{3}(T^3 - t^3) + \frac{a_{4L}}{4}(T^4 - t^4)\right] - \frac{\alpha_L^2 t^{\beta^L}}{2}\left[\frac{a_{1L}}{\beta_L+1}(T^{\beta_L+1} - t^{\beta_L+1}) + \frac{a_{2L}}{\beta_L+2}(T^{\beta_L+2} - t^{\beta_L+2}) + \frac{a_{3L}}{\beta_L+3}(T^{\beta_L+3} - t^{\beta_L+3}) + \frac{a_{4L}}{\beta_L+4}(T^{\beta_L+4} - t^{\beta_L+4})\right] \quad (6)$$

$$Q_0 = \frac{1}{2}\left[\left(a_{1L}T + \frac{a_{2L}}{2}T^2 + \frac{a_{3L}}{3}T^3 + \frac{a_{4L}}{4}T^4\right) + \left(a_{1U}T + \frac{a_{2U}}{2}T^2 + \frac{a_{3U}}{3}T^3 + \frac{a_{4U}}{4}T^4\right) + \alpha_L\left(\frac{a_{1L}}{\beta_L+1}T^{\beta_L+1} + \frac{a_{2L}}{\beta_L+2}T^{\beta_L+2} + \frac{a_{3L}}{\beta_L+3}T^{\beta_L+3} + \frac{a_{4L}}{\beta_L+4}T^{\beta_L+4}\right) + \alpha_U\left(\frac{a_{1U}}{\beta_U+1}T^{\beta_U+1} + \frac{a_{2U}}{\beta_U+2}T^{\beta_U+2} + \frac{a_{3U}}{\beta_U+3}T^{\beta_U+3} + \frac{a_{4U}}{\beta_U+4}T^{\beta_U+4}\right)\right] \quad (7)$$

The costs associated with the inventory per cycle are as follows:

Ordering cost $OC = c_0$ (8)

Fuzzy Holding cost $\widetilde{HC} = \breve{h}\int_0^T I(t)dt$ and the holding cost after defuzzification is

$$HC = \frac{h_1 + h_3 + \alpha(2h_2 - h_1 - h_3)}{2}\left[\left(\frac{a_{1U}T^2}{2} + \frac{a_{2U}T^3}{3} + \frac{a_{3U}T^4}{4} + \frac{a_{4U}T^5}{5} + \frac{a_{1U}T^2}{2} + \frac{a_{2U}T^3}{3} + \frac{a_{3U}T^4}{4} + \frac{a_{4U}T^5}{5}\right) + \alpha_L\left(\frac{a_{1U}T^{\beta_U+2}}{\beta_U+2} + \frac{a_{2U}T^{\beta_U+3}}{\beta_U+3} + \frac{a_{3U}T^{\beta_U+4}}{\beta_U+4} + \frac{a_{4U}T^{\beta_U+5}}{\beta_U+5}\right) + \alpha_L\left(\frac{a_{1L}T^{\beta_L+2}}{\beta_L+2} + \frac{a_{2L}T^{\beta_L+3}}{\beta_L+3} + \frac{a_{3L}T^{\beta_L+4}}{\beta_L+4} + \frac{a_{4L}T^{\beta_L+5}}{\beta_L+5}\right) - \alpha_U\left(\frac{a_{1U}T^{\beta_U+2}}{(\beta_U+1)(\beta_U+2)} + \frac{a_{2U}T^{\beta_U+3}}{(\beta_U+1)(\beta_U+3)} + \frac{a_{3U}T^{\beta_U+4}}{(\beta_U+1)(\beta_U+4)} + \right.$$

$\frac{a_{4U}T^{\beta_U+5}}{(\beta_U+1)(\beta_U+5)}\Big) - \alpha_L\Big(\frac{a_{1L}T^{\beta_L+2}}{(\beta_L+1)(\beta_L+2)} + \frac{a_{2L}T^{\beta_L+3}}{(\beta_L+1)(\beta_L+3)} + \frac{a_{3L}T^{\beta_L+4}}{(\beta_L+1)(\beta_L+4)} + \frac{a_{4L}T^{\beta_L+5}}{(\beta_L+1)(\beta_L+5)}\Big)\Big]$

$DC = \frac{1}{2}\Big[(d_1(1-\alpha)+\alpha d_2)\alpha_U\beta_U\Big(\frac{a_{1U}T^{\beta_U+1}}{\beta_U(\beta_U+1)} + \frac{a_{2U}T^{\beta_U+2}}{\beta_U(\beta_U+2)} + \frac{a_{3U}T^{\beta_U+3}}{\beta_U(\beta_U+3)} + \frac{a_{4U}T^{\beta_U+4}}{\beta_U(\beta_U+4)}\Big) + (d_3(1-\alpha) + \alpha d_2)\alpha_L\beta_L\Big(\frac{a_{1L}T^{\beta_L+1}}{\beta_L(\beta_L+1)} + \frac{a_{2L}T^{\beta_L+2}}{\beta_L(\beta_L+2)} + \frac{a_{3L}T^{\beta_L+3}}{\beta_L(\beta_L+3)} + \frac{a_{4L}T^{\beta_L+4}}{\beta_L(\beta_L+4)}\Big)\Big]$

Purchasing cost $PC = p_c Q_0$

$PC = \frac{p_c}{2}\Big[\Big(a_{1L}T + \frac{a_{2L}}{2}T^2 + \frac{a_{3L}}{3}T^3 + \frac{a_{4L}}{4}T^4\Big) + \Big(a_{1U}T + \frac{a_{2U}}{2}T^2 + \frac{a_{3U}}{3}T^3 + \frac{a_{4U}}{4}T^4\Big) + \alpha_L\Big(\frac{a_{1L}}{\beta_L+1}T^{\beta_L+1} + \frac{a_{2L}}{\beta_L+2}T^{\beta_L+2} + \frac{a_{3L}}{\beta_L+3}T^{\beta_L+3} + \frac{a_{4L}}{\beta_L+4}T^{\beta_L+4}\Big) + \alpha_U\Big(\frac{a_{1U}}{\beta_U+1}T^{\beta_U+1} + \frac{a_{2U}}{\beta_U+2}T^{\beta_U+2} + \frac{a_{3U}}{\beta_U+3}T^{\beta_U+3} + \frac{a_{4U}}{\beta_U+4}T^{\beta_U+4}\Big)\Big]$

$$TC = \frac{1}{T}[OC + HC + DC + PC]$$

$TC = \frac{1}{T}\Big|c_0 + \frac{h_1+h_3+\alpha(2h_2-h_1-h_3)}{2}\Big[\Big(\frac{a_{1U}T^2}{2} + \frac{a_{2U}T^3}{3} + \frac{a_{3U}T^4}{4} + \frac{a_{4U}T^5}{5} + \frac{a_{1U}T^2}{2} + \frac{a_{2U}T^3}{3} + \frac{a_{3U}T^4}{4} + \frac{a_{4U}T^5}{5}\Big) + \alpha_L\Big(\frac{a_{1U}T^{\beta_U+2}}{\beta_U+2} + \frac{a_{2U}T^{\beta_U+3}}{\beta_U+3} + \frac{a_{3U}T^{\beta_U+4}}{\beta_U+4} + \frac{a_{4U}T^{\beta_U+5}}{\beta_U+5}\Big) + \alpha_L\Big(\frac{a_{1L}T^{\beta_L+2}}{\beta_L+2} + \frac{a_{2L}T^{\beta_L+3}}{\beta_L+3} + \frac{a_{3L}T^{\beta_L+4}}{\beta_L+4} + \frac{a_{4L}T^{\beta_L+5}}{\beta_L+5}\Big) - \alpha_U\Big(\frac{a_{1U}T^{\beta_U+2}}{(\beta_U+1)(\beta_U+2)} + \frac{a_{2U}T^{\beta_U+3}}{(\beta_U+1)(\beta_U+3)} + \frac{a_{3U}T^{\beta_U+4}}{(\beta_U+1)(\beta_U+4)} + \frac{a_{4U}T^{\beta_U+5}}{(\beta_U+1)(\beta_U+5)}\Big) - \alpha_L\Big(\frac{a_{1L}T^{\beta_L+2}}{(\beta_L+1)(\beta_L+2)} + \frac{a_{2L}T^{\beta_L+3}}{(\beta_L+1)(\beta_L+3)} + \frac{a_{3L}T^{\beta_L+4}}{(\beta_L+1)(\beta_L+4)} + \frac{a_{4L}T^{\beta_L+5}}{(\beta_L+1)(\beta_L+5)}\Big)\Big] + \frac{1}{2}\Big[(d_1(1-\alpha) + \alpha d_2)\alpha_U\beta_U\Big(\frac{a_{1U}T^{\beta_U+1}}{\beta_U(\beta_U+1)} + \frac{a_{2U}T^{\beta_U+2}}{\beta_U(\beta_U+2)} + \frac{a_{3U}T^{\beta_U+3}}{\beta_U(\beta_U+3)} + \frac{a_{4U}T^{\beta_U+4}}{\beta_U(\beta_U+4)}\Big) + (d_3(1-\alpha)+\alpha d_2)\alpha_L\beta_L\Big(\frac{a_{1L}T^{\beta_L+1}}{\beta_L(\beta_L+1)} + \frac{a_{2L}T^{\beta_L+2}}{\beta_L(\beta_L+2)} + \frac{a_{3L}T^{\beta_L+3}}{\beta_L(\beta_L+3)} + \frac{a_{4L}T^{\beta_L+4}}{\beta_L(\beta_L+4)}\Big)\Big] + \frac{p_c}{2}\Big[\Big(a_{1L}T + \frac{a_{2L}}{2}T^2 + \frac{a_{3L}}{3}T^3 + \frac{a_{4L}}{4}T^4\Big) + \Big(a_{1U}T + \frac{a_{2U}}{2}T^2 + \frac{a_{3U}}{3}T^3 + \frac{a_{4U}}{4}T^4\Big) + \alpha_L\Big(\frac{a_{1L}}{\beta_L+1}T^{\beta_L+1} + \frac{a_{2L}}{\beta_L+2}T^{\beta_L+2} + \frac{a_{3L}}{\beta_L+3}T^{\beta_L+3} + \frac{a_{4L}}{\beta_L+4}T^{\beta_L+4}\Big) + \alpha_U\Big(\frac{a_{1U}}{\beta_U+1}T^{\beta_U+1} + \frac{a_{2U}}{\beta_U+2}T^{\beta_U+2} + \frac{a_{3U}}{\beta_U+3}T^{\beta_U+3} + \frac{a_{4U}}{\beta_U+4}T^{\beta_U+4}\Big)\Big]\Big|$ (9) (10)

(12)

The necessary and sufficient conditions for total cost minimization per unit time are

$\frac{dTC}{dT} = 0$ and $\frac{d^2TC}{dT^2} > 0$for all $T > 0$

IV. NUMERICAL ILLUSTRATION

Let $c_0 = 500, a_1 = (4,5,6)$, $a_2 = (8,9,10)$, $a_3 = (14,15,16)$, $a_4 = (19,20,21)$, $h = (0.3,0.4,0.5)$, $\alpha = (0.4,0.5,0.6)$, $\beta = (0.1,0.2,0.3), p_c = (0.5,0.6,0.7)$, $\widetilde{d_c} = (0.1,0.2, 0.3)$. We have optimal inventory cycle length $T^* = 1.7270$, total inventory cost $TC = 397.3225$and optimal ordered quantity $Q_0^* = 141.2169$ (11)

V. SENSITIVITY ANALYSIS

In this section, we study the effects of major changes in parameters such as $c_0, \tilde{a}_1, \tilde{a}_2$, $\tilde{a}_3$, $\tilde{a}_4$, $\tilde{h}$, $\tilde{\alpha}$, $\tilde{\beta}, \widetilde{p_c}$, $\widetilde{d_c}$on T^*, TC and Q_0. One parameter is changed at a time and the other parameters remain fixed to examine the sensitivity analysis. The results are summarized in table 1 below.

Table 1. Sensitivity analysis of various parameters

Parameters	Parameter change	Optimal T^*	Optimal TC	Optimal Qty Q_0
	400	1.627108	337.548508	116.637494
c_0	500	1.716972	397.322487	138.580299
	600	1.793477	454.274212	159.706596
	700	1.860405	508.996108	180.185531
	(3, 4, 5)	1.718033	395.677846	136.343246
	(4, 5, 6)	1.716972	397.322487	138.580299
$\tilde{a}_1$	(5, 6, 7)	1.715912	398.966664	140.814554
	(6, 7, 8)	1.714852	400.610379	143.046014
	(7, 8, 9)	1.720483	395.685744	137.266803
$\tilde{a}_2$	(8, 9, 10)	1.716972	397.322487	138.580299

	(9, 10, 11)	1.713476	398.954176	139.883423
	(10, 11, 12)	1.709994	400.580840	141.176272
	(13, 14, 15)	1.724195	395.318008	137.868880
	(14, 15, 16)	1.716972	397.322487	138.580299
$\tilde{a}_3$	(15, 16, 17)	1.709867	399.305834	139.276065
	(16, 17, 18)	1.702876	401.268530	139.956734
	(18, 19, 20)	1.730666	394.623353	138.747594
	(19, 20, 21)	1.716972	397.322487	138.580299
$\tilde{a}_4$	(20, 21, 22)	1.703894	399.947892	138.430844
	(21, 22, 23)	1.691382	402.504214	138.297302
	(0.2, 0.3, 0.4)	1.784256	383.364909	157.034353
	(0.3, 0.4, 0.5)	1.716972	397.322487	138.580299
$\tilde{h}$	(0.4, 0.5, 0.6)	1.661988	409.762909	124.802588
	(0.5, 0.6, 0.7)	1.615655	421.043871	114.050407
	(0.3, 0.4, 0.5)	1.737253	391.848676	134.157815
	(0.4, 0.5, 0.6)	1.716972	397.322487	138.580299
$\tilde{\alpha}$	(0.5, 0.6, 0.7)	1.697716	402.646060	142.686040
	(0.6, 0.7, 0.8)	1.679393	407.830068	146.511324
	(0.1, 0.2, 0.3)	1.716972	397.322487	138.580299
	(0.2, 0.3, 0.4)	1.702999	399.581284	135.922529
$\tilde{\beta}$	(0.3, 0.4, 0.5)	1.689771	401.649134	133.481408
	(0.4, 0.5, 0.6)	1.677135	403.567312	131.208688
	(0.4, 0.5, 0.6)	1.743260	389.113523	145.575431
	(0.5, 0.6, 0.7)	1.716972	397.322487	138.580299
$\widetilde{p_c}$	(0.6, 0.7, 0.8)	1.692212	405.263491	132.235614
	(0.7, 0.8, 0.9)	1.668838	412.957894	126.457711
	(0.0, 0.1, 0.2)	1.726583	394.509953	141.106381
$\widetilde{d_c}$	(0.1, 0.2, 0.3)	1.716972	397.322487	138.580299
	(0.2, 0.3, 0.4)	1.707586	400.097879	136.147657
	(0.3, 0.4, 0.5)	1.698416	402.837347	133.803462

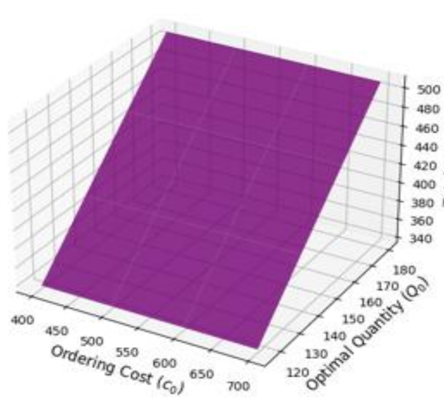

Figure 2. Total cost against ordering quantity and ordering cost.

5.1 Observations:

5.2

The following observations are based on table 1 above

- Optimal cycle length T
- Total cost TC
- Optimal order quantity Q

(a) Ordering Cost (OC)

As the ordering cost increases, the optimal inventory cycle length increases. This occurs because firms attempt to reduce the frequency of replenishment to avoid repeated ordering expenses. Consequently, a larger order quantity is maintained, leading to higher holding and deterioration effects, thereby increasing the total inventory cost.

(b) Fuzzy Demand Parameters $\widetilde{D}(t)$

An increase in fuzzy demand parameters leads to higher customer demand over time, requiring larger replenishment quantities. Consequently, the optimal order quantity increases. Since inventory depletes faster, the replenishment cycle becomes shorter. The increase in demand also raises inventory-related expenses, causing total cost to increase. This shows that the model responds realistically to market demand uncertainty, hence this strengthens model validation.

(c). Fuzzy Weibull Parameters $(\tilde{\alpha}, \tilde{\beta})$

Increasing the Weibull shape parameter increases the deterioration intensity of products over time. As deterioration becomes more severe, firms are forced to replenish inventory more frequently to minimize spoilage losses. Consequently, the optimal cycle length decreases while deterioration-related costs contribute to an increase in total inventory cost.

(d) Fuzzy Holding Cost $(\tilde{h})$

Higher holding cost discourages maintaining large inventories. Therefore, decision makers reduce order quantities and replenish more frequently. Although inventory storage decreases, the total inventory cost still rises because holding charges constitute a major component of the objective function.

(e) Fuzzy Deterioration Cost $(\widetilde{d_c})$

As deterioration cost increases, managers tend to minimize the exposure of items to spoilage by reducing inventory residence time. This leads to shorter replenishment cycles and lower order quantities to reduce deterioration losses.

VI. CONCLUSION

In this paper, we developed a Fuzzy inventory model for two-parameter Weibull deteriorating items with cubic time dependent demand under no shortage condition. A numerical example was presented and sensitivity analysis carried out to the effect of the deterioration, demand and cost parameters on the optimal inventory cycle length, total cost and optimal ordering quantity. The analysis shows that an increase in demand and deterioration parameters slightly decreases the optimal cycle length, and increased the total inventory cost and the ordering quantity. In future we extend the study further by considering shortages partially or fully backlogged.

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