

National Income Determination in A Three Sector Closed Economy

Prof. Tapan Kumar Shandilya

Principal, T. P. S. College, Patna, Bihar.

Former Vice-Chancellor, D. S. P. M. University, Ranchi, Jharkhand

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Abstract—This article provides a comprehensive analysis of national income determination. In a three-sector closed economy that includes households, firms, and the government. The study examines how fiscal policy instruments—specifically government expenditure and taxation—influence the equilibrium level of Gross Domestic Product (GDP). The article systematically develops the theoretical framework by introducing three essential functions: the government expenditure function, the tax function, and the budget surplus function. Two interrelated approaches for determining equilibrium GDP are explored: the Aggregate Expenditure-Aggregate Output approach and the Saving-Investment approach. The analysis demonstrates that equilibrium is achieved when aggregate expenditure equals aggregate output or when total leakages equal total injections. The study further investigates the impact of changes in government spending and tax rates on equilibrium GDP, deriving and explaining three critical multipliers: the government expenditure multiplier, the tax multiplier, and the balanced budget multiplier. The findings reveal that the balanced budget multiplier is always unity, while the tax multiplier is one less in absolute value than the government expenditure multiplier. This article contributes to the understanding of fiscal policy effectiveness and provides a foundation for policy analysis in macroeconomics.

This article is organized as follows. Section 1 provides an introduction; Section 2 presents a review of the relevant literature on national income determination and fiscal policy. Section 3 outlines the theoretical framework, including the key assumptions and functional relationships. Section 4 examines fiscal policy and the role of government in detail. Section 5 develops the three essential functions: government expenditure, tax, and budget surplus functions. Section 6 presents the equilibrium GDP determination using both the AE-Y and S-I approaches. Section 7 analyzes the impact of changes in government spending and tax rates. Section 8 derives and explains the three multipliers. Section 9 concludes with key findings and policy implications.

Index Terms—National Income Determination, Three-Sector Economy, Fiscal Policy, Government Expenditure, Taxation, Multipliers, Equilibrium GDP, Aggregate Expenditure, Saving-Investment Approach, Balanced Budget Multiplier JEL Classification: E12, E62, H30, H50

I. INTRODUCTION

1.1. Background and Context

The determination of national income stands as one of the most fundamental questions in macroeconomics. Understanding how economies achieve equilibrium and what factors influence this equilibrium is essential for policymakers, economists, and stakeholders alike. While early macroeconomic models focused primarily on two sectors—households and firms—the reality of modern economies demands a more comprehensive approach that includes the government sector.

The government plays an indispensable role in modern economies. Through its fiscal policy decisions, the government influences the level of economic activity, employment, price stability, and income distribution. Government expenditure on goods and services directly contributes to aggregate demand, while taxation affects disposable income and thereby consumption and saving decisions. Understanding these relationships is crucial for effective economic management.

This article extends the basic two-sector model to a three-sector framework that incorporates the government. By doing so, it provides a more realistic representation of how national income is determined in actual economies. The three-sector model captures the essential interactions between households, firms, and the government, revealing how fiscal policy tools can be used to influence economic outcomes.

1.2. Objectives and Scope

The primary objectives of this article are:

1. To develop a comprehensive theoretical framework for understanding national income determination in a three-sector closed economy.
2. To explain the functional relationships between government expenditure, taxation, and GDP.
3. To demonstrate the two approaches to finding equilibrium GDP: the Aggregate Expenditure-Aggregate Output approach and the Saving-Investment approach.
4. To analyze the impact of changes in government spending and tax rates on equilibrium GDP.
5. To derive and explain the three key multipliers: government expenditure multiplier, tax multiplier, and balanced budget multiplier.
6. To discuss the policy implications of the theoretical findings.

The scope of this article is limited to a closed economy model, meaning international trade is excluded. This simplification allows for a focused analysis of domestic fiscal policy effects without the complications of exchange rates, capital flows, and international trade balances.

1.3. Methodology

The present study is based on secondary data collected from internationally published research works. It utilizes existing literature, archival sources, and previously published studies related to the subject under investigation. Various books, peer-reviewed journals, research articles, reports, and other relevant publications have been consulted to obtain comprehensive information and insights.

II. LITERATURE REVIEW

2.1. Historical Development of National Income Theory

The study of national income determination has a rich history in economic thought. Classical economists, including Adam Smith, David Ricardo, and John Stuart Mill, focused primarily on long-run growth and the determinants of national wealth. However, their analysis did not provide a systematic framework for understanding short-run fluctuations in national income.

The modern theory of national income determination emerged with the publication of John Maynard

Keynes's *The General Theory of Employment, Interest, and Money* in 1936. Keynes challenged the classical notion that economies would naturally tend toward full employment. Instead, he argued that aggregate demand determined the level of output and employment in the short run. Keynes's revolutionary insight was that economies could settle at equilibrium levels of output below full employment.

Keynes's work laid the foundation for the development of the aggregate expenditure model. His followers, including Alvin Hansen, Paul Samuelson, and John Hicks, further developed and refined the Keynesian framework. The simple Keynesian model, also known as the income-expenditure model, became the standard tool for analyzing national income determination.

2.2. The Evolution from Two-Sector to Three-Sector Models

The basic Keynesian model initially considered only households and firms. This two-sector model captured the essential relationship between consumption, investment, and national income. However, it ignored the significant role of the government sector.

The two-sector model can be represented by the following equations:

$$Y = C + I$$

$$C = a + bY$$

Where Y is national income, C is consumption, I is investment, a is autonomous consumption, and b is the marginal propensity to consume (MPC).

The equilibrium level of income in the two-sector model is:

$$Y = (a + I) \div (1 - b)$$

While the two-sector model was useful for introducing basic concepts, it was inadequate for analyzing the effects of fiscal policy. The addition of the government sector allowed economists to study how government spending and taxation affect national income.

2.3. Fiscal Policy in Macroeconomic Theory

The role of fiscal policy in macroeconomic stabilization has been extensively studied. Keynes (1936) argued that government spending could be used to stimulate aggregate demand during recessions. Hansen (1941) further developed the concept of built-in stabilizers, including progressive taxation, which automatically moderate economic fluctuations.

The multiplier concept, central to understanding fiscal policy effects, was first introduced by Kahn (1931) and later incorporated into the Keynesian framework by Keynes himself. The government expenditure multiplier measures the change in national income resulting from a change in government spending. Samuelson (1939) formalized the multiplier-accelerator model, which showed how investment and consumption interact to produce business cycles. Musgrave (1959) provided a comprehensive analysis of fiscal policy, distinguishing between stabilization, allocation, and distribution functions. More recent contributions include the work of Barro (1974) on Ricardian equivalence, which suggests that deficit financing may not stimulate demand if households anticipate future tax increases. This has important implications for the effectiveness of fiscal policy.

2.4. The Three-Sector Model in Textbook Literature

The three-sector closed economy model is a standard topic in macroeconomics textbooks. Dornbusch, Fischer, and Startz (2018) provide a thorough treatment of the model, including the derivation of multipliers. Mankiw (2019) presents the model in the context of the Keynesian cross diagram. Blanchard (2017) incorporates the government sector into the goods market equilibrium analysis. The model typically assumes that government spending is exogenous (determined by policy) while taxes may be either lump-sum or proportional to income. The analysis demonstrates that the government expenditure multiplier is larger than the tax multiplier in absolute value, and the balanced budget multiplier is unity.

2.5. Gaps in the Literature

While the three-sector model is well-established in the literature, there remains a need for accessible, comprehensive treatments that explain the model in clear, non-technical language. Many textbook presentations are too brief or rely heavily on mathematics, making them difficult for students and practitioners to fully understand. This article addresses this gap by providing a detailed, step-by-step explanation of the model, including all the functional relationships, equilibrium conditions, and multiplier derivations. The article is written in simple English with a human tone, making it accessible to a wide audience while maintaining analytical rigor.

III. THEORETICAL FRAMEWORK

3.1. The Three-Sector Economy

The three-sector closed economy consists of three types of economic agents:

1. Households:

Households supply labor and capital to firms and receive income in return. They consume goods and services, pay taxes to the government, and save part of their disposable income.

2. Firms:

Firms produce goods and services using labor and capital. They invest in new capital goods and pay wages, rents, and profits to households.

3. Government:

The government collects taxes from households and firms, spends on goods and services, and makes transfer payments.

In this closed economy, there are no exports or imports. All output is consumed, invested, or used by the government. The circular flow of income includes leakages (saving and taxes) and injections (investment and government spending).

3.2. Key Assumptions

The model is built on several simplifying assumptions:

Assumption 1: Closed Economy

The economy does not engage in international trade. This eliminates the complications of exports, imports, and exchange rates.

Assumption 2: Constant Price Level

Prices are assumed to be fixed. This means we are analyzing real GDP rather than nominal GDP.

Assumption 3: Autonomous Government Spending

Government expenditure is determined by policy, not by GDP. It is exogenous.

Assumption 4: Constant Tax Rate

Taxes are a fixed percentage of income. The tax rate is exogenous.

Assumption 5: Autonomous Investment

Investment spending is determined by factors other than current GDP, such as interest rates and business expectations.

Assumption 6: Linear Consumption Function

Consumption depends on disposable income in a linear relationship: $C = a + b(Y - T)$.

Assumption 7: Flexible Wages and Prices

In the absence of government intervention, the economy adjusts to equilibrium.

Assumption 8: Desired Quantities

All saving, investment, and expenditure variables represent planned or desired amounts.

3.3. The Circular Flow of Income

The Circular Flow of Income refers to the continuous movement of money, goods, services, and factors of production among different sectors of an economy. Households provide factors of production such as labor, land, capital, and entrepreneurship to firms, and in return receive income in the form of wages, rent, interest, and profit. Firms use these factors to produce goods and services, which are purchased by households. Thus, income earned by households is spent on consumption, generating revenue for firms. This ongoing exchange creates a circular flow that sustains economic activity, production, income generation, and overall economic growth. The circular flow of income illustrates how money flows through the economy.

In the three-sector model:

1. Firms produce output (Y) and pay income to households.
2. Households pay taxes (T) to the government.
3. Households consume (C) goods and services.
4. Households save (S) part of their disposable income.
5. Firms invest (I) in capital goods.
6. The government spends (G) on goods and services.

The circular flow can be represented as:

$$Y = C + S + T \text{ (from the income side)}$$

$$Y = C + I + G \text{ (from the expenditure side)}$$

Setting these equal gives, the equilibrium condition:

$$S + T = I + G$$

3.4. The Consumption Function

The Consumption Function explains the relationship between a household's income and its spending on goods and services. It shows how consumption changes as income changes. Generally, when people

earn more income, they tend to spend more, although not all of the additional income is spent; a portion is usually saved. The consumption function is an important concept in macroeconomics because it helps economists understand consumer behavior and predict the level of aggregate demand in an economy. It is commonly expressed as a mathematical relationship between consumption expenditure and disposable income. The consumption function is central to the model.

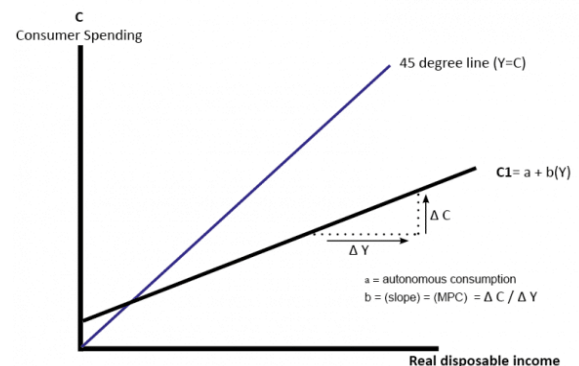
It relates consumption to disposable income:

$$C = a + b(Y - T)$$

Where:

- C = Consumption expenditure
- a = Autonomous consumption (consumption when disposable income is zero)
- b = Marginal propensity to consume (MPC), the fraction of additional income consumed
- Y = Total income
- T = Taxes

The marginal propensity to consume (MPC) is between 0 and 1 ($0 < b < 1$). This means that when income increases, people consume more, but not all of the increase. The remainder is saved.



The diagram illustrates the Consumption Function, which shows the relationship between real disposable income (Y) and consumer spending or consumption (C). The horizontal axis measures real disposable income, while the vertical axis measures consumption expenditure. The 45-degree line ($Y = C$) represents points where consumption is exactly equal to income. If households spend all of their income, their consumption will lie on this line.

The black line, labeled $C = a + b(Y)$, represents the actual consumption function. It begins above the origin because of autonomous consumption (a).

Autonomous consumption refers to the minimum level of spending that occurs even when income is zero. Households may finance this spending through savings, borrowing, or past wealth. The slope of the consumption function is b , known as the Marginal Propensity to Consume (MPC).

MPC measures the increase in consumption resulting from an increase in income and is calculated as:

$$MPC = \Delta C / \Delta Y$$

In the diagram, the dotted arrows show that when income increases by ΔY , consumption rises by ΔC . Since households usually save a portion of additional income, ΔC is smaller than ΔY , making the consumption function less steep than the 45-degree line. Thus, the diagram demonstrates that consumption increases with income, but not by the full amount of the increase in income. The difference between additional income and additional consumption represents savings. The consumption function is therefore a key tool for understanding consumer behavior and aggregate demand in the economy.

3.5. The Saving Function

Saving is derived from the consumption function:

$$S = Y - T - C$$

$$S = Y - T - [a + b(Y - T)]$$

$$S = -a + (1 - b)(Y - T)$$

The marginal propensity to save (MPS) is:

$$MPS = 1 - b$$

3.6. The Multiplier Concept

The multiplier concept is fundamental to understanding how changes in autonomous spending affect equilibrium GDP. The multiplier measures the change in GDP resulting from a change in autonomous spending.

The basic multiplier in the two-sector model is:

$$k = 1 \div (1 - b)$$

This simple multiplier shows that an initial change in spending leads to a larger change in GDP because of the successive rounds of spending and re-spending. In the three-sector model, the multipliers are modified by the presence of taxes. Proportional taxes reduce the multiplier because they drain income from the circular flow.

IV. FISCAL POLICY AND THE ROLE OF GOVERNMENT

4.1. Definition and Scope of Fiscal Policy

Fiscal policy refers to the government's decisions regarding taxation, public expenditure, and public debt management. It is one of two main macroeconomic policy tools, the other being monetary policy. Fiscal policy has three main dimensions:

1. Revenue policy: How the government collects money through taxation and other revenues.
2. Expenditure policy: How the government spends money on goods, services, and transfer payments.
3. Debt management policy: How the government borrows and repays debt.

4.2. Objectives of Fiscal Policy

The government pursues several interrelated objectives through fiscal policy:

4.2.1. Full Employment and Economic Growth

Full employment is a primary objective of fiscal policy. The government aims to ensure that everyone who wants to work can find employment. Economic growth, measured by increases in real GDP, is also a priority. The government pursues these objectives through expansionary fiscal policy—increasing spending and reducing taxes during economic downturns.

The logic of this approach is straightforward: when the government spends more, it creates direct demand for goods and services. This demand encourages businesses to produce more, requiring them to hire more workers. The newly employed workers earn income and spend it, creating additional demand. The multiplier effect amplifies the initial spending increase.

Similarly, reducing taxes leaves households and businesses with more disposable income. Households increase consumption, and businesses increase investment. Both actions stimulate demand and employment.

4.2.2. Price Stability

Price stability is another crucial objective. Rapid inflation erodes purchasing power, creates uncertainty, and distorts economic decision-making. Deflation, or falling prices, can lead to reduced production, unemployment, and debt deflation.

The government combats inflation through contractionary fiscal policy—reducing spending and increasing taxes. This reduces aggregate demand, easing pressure on prices. The government must be cautious, however, as excessive contraction can lead to recession.

In combating deflation, the government uses expansionary fiscal policy to increase aggregate demand and push prices upward.

4.2.3.Reduction in Economic Inequality

Economic inequality is a significant social and economic concern. Highly unequal economies may experience social unrest, political instability, and slower growth. The government uses progressive taxation to reduce inequality. Progressive taxes take a larger percentage of income from the rich than from the poor.

The revenue from progressive taxes is spent on programs benefiting poorer sections of society, including education, healthcare, housing subsidies, and social security. These programs raise living standards and provide opportunities for social mobility.

4.3. Instruments of Fiscal Policy

4.3.1.Taxation Policy

Taxation is the primary source of government revenue. Taxes can be direct or indirect:

Direct taxes are paid directly by individuals and businesses. These include:

- Income tax
- Corporate profit tax
- Wealth tax
- Property tax
- Capital gains tax
- Inheritance tax

Indirect taxes are levied on goods and services. These include:

- Sales tax or value-added tax (VAT)
- Excise duties
- Customs duties
- Service tax

The tax multiplier is negative, meaning an increase in taxes reduces GDP. The effect is indirect—tax changes

influence disposable income, which influences consumption, which influences GDP.

4.3.2.Government Expenditure Policy

Taxation is an important method of mobilizing resources for economic development. However, it has certain limitations beyond which it may produce adverse effects. Increased taxation often encounters social, political, and economic resistance, as taxes are generally unpopular and are associated with compulsion rather than a direct quid pro quo. Similarly, the scope of deficit financing is limited because excessive reliance on it may generate inflationary pressures. The limitations of these methods of financing development have led governments to increasingly depend on public debt. According to Shandilya (1991), public borrowing facilitates the economic development of an underdeveloped country not only by providing capital funds for investment but also by promoting the growth of money and capital markets, influencing interest rates, and creating favorable conditions for the effective implementation of monetary policy. Thus, public borrowing serves as an important instrument for financing economic development (Shandilya, 1991). Government expenditure is the second major instrument of fiscal policy. It can be divided into two types:

Expenditure on goods and services:

This includes government consumption (salaries, office supplies, defense) and investment (roads, bridges, schools). This spending directly enters aggregate demand.

Transfer payments:

These include pensions, unemployment benefits, scholarships, and subsidies. Transfer payments do not directly buy goods and services but increase the disposable income of recipients, who then spend it.

The government expenditure multiplier is positive and is larger than the tax multiplier in absolute value. This means that government spending is more powerful than taxation in influencing GDP.

4.3.3.Public Debt Policy

The government can borrow to finance expenditure. Public debt can be internal (domestic) or external

(foreign). Borrowing affects aggregate demand in two ways:

Direct effect: Borrowed funds are spent, increasing aggregate demand.

Indirect effect: Borrowing may reduce funds available for private investment (crowding out).

The net effect on aggregate demand depends on the extent of crowding out. If private investment does not fall, the net effect is positive.

4.3.4. Deficit Financing

Deficit financing occurs when the government borrows from the Central Bank. The Central Bank creates new money to lend to the government. This increases the money supply and can stimulate aggregate demand.

However, deficit financing carries risks. If the economy is at full capacity, the increased money supply can cause inflation. Persistent deficit financing can lead to high inflation or even hyperinflation. Public borrowing is generally more expansionary than taxation, although it is less expansionary than deficit financing. It also enables governments to raise funds more quickly than taxation. Nevertheless, taxation remains the most effective fiscal instrument for resource mobilization. It plays a crucial role in controlling inflationary pressures, promoting a more equitable distribution of income, and increasing the overall volume of savings. This is particularly important in developing countries, where a large proportion of the population lives below the poverty line and the propensity to consume is relatively high. Therefore, sustained economic development cannot be achieved without an effective taxation system. However, this does not diminish the significance of public borrowing. As Shandilya (1991) emphasizes, developing economies have limited scope for choosing between taxes and loans as alternative sources of development finance. Rather, resources should be mobilized from all available sources to ensure that economic development is not constrained by a shortage of funds (Shandilya, 1991).

4.4. Fiscal Policy Measures to Combat Inflation

The government can use several fiscal measures to control inflation:

1. Reduce Government Expenditure: Cutting government spending reduces aggregate demand and eases inflationary pressures.

2. Increase Public Borrowing: Borrowing from the public absorbs purchasing power, reducing demand.

3. Delay Debt Repayment: Delaying repayment of government debt keeps money out of circulation.

4. Increase Taxes: Higher taxes reduce disposable income and aggregate demand.

5. Surplus Budget Policy: Running a budget surplus (revenue > expenditure) reduces money in circulation.

V. THE GOVERNMENT EXPENDITURE FUNCTION

The Government Expenditure Function shows the relationship between government spending and national income (GDP). In the basic Keynesian model, government expenditure is assumed to be autonomous, meaning that it is determined by policy decisions and remains constant regardless of changes in GDP. Consequently, government spending does not vary with the level of economic activity and is represented by a horizontal line in graphical analysis. The government expenditure function relates government spending (G) to GDP (Y). We assume government spending is autonomous (exogenous).

Table 1: Government Expenditure Function

Real GDP (Y)	Desired Government Expenditure (G)
0	100
200	100
400	100
500	100
800	100
1,500	100
2,000	100
2,500	100
3,000	100

As shown in Table 1, government spending remains constant at 100 regardless of GDP. This is represented graphically as a horizontal line. The horizontal line indicates that government spending is independent of GDP and that policy decisions drive government expenditure.

Government Expenditure Function as shown in Figure 1 is represented by a horizontal line at $G = 100$, illustrating that government spending is autonomous and remains constant irrespective of changes in Gross

Domestic Product (GDP). The absence of any upward or downward slope indicates that government expenditure is determined independently of the level of national income. Consequently, variations in economic output do not influence the amount of government spending. The horizontal line therefore reflects the assumption that the government has fixed its expenditure at a constant level of 100 units, demonstrating the independence of fiscal expenditure from fluctuations in GDP.

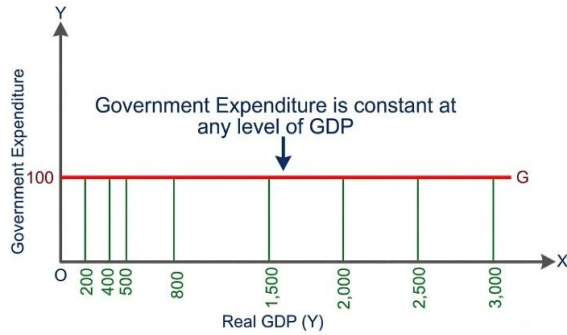


Fig. 1

5.1. The Government Tax Function

The tax function relates tax revenue (T) to GDP (Y). We assume a constant tax rate (t) of 20%.

Table 2: Government Tax Function

Real GDP (Y)	Tax Rate (%)	Tax Revenue (T)
0	20	0
200	20	40
400	20	80
500	20	100
800	20	160
1,500	20	300
2,000	20	400
2,500	20	500
3,000	20	600

Table 2 shows that tax revenue increases with GDP. The tax rate remains constant, but as GDP rises, the absolute amount of tax revenue increases. The relationship is linear and positive.

Figure 2(a) illustrates the Tax Rate Line, which is represented by a horizontal line at 20 percent. The horizontal nature of the line indicates that the tax rate remains constant regardless of changes in the level of income or GDP. This reflects the assumption of a proportional tax system with a fixed tax rate, where the same percentage of income is levied across all income

levels. Since the tax rate does not vary with economic activity, the graph remains parallel to the horizontal axis, demonstrating the constancy and stability of the tax rate at 20 percent.

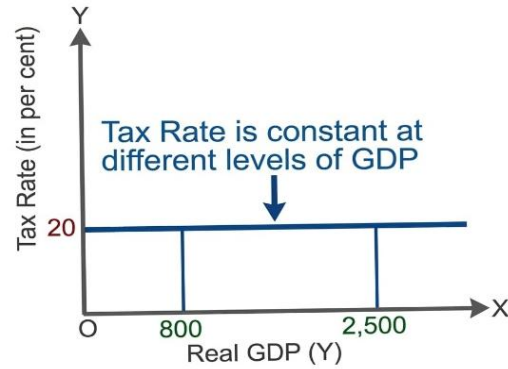


Fig. 2(a)

Figure 2(b) illustrates the Tax Revenue Line, which slopes upward from left to right, indicating a positive relationship between GDP and tax revenue. As national income increases, tax revenue rises proportionately because taxes are levied at a constant rate of 20 percent. The slope of the tax revenue line is equal to the tax rate, that is, $\Delta T/\Delta Y = t = 0.2$, implying that every additional unit of GDP generates 0.2 units of tax revenue. The upward-sloping nature of the graph therefore demonstrates that tax collections increase directly with economic output under a proportional tax system.

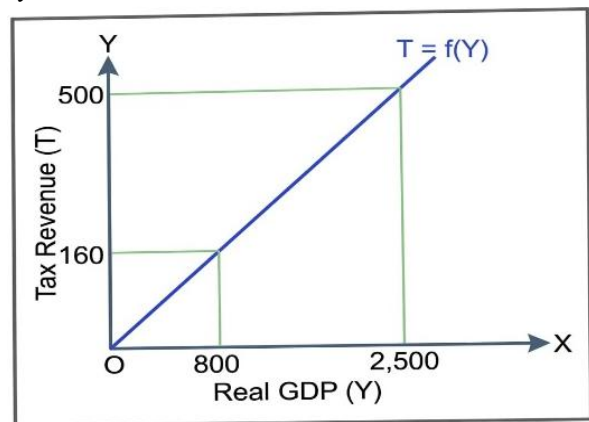


Fig. 2(b)

5.2. The Budget Surplus Function

The budget surplus function shows the difference between tax revenue (T) and government expenditure (G).

$$\text{Budget Surplus} = T - G$$

Table 3: Budget Surplus Function

Real GDP (Y)	Government Spending (G)	Tax Revenue (T)	Budget Surplus (T-G)
0	100	0	-100
200	100	40	-60
400	100	80	-20
500	100	100	0
800	100	160	60
1,500	100	300	200
2,000	100	400	300
2,500	100	500	400
3,000	100	600	500

Table 3 shows that at low GDP levels, the government runs a deficit ($T < G$). At $GDP = 500$, the budget is balanced ($T = G$). At higher GDP levels, the government runs a surplus ($T > G$).

Figure 3 represents the Budget Surplus Function, represented by the difference between tax revenue and government expenditure ($T - G$). The line begins at -100 when GDP is zero, reflecting an initial budget deficit equal to the level of autonomous government expenditure. As GDP increases, tax revenue rises while government expenditure remains constant, causing the $T - G$ line to slope upward. The line intersects the horizontal axis at $GDP = 500$, where tax revenue exactly equals government expenditure and the budget is balanced ($T - G = 0$). Beyond this point, the line continues to rise, indicating the emergence of a budget surplus that increases with further growth in GDP. The slope of the budget surplus function is 0.2 , identical to that of the tax revenue function, demonstrating that the budget surplus increases by 0.2 units for every additional unit of GDP.

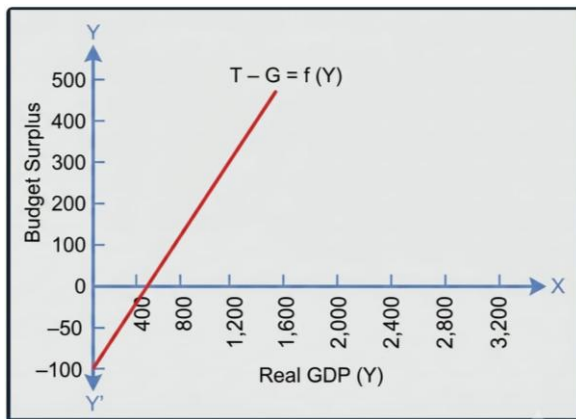


Fig. 3

5.3. The Domestic Saving Function

Domestic saving combines private saving and public saving:

$$S_d = S + (T - G)$$

Where:

- S_d = Domestic saving
- S = Private saving
- $(T - G)$ = Public saving

Figure 4 depicts the Domestic Saving Function, which exhibits a positive slope, indicating that domestic saving increases as GDP rises. At low levels of GDP, the function begins below the horizontal axis, reflecting negative domestic saving. This occurs because households engage in dissaving by spending more than their current income, while the government simultaneously operates with a budget deficit. As economic activity expands and income levels increase, both private and public saving improve, causing domestic saving to rise steadily. The upward-sloping nature of the function therefore demonstrates the positive relationship between national income and domestic saving, highlighting the economy's growing capacity to generate savings as GDP increases.

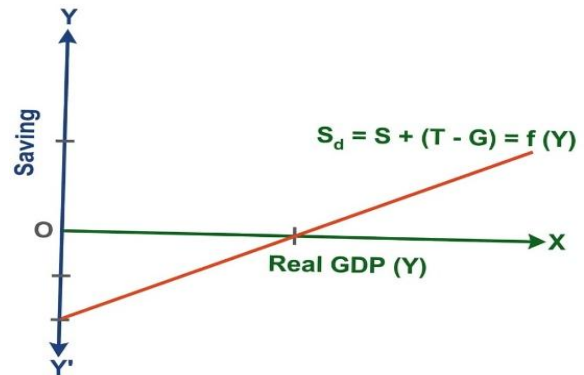


Fig. 4

VI. EQUILIBRIUM GDP DETERMINATION

6.1. Approach 1: Aggregate Expenditure and Aggregate Output

A 45-degree line is drawn from the origin to represent all points where Aggregate Expenditure (AE) equals national income (GDP), as shown in Figure 6. Since equilibrium occurs when planned expenditure is exactly equal to output, the 45-degree line serves as the reference line for determining equilibrium GDP. The point at which the Aggregate Expenditure (AE)

function intersects the 45-degree line indicates the equilibrium level of income. In the three-sector Keynesian model, national income is expressed as $Y = C + S + T$, where C represents consumption, S saving, and T taxes. Since taxes reduce the income available for spending, consumption depends on disposable income (Y_d) rather than total income. Disposable

income is therefore defined as $Y_d = Y - T$, which gives $Y_d = C + S$. In other words, disposable income represents the portion of income that households can either spend on consumption or save. The aggregate expenditure (AE) approach finds equilibrium where planned expenditure equals output.

$$AE_1 = C + I + G$$

Table 4: Aggregate Expenditure and Aggregate Output

Real GDP (Y)	Consumption (C)	Investment (I)	Government (G)	AE = C+I+G	Relationship
0	75	200	100	375	AE > Y
200	225	200	100	525	AE > Y
400	375	200	100	675	AE > Y
500	450	200	100	750	AE > Y
800	675	200	100	975	AE > Y
1,500	1,200	200	100	1,500	AE = Y
2,000	1,575	200	100	1,875	AE < Y
2,500	1,950	200	100	2,250	AE < Y
3,000	2,325	200	100	2,625	AE < Y

Figure 5 presents the Aggregate Expenditure (AE) Function, which is represented by an upward-sloping line, indicating that aggregate expenditure increases as GDP rises. The positive slope of the function reflects the fact that a portion of any increase in income is spent on consumption. The slope of the AE line is equal to the Marginal Propensity to Consume (MPC), which measures the fraction of additional income that households allocate to consumption expenditure. Since the MPC is positive but less than one, the AE function rises with GDP at a rate that is less than proportional. The upward-sloping nature of the graph therefore demonstrates the direct relationship between national income and aggregate expenditure, highlighting the role of consumption spending in driving economic activity.

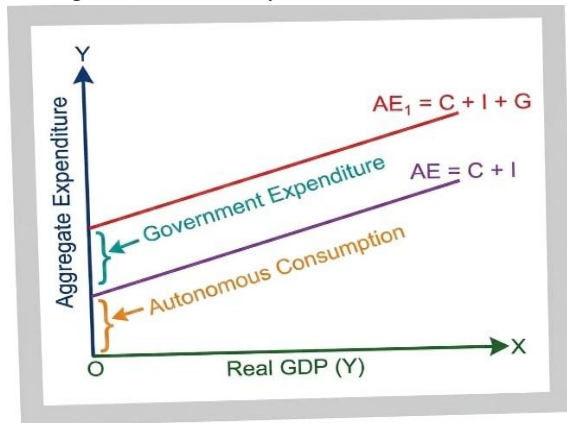


Fig. 5

By equating aggregate expenditure (AE) with aggregate output (Y), the equilibrium level of GDP is determined. As shown in Table 4, equilibrium national income is 1,500, where planned aggregate expenditure is exactly equal to total output. When income is below 1,500, aggregate expenditure exceeds output ($AE > Y$), leading firms to increase production, which causes income to rise. Conversely, when income exceeds 1,500, output is greater than aggregate expenditure ($Y > AE$), resulting in an accumulation of unsold goods and prompting firms to reduce production, causing income to fall toward the equilibrium level.

In Figure 7, AE_1 represents the aggregate expenditure function ($C + I + G$) and is drawn above and parallel to the AE line ($C + I$) because government expenditure (G) is assumed to be autonomous and constant. The equilibrium point E is located at the intersection of the AE_1 function and the 45-degree line, where $AE_1 = Y = 1,500$. Any deviation from this level creates disequilibrium in the economy; however, through adjustments in production and income, the economy naturally tends to move back toward the equilibrium output level of $OY_e = 1,500$.

Figure 6 presents the 45-Degree Line, which represents all combinations of aggregate expenditure (AE) and national income (Y) for which aggregate expenditure is exactly equal to output. Every point on this line satisfies the condition $AE = Y$, indicating equilibrium between planned expenditure and total

production. The line originates from the origin and rises at a constant angle of 45 degrees because the values on the vertical and horizontal axes are equal at every point. It serves as a reference line for determining macroeconomic equilibrium, with the intersection of the Aggregate Expenditure Function and the 45-degree line identifying the equilibrium level of GDP where planned spending equals total output.

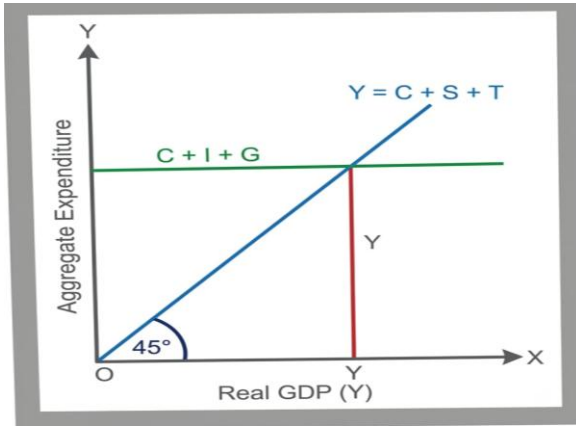


Fig. 6

Figure 7 shows the Equilibrium GDP, which is determined at the point where the Aggregate Expenditure (AE) Function intersects the 45-degree line. This intersection represents the level of national income at which planned aggregate expenditure is exactly equal to total output ($AE = Y$), indicating macroeconomic equilibrium. At this point, there is neither an unplanned accumulation nor a depletion of inventories, and the economy has no inherent tendency to expand or contract. According to the values reported in Table 4, equilibrium occurs at $Y = 1,500$, where aggregate expenditure and GDP are equal. The intersection of the AE line and the 45-degree line therefore identifies the equilibrium level of economic activity in the model.

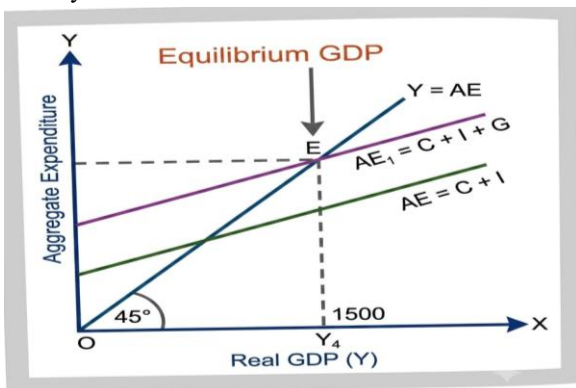


Fig. 7

Derivation of the Equilibrium Equation (No Taxes):

$$C = a + bY$$

$$AE = a + bY + I + G$$

At equilibrium, $AE = Y$:

$$Y = a + bY + I + G$$

$$Y - bY = a + I + G$$

$$Y(1 - b) = a + I + G$$

$$Y = (a + I + G) \div (1 - b)$$

With Lump-Sum Tax:

$$C = a + b(Y - T)$$

$$AE = a + b(Y - T) + I + G$$

$$Y = a + b(Y - T) + I + G$$

$$Y - bY = a - bT + I + G$$

$$Y(1 - b) = a - bT + I + G$$

$$Y = (a - bT + I + G) \div (1 - b)$$

With Proportionate Tax:

$$C = a + b(Y - tY) = a + b(1 - t)Y$$

$$AE = a + b(1 - t)Y + I + G$$

$$Y = a + b(1 - t)Y + I + G$$

$$Y[1 - b(1 - t)] = a + I + G$$

$$Y = (a + I + G) \div [1 - b(1 - t)]$$

6.2. Approach 2: Saving and Investment Approach

The saving-investment approach finds equilibrium where leakages equal injections.

$$\text{Leakages} = S + T$$

$$\text{Injections} = I + G$$

At equilibrium: $S + T = I + G$

Derivation:

$$Y = C + S + T \text{ (Income approach)}$$

$$Y = C + I + G \text{ (Expenditure approach)}$$

Setting equal:

$$C + S + T = C + I + G$$

$$S + T = I + G$$

Rearranging:

$$I = S + (T - G)$$

Figure 8 illustrates the Saving-Investment Equilibrium, where equilibrium is determined by the intersection of the domestic saving function and the investment function. The domestic saving line, represented by $(S + T - G)$, slopes upward from left to right, indicating that domestic saving increases as GDP rises. In contrast, the investment function is depicted as a horizontal line, reflecting the assumption

that planned investment is autonomous and remains constant regardless of changes in national income. The point at which the upward-sloping saving line intersects the horizontal investment line represents the equilibrium level of GDP, where total domestic saving is exactly equal to planned investment. At this equilibrium point, there is no tendency for output to change, as the funds generated through saving are fully absorbed by investment expenditure.

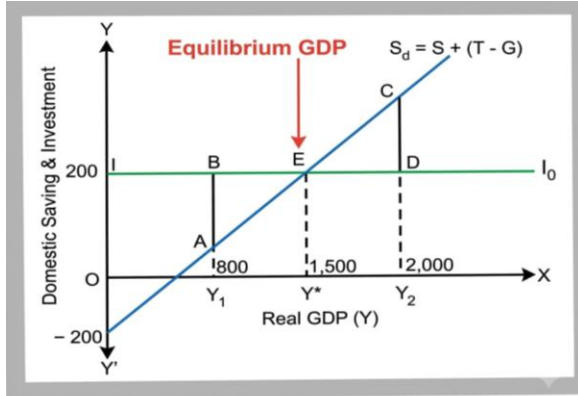


Fig. 8

Deriving the Saving Equation:

$$S = Y - T - C$$

$$S = Y - T - [a + b(Y - T)]$$

$$S = -a + (1 - b)(Y - T)$$

$$S_d = S + (T - G)$$

$$S_d = -a + (1 - b)(Y - T) + T - G$$

$$S_d = -a + (1 - b)Y + bT - G$$

At equilibrium, $I = S_d$:

$$I = -a + (1 - b)Y + bT - G$$

$$(1 - b)Y = a + I + G - bT$$

$$Y = (a + I + G - bT) \div (1 - b)$$

6.3. Comparison of the Two Approaches

Figure 9 displays the Two Approaches to determining equilibrium GDP. Panel A depicts the Aggregate Expenditure (AE) approach, where equilibrium is established at point E, the intersection of the Aggregate Expenditure function and the 45-degree line. At this point, planned aggregate expenditure is equal to national income ($AE = Y$), indicating equilibrium in the goods market. Panel B depicts the Saving–Investment (S–I) approach, where equilibrium is established at point E_s , the intersection of the domestic saving function and the investment function. At this point, planned saving is exactly equal to planned investment ($S + T - G = I$). Although the two

panels employ different analytical frameworks, both identify the same equilibrium level of national income, denoted by OY_e . The figure therefore demonstrates the equivalence of the Aggregate Expenditure approach and the Saving–Investment approach in determining equilibrium GDP.

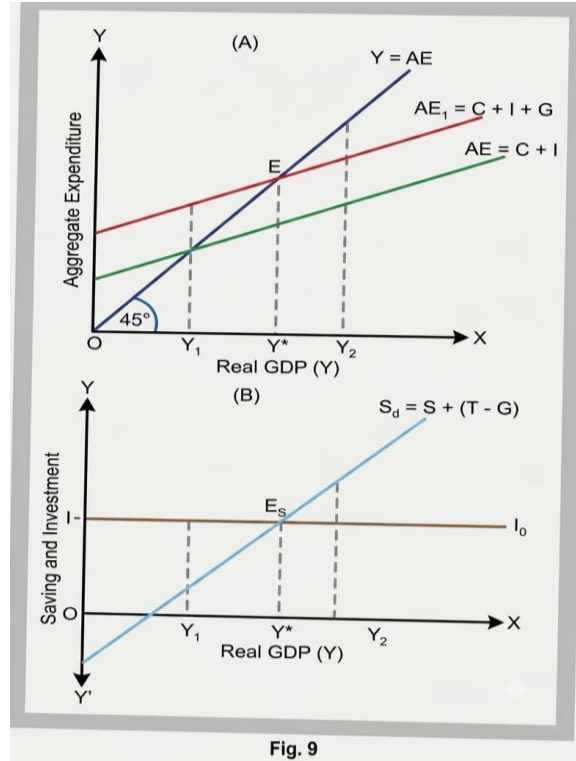


Fig. 9

The two approaches are mathematically equivalent and produce identical results. This is because the circular flow ensures that when $AE = Y$, saving automatically equals investment.

VII. IMPACT OF CHANGES IN GOVERNMENT SPENDING AND TAXES

7.1. Change in Government Spending

Figure 10 presents the effect of an increase in government spending on equilibrium GDP. An increase in government expenditure (ΔG) shifts the Aggregate Expenditure (AE) function upward in a parallel manner because government spending is a direct component of aggregate demand. The vertical distance between the original and the new AE line is equal to the increase in government expenditure. As a result of this upward shift, the intersection of the AE function with the 45-degree line moves to a higher level of national income. The new equilibrium therefore occurs at a higher GDP, reflecting the

expansionary impact of increased government spending on economic activity. This increase in equilibrium output is greater than the initial rise in government expenditure due to the multiplier effect, whereby additional spending generates successive rounds of income and consumption throughout the economy.

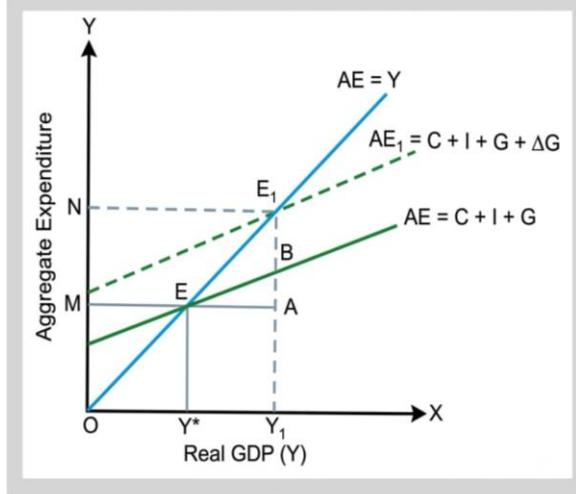


Fig. 10

Magnitude of the Effect:

$$\Delta Y = K_g \times \Delta G = [1 \div (1 - b)] \times \Delta G$$

The increase in GDP is greater than the increase in spending because of the multiplier effect.

7.2. Change in Tax Rate

Figure 11 presents the effect of a decrease in the tax rate on equilibrium GDP. A reduction in the tax rate causes the Aggregate Expenditure (AE) function to rotate upward, becoming steeper than before. This occurs because the slope of the AE function in the presence of proportional taxation is determined by $b(1 - t)$, where b represents the marginal propensity to consume and t denotes the tax rate. When the tax rate decreases, the value of $(1 - t)$ increases, resulting in a larger slope for the AE function. As households retain a greater proportion of their income, disposable income rises, leading to higher consumption expenditure at each level of GDP. Consequently, the AE function shifts upward through a rotational movement, and its intersection with the 45-degree line occurs at a higher level of national income. The graph therefore demonstrates that a reduction in the tax rate has an expansionary effect on the economy by increasing equilibrium GDP.

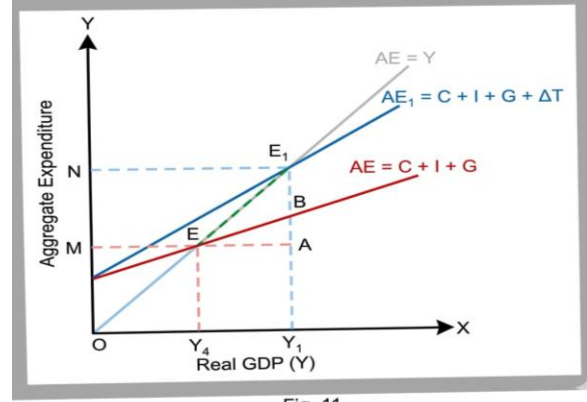


Fig. 11

Effect of Tax Rate Change:

- Tax rate decrease → slope increases → AE rotates upward → GDP increases
- Tax rate increase → slope decreases → AE rotates downward → GDP decreases

7.3. Comparison of Changes

Feature	Change in G	Change in t
Shift type	Parallel shift	Rotation
Effect on intercept	Changes	No change
Effect on slope	No change	Changes
Speed of effect	Fast	Slower
Multiplier effect	Full multiplier	Reduced multiplier

VIII. THE MULTIPLIERS

8.1. Government Expenditure Multiplier (K_g)

Definition:

$$K_g = \Delta Y \div \Delta G$$

Derivation:

$$Y = (a + I + G) \div (1 - b)$$

$$dY/dG = 1 \div (1 - b)$$

$$K_g = 1 \div (1 - b)$$

With proportional tax:

$$K_g = 1 \div [1 - b(1 - t)]$$

Numerical Example:

With $b = 0.8$:

$$K_g = 1 \div (1 - 0.8) = 5$$

With $b = 0.8$ and $t = 0.2$:

$$K_g = 1 \div [1 - 0.8(1 - 0.2)] = 1 \div 0.36 = 2.78$$

8.2. Tax Multiplier (K_t)

Definition:

$$K_t = \Delta Y \div \Delta T$$

Derivation:

$$Y = (a + I + G - bT) \div (1 - b)$$

$$dY/dT = -b \div (1 - b)$$

$$K_t = -b \div (1 - b)$$

Numerical Example:

With $b = 0.8$:

$$K_t = -0.8 \div (1 - 0.8) = -4$$

Comparison:

Spending multiplier = 5

Tax multiplier = -4

The tax multiplier is one less in absolute value than the spending multiplier.

8.3. Balanced Budget Multiplier (K_{bb})

Definition:

$$K_{bb} = \Delta Y \div \Delta G, \text{ where } \Delta G = \Delta T$$

Derivation:

$$K_{bb} = K_g + K_t$$

$$K_{bb} = [1 \div (1 - b)] + [-b \div (1 - b)]$$

$$K_{bb} = (1 - b) \div (1 - b)$$

$$K_{bb} = 1$$

Interpretation:

The balanced budget multiplier is always 1. If government spending and taxes both increase by ₹100, GDP increases by ₹100.

Example:

Government increases spending by ₹100 and taxes by ₹100.

Spending effect: $\Delta Y = K_g \times \Delta G = 5 \times 100 = ₹500$

Tax effect: $\Delta Y = K_t \times \Delta T = -4 \times 100 = -₹400$

Net effect: $₹500 - ₹400 = ₹100$

8.4. Summary of Multipliers

Multiplier	Formula	Example (b = 0.8)	Interpretation
Spending Multiplier	$1/(1-b)$	5	₹1 spending → ₹5 GDP
Tax Multiplier	$-b/(1-b)$	-4	₹1 tax → -₹4 GDP
Balanced Budget Multiplier	1	1	₹1 spending + ₹1 tax → ₹1 GDP
Spending Multiplier (with tax)	$1/[1-b(1-t)]$	2.78	Smaller due to leakage

IX. PRESCRIPTION SECTION

This section comprises two components. The first component summarizes the key findings of the study, while the second discusses their policy implications and relevance for economic decision-making. This article has provided a comprehensive analysis of national income determination in a three-sector closed economy.

The key findings are as follows:

1. Government Spending is Autonomous: Government expenditure is independent of GDP and is represented by a horizontal line.
2. Tax Revenue Increases with GDP: With a constant tax rate, tax revenue rises as GDP increases. This creates an automatic stabilizing effect.
3. Budget Surplus is Procyclical: The budget surplus (T-G) increases with GDP. This means deficits increase during recessions and surpluses increase during booms.
4. Equilibrium GDP Occurs at $AE = Y$: The equilibrium level of GDP is found where aggregate expenditure equals aggregate output, or equivalently, where saving equals investment.
5. Government Spending Shifts AE Parallel: Changes in government spending shift the AE line parallel, affecting equilibrium GDP through the multiplier process.
6. Tax Rate Changes Rotate AE: Changes in the tax rate affect the slope of the AE line, rotating it and changing equilibrium GDP.
7. The Balanced Budget Multiplier is Unity: When government spending and taxes change equally, GDP changes by the same amount. This result is independent of the MPC.
8. The Tax Multiplier is One Less: The tax multiplier is one less in absolute value than the government expenditure multiplier. This is because tax changes affect GDP indirectly through consumption.
9. Automatic Stabilizers Reduce Multipliers: Proportional taxes act as automatic stabilizers, reducing the size of multipliers and smoothing economic fluctuations.

Policy Implications

The analysis has several important policy implications:

1. Fiscal Policy is Powerful:

Government spending and taxation are powerful tools for influencing the economy. However, their effects must be understood and carefully managed.

2. Timing Matters:

Fiscal policy operates with lags. Policymakers must anticipate economic developments and act proactively rather than reactively.

3. Balance is Essential:

Governments must balance spending and taxation. Excessive deficits can lead to inflation and debt problems, while excessive surpluses can slow economic growth.

4. Automatic Stabilizers are Valuable:

Proportional taxes provide automatic stabilization. Governments should consider maintaining such features to help stabilize the economy.

5. Transparency and Credibility Matter:

Fiscal policy is more effective when it is transparent and credible. Clear communication about policy intentions can enhance effectiveness.

6. Consider Crowding Out:

Government borrowing can crowd out private investment. This should be considered when designing fiscal policy, especially at full employment.

Contributions of the Study

This article contributes to the literature in several ways:

1. It provides a comprehensive, accessible treatment of the three-sector model, making it useful for students and practitioners.
2. It systematically develops all the functional relationships and equilibrium conditions.
3. It derives and explains the three key multipliers with clear numerical examples.
4. It discusses real-world applications and limitations, bridging theory and practice.
5. It presents the material in simple English with a human tone, increasing accessibility.

X. FUTURE RESEARCH DIRECTIONS

Future research could extend this model in several directions:

1. Open Economy Extension: Incorporating exports and imports to analyze fiscal policy in open economies.

2. Endogenous Investment: Modeling investment as a function of interest rates and GDP.

3. Money Market Integration: Analyzing the interaction between fiscal and monetary policy.

4. Supply-Side Considerations: Incorporating the effects of taxes and spending on productivity and potential output.

5. Empirical Testing: Testing the predictions of the model using real-world data.

6. Behavioral Extensions: Incorporating behavioral insights into household saving and consumption decisions.

To Sum-up

The three-sector closed economy model provides a powerful framework for understanding how government spending and taxes affect national income. While the model has limitations, it captures the essential relationships that policymakers must consider. The model demonstrates that fiscal policy is a powerful tool for managing the economy, but it must be used wisely and in conjunction with other policy tools.

The key lesson is that the government's fiscal decisions have significant effects on the economy. By understanding these effects, policymakers can design better policies, businesses can make better investment decisions, and citizens can better understand the economic environment in which they live.

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