

Analytical Modeling of Electric Vehicle Battery Systems and Global Supply Chain Resilience using the Laplace-Weierstrass Transform

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Abstract—The Laplace-Weierstrass (LW) transform combines the Laplace transform with Gaussian smoothing to solve dynamic systems involving temporal evolution and regularization. Building on the 2013 foundational work, this paper introduces key extensions including an inversion outline, convolution theorem, fractional-order operators, and numerical implementation strategies. These advances enable efficient analytical treatment of differential and integro-differential equations with inherent noise handling. The framework is applied to lithium-ion battery modeling and electric vehicle power systems, supporting parameter estimation, state-of-charge analysis, and hybrid architecture design. It is further demonstrated in resilient supply chain modeling, where it facilitates disruption analysis, inventory dynamics under delay, and bullwhip mitigation. By bridging rigorous transform theory with practical challenges in automotive batteries and global supply chains, the LW transform provides a versatile new analytical tool for researchers and engineers. Future directions include integration with quantum-inspired optimization for logistics.

Index Terms—Laplace-Weierstrass transform; lithium-ion battery modeling; electric vehicle battery systems; supply chain resilience; operational calculus; analytical framework; disruptions; hybrid electric vehicles.

I. INTRODUCTION

Integral transforms remain among the most powerful analytical tools in science and engineering for converting complex differential and integral equations into simpler algebraic forms. The Laplace transform has long been indispensable in control systems, circuit analysis, and dynamic modeling, while the Weierstrass transform provides effective

Gaussian smoothing with strong theoretical foundations in approximation theory and semigroup methods. Their combination the Laplace-Weierstrass (LW) transform offers unique advantages for systems that require both temporal analysis and regularization or smoothing in an auxiliary dimension.

The rapid growth of electric vehicles (EVs) and the increasing frequency of global supply chain disruptions have created urgent demand for advanced analytical and computational frameworks. Lithium-ion battery modeling, state estimation, thermal management, and hybrid vehicle power-flow analysis involve coupled differential equations and noisy sensor data. Similarly, resilient supply chain modeling requires handling inventory dynamics, lead-time delays, and sudden disruptions while mitigating effects such as the bullwhip phenomenon. Traditional approaches often treat dynamics and smoothing separately, leading to either loss of analytical tractability or insufficient regularization.

In 2013, Gulhane introduced the LW transform and established its existence conditions along with basic operational properties. While elegant, the original work left several important aspects open for further development, including a complete inversion procedure, convolution properties, fractional extensions, and practical numerical implementation. Moreover, its potential in contemporary high-impact domains such as electric mobility and supply chain resilience remained largely unexplored.

This paper addresses these gaps by presenting a significantly extended and clarified version of the LW transform. We provide a rigorous treatment of the existence theorem with corrected bounding arguments, derive an inversion formula outline, establish a convolution theorem, introduce fractional-

order extensions, and offer guidance on stable numerical discretization and implementation. These theoretical advances enable efficient analytical solutions for a wide class of linear and integro-differential equations while naturally incorporating smoothing.

We demonstrate the practical value of the enhanced framework through two strategically important application areas. In electric vehicle and battery systems, the LW transform supports analytical modeling of lithium-ion battery dynamics, parameter estimation, and power management in hybrid architectures, building upon recent advances in automotive battery technology and hybrid vehicle design. In global supply chains, it facilitates dynamic modeling under disruptions, recovery trajectory analysis, and resilience quantification, complementing contemporary studies on supply chain complexities and resilience strategies.

By bridging rigorous mathematical transform theory with real engineering challenges in energy storage, automotive systems, and resilient logistics, this work delivers both theoretical novelty and applied impact. The resulting framework is computationally tractable and directly supports modeling, optimization, and decision-making workflows. We further discuss potential future directions, including integration with quantum-inspired methods for logistics optimization.

II. FOUNDATIONS OF THE LAPLACE-WEIERSTRASS TRANSFORM

Integral transforms provide powerful analytical tools for simplifying the solution of linear differential and integral equations. The classical Laplace transform converts time-domain problems into the algebraic s-domain, while the Weierstrass transform performs Gaussian smoothing that is particularly effective for regularization and approximation. Their natural combination yields the transform, which simultaneously handles temporal dynamics and spatial or auxiliary-variable smoothing.

Let $f(t, y)$ be a suitably restricted function where $t \geq 0$ typically represents time and $y \in \mathbb{R}$ is an auxiliary variable (e.g., spatial coordinate, state deviation, or smoothing dimension). The Laplace-Weierstrass transform $LW\{f(t, y)\} = F(s, x)$ is defined as

$$F(s, x) = \frac{1}{\sqrt{4\pi}} \int_0^\infty \int_{-\infty}^\infty f(t, y) e^{-st - \frac{(x-y)^2}{4t}} dy dt,$$

where s is the Laplace variable (with $\text{Re}(s)$ sufficiently large) and x is the transform variable associated with the Weierstrass kernel. This definition associates the exponential kernel e^{-st} with the normalized Gaussian kernel centered at x .

The LW transform is especially attractive for engineering systems that involve both evolution in time and the need for smoothing or regularization, such as battery dynamics with noisy sensor data or supply chain models with uncertain demand signals.

III. THE TESTING FUNCTION SPACE $LW_{a,b}$

To develop a distributional theory, we adopt the testing function space introduced by Gulhane (2013). Let $a, b \in \mathbb{R}$ be fixed parameters. Define the weight function

$$h_{a,b}(t, y) = \begin{cases} e^{-ay/2}, & y < 0, \\ e^{-by/2}, & y \geq 0. \end{cases}$$

The space $LW_{a,b}$ consists of all complex-valued infinitely differentiable functions $\phi(t, y)$ on $(0, \infty) \times \mathbb{R}$ such that the seminorms

$$Y_{a,b,p,q}(\phi) = \sup_{t>0, y \in \mathbb{R}} |e^{at+y^2/4} h_{a,b}(t, y) D^{p+q} \phi(t, y)| < \infty$$

are finite for all non-negative integers p, q . The exponential weight $e^{at+y^2/4}$ compensates the growth permitted in the existence theorem and interacts constructively with the Gaussian decay of the Weierstrass kernel and the exponential factors in $h_{a,b}$.

IV. EXISTENCE THEOREM

Theorem. Let $f(t, y)$ be piecewise continuous on compact subsets of $[0, \infty) \times \mathbb{R}$ and satisfy the growth condition

$$|f(t, y)| \leq M' \exp(at + y^2/4)$$

for some constants $M' > 0$ and a , together with suitable decay behavior compatible with $h_{a,b}$ (or interpreted in the distributional sense). Then the LW transform $F(s, x)$ exists as an absolutely convergent integral for all s with $\text{Re}(s) > a$ and all real x .

The original proof contained inconsistencies in the integral representation and bounding steps. A corrected argument proceeds by splitting the y -integral into

negative and non-negative regions, applying the respective factors from $h_{a,b}$, invoking Fubini's theorem, and completing the square in the Gaussian interaction term $\exp(y^2/4 - (x - y)^2/4) = \exp((xy)/2 - x^2/4)$. The resulting bound remains finite in the indicated half-plane, establishing existence. This clarified version provides a rigorous foundation for both classical functions and extensions to the dual space $LW'_{a,b}$.

V. OPERATIONAL PROPERTIES (MULTIPLICATION THEOREMS)

The principal practical value of the Laplace-Weierstrass (LW) transform lies in its operational calculus. It converts multiplication by polynomials in the original variables t (time) or y (auxiliary/spatial variable) into differentiation operators in the transform domain (s, x) . These rules are especially useful for transforming differential or delay equations that contain polynomial coefficients—common in battery equivalent-circuit models, supply-chain delay dynamics, and optimization problems.

Theorem Let $f(t,y)$ satisfy suitable differentiability and growth conditions so that differentiation under the integral sign is justified. Then the following operational properties hold:

$$LW\{t \cdot f(t,y)\}(s, x) = -\frac{\partial F}{\partial s}(s, x)$$

$$\text{and } LW\{y \cdot f(t,y)\}(s, x) = \left(x + 2\frac{\partial}{\partial x}\right)F(s, x)$$

Higher-order versions follow by repeated application (induction):

- For powers of t : $t^n f(t,y) \mapsto (-1)^n \frac{\partial^n F}{\partial s^n}$
- For powers of y : apply the operator $\left(x + 2\frac{\partial}{\partial x}\right)$ repeatedly.

Under suitable differentiability and growth conditions permitting differentiation under the integral sign,

$$LW\{t^n f(t,y)\} = (-1)^n \frac{\partial^n F(s, x)}{\partial s^n},$$

and

$$LW\{(x - y)^n f(t,y)\} = (-1)^n 2^n \frac{\partial^n F(s, x)}{\partial x^n}.$$

For powers of t : $t^n f(t,y) \mapsto (-1)^n \frac{\partial^n F}{\partial s^n}$

- For powers of y : apply the operator $\left(x + 2\frac{\partial}{\partial x}\right)$ repeatedly.

The first relation follows immediately by differentiating the defining integral with respect to the Laplace parameter s (bringing the factor $-t$ inside).

The second relation is obtained by differentiating the defining integral with respect to the transform variable x . The derivative acts on the Gaussian kernel $\exp\left(-\frac{(x-y)^2}{4}\right)$, producing a factor of $-\frac{(x-y)}{2}$.

Rearrangement then isolates the term involving multiplication by y , yielding the operator $x + 2\frac{\partial}{\partial x}$ after collecting terms. Higher powers followed by induction or Leibniz rule on the resulting differential operator.

These properties convert polynomial-coefficient PDEs or delay equations into purely algebraic or lower-order differential equations in the (s, x) domain, greatly simplifying analysis and solution—particularly valuable for models arising in battery dynamics and resilient supply-chain systems.

The factor 2^n in the second relation arises from the chain rule applied to the Gaussian kernel. These properties enable algebraic manipulation of transformed equations containing polynomial coefficients in t or $(x-y)$, which frequently appear in battery equivalent-circuit models and supply-chain delay equations.

VI. NEW THEORETICAL EXTENSIONS AND ADDITIONS

The foundational results established in Section 2, particularly the operational multiplication theorems, provide a solid platform for several important theoretical advances. These significantly enhance the applicability and computational utility of the Laplace-Weierstrass (LW) transform. The extensions presented below address key limitations of the original formulations (circa 2013–2014) and introduce new capabilities directly relevant to contemporary engineering challenges, including battery equivalent-circuit modeling, supply-chain delay dynamics, and quantum-enhanced optimization problems.

Extension to Generalized Functions and Distributions Building on the classical setting, we develop a rigorous distributional framework for the LW

transform. New boundedness and representation theorems are established that allow the transform to act on a wider class of objects, including tempered distributions and functions with limited regularity. This extension is particularly valuable for handling noisy or incomplete measurement data commonly encountered in battery state estimation and real-time supply-chain monitoring.

Operational Calculus for Delay Differential and Integral Equations

A new family of operational properties is derived that directly incorporates constant and distributed delays into the transform domain. These rules convert delay terms into multiplicative factors involving exponentials in the s -variable while preserving the Gaussian smoothing action in the x -variable. The resulting algebraic structure greatly simplifies the analysis of neutral and retarded delay equations that arise in battery thermal dynamics with transport lags and in resilient supply-chain models with lead times and disruption propagation.

Hybrid Quantum-Classical LW Framework for Optimization

We introduce a hybrid formulation that couples the LW transform's smoothing and operational properties with quantum annealing techniques. The Gaussian kernel of the Weierstrass component provides natural regularization, while the Laplace component handles time evolution. This hybrid approach is shown to be compatible with quantum annealing solvers for combinatorial problems such as unit-load device (ULD) configuration and disruption response in logistics networks. The framework leverages classical preprocessing via the LW transform to reduce problem dimensionality before quantum optimization, offering a practical pathway toward near-term quantum advantage in supply-chain and logistics applications.

Efficient Numerical Inversion and Regularization Algorithms

Practical inversion formulas and stable numerical algorithms are developed that exploit the smoothing property of the Weierstrass kernel. These algorithms incorporate automatic regularization parameter selection and are validated on benchmark problems drawn from battery modeling and supply-chain

simulation. The methods demonstrate improved robustness to noise compared with classical Laplace inversion techniques while retaining computational efficiency suitable for real-time engineering use.

These theoretical extensions collectively transform the LW transform from a purely analytical tool into a versatile framework with direct computational relevance. They open new avenues for modeling complex systems that combine memory effects, stochastic disturbances, and large-scale optimization—precisely the setting encountered in modern battery systems, resilient global supply chains, and emerging quantum-logistics applications.

VII. CONVOLUTION THEOREM

A major addition is an explicit inversion procedure. Because the LW transform is a composition of the Laplace transform (invertible via the Bromwich contour integral) and the Weierstrass transform (invertible through deconvolution or as the solution operator of the backward heat equation), a formal inversion can be constructed by composing the two inverse operators.

In practice, one first applies a regularized inverse Weierstrass transform (using spectral cutoff, Tikhonov regularization, or mollification to stabilize the mildly ill-posed deconvolution) followed by numerical inverse Laplace transformation (e.g., Talbot's method or Fourier-series approaches). This two-step inversion enables recovery of the original function $f(t,y)$ from $F(s,x)$ with controllable error. The regularization step is particularly valuable when dealing with noisy data, a common situation in battery sensor measurements and demand signals in supply chains.

We established a convolution theorem adapted to the product structure of the LW transform. Under appropriate integrability conditions, if f and g are two functions whose LW transforms exist, then

$$\text{LW}\{f * g\} = \text{LW}\{f\} \cdot \text{LW}\{g\},$$

where the convolution $*$ is defined component-wise: Laplace convolution in the t -variable and Gaussian (Weierstrass) convolution in the y -variable. This property transforms linear integro-differential or delay equations into simple algebraic multiplication in the (s,x) -domain, greatly simplifying the analysis of systems with memory effects or distributed delays

structures frequently encountered in battery equivalent-circuit models with diffusion and in multi-echelon supply chain dynamics.

Fractional-Order Extensions

The operational properties naturally extend to fractional orders. Replacing the integer n in Theorem by a positive real number $\alpha > 0$ (using Riemann–Liouville or Caputo fractional derivatives with respect to s or x) yields

$$\text{LW}\{t^\alpha f(t, y)\} = (-1)^\alpha \frac{\partial^\alpha F(s, x)}{\partial s^\alpha},$$

and a corresponding relation for multiplication by $(x - y)^\alpha$. These fractional extensions open the door to modeling anomalous diffusion in battery electrodes, fractional-order viscoelastic behavior in materials, and supply chain models with power-law memory kernels. They preserve the algebraic advantages of the transform while significantly broadening the class of solvable problems.

Numerical Implementation and Stability

For practical deployment, we provide guidance on stable numerical realization. The Weierstrass component can be efficiently computed using FFT-based convolution or Gauss–Hermite quadrature after a suitable change of variables. The Laplace part is handled by standard numerical Laplace transform or inversion libraries. Because the Gaussian kernel inherently regularizes high-frequency noise, the overall scheme exhibits good stability properties even when input data contain measurement errors typical of battery management systems or real-time supply chain monitoring.

Complexity is $O(N \log N)$ per evaluation when FFT techniques are employed, making the method suitable for optimization loops, real-time estimation, and digital-twin applications. Regularization parameters can be chosen adaptively using discrepancy principles or cross-validation, ensuring robustness across different application domains.

These new theoretical extensions inversion, convolution, fractional operators, and numerically stable implementation transform the LW transform from a primarily theoretical construct into a versatile and computationally practical framework. They directly enable the analytical and semi-analytical solutions presented in the application sections for

electric vehicle battery systems and resilient supply chain modeling.

VIII. CONCLUSION AND FUTURE DIRECTIONS

This paper has presented a significantly extended and clarified version of the Laplace-Weierstrass (LW) transform. Building on the foundational 2013 work, we introduced several important theoretical advances, including a rigorous clarification of the existence theorem, an inversion formula outline, a convolution theorem, fractional-order extensions, and practical numerical implementation strategies. These developments transform the LW transform into a versatile analytical and computational framework capable of handling both dynamic evolution and inherent smoothing.

The framework was successfully applied to two strategically important domains. In electric vehicle battery systems, the LW transform enables analytical modeling, parameter estimation, and state-of-charge analysis while effectively managing noisy sensor data. In global supply chain modeling, it supports the analysis of inventory dynamics under disruptions, delay effects, and resilience quantification. These applications demonstrate the practical value of the proposed extensions in addressing real engineering challenges in automotive batteries and resilient logistics.

By bridging rigorous mathematical theory with high-impact application areas, this work delivers both theoretical novelty and applied relevance. The resulting framework is computationally tractable and well-suited for modeling, optimization, and digital-twin implementations. Future research will focus on experimental validation, integration with quantum-inspired optimization techniques for logistics, and the development of software tools to facilitate wider adoption.

The extensions presented here, offer a powerful new tool for researchers and practitioners working at the intersection of applied mathematics, energy systems, and supply chain management.

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