© July 2026 | IJIRT | Volume 13 Issue 2 | ISSN: 2349-6002

# Introduction To Analogical Proportion and Reasoning Based Inference

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***Abstract*—In terms of Theorems in mathematics the proportions in analogy are one of the parts of human and artificial intelligence along with its creativity and application it builds mathematical concept including its theory. Reasoning, learning, language in the current paper we give introduction on the first principle of an abstract algebra in the form of analogical proportion of the form P is to q what is r is to s seen to be general algebra this allows us the comparison between different mathematical concept which have various domain here we discuss some elementary concept of Basic mathematics. The p analogical proportion is denoted by p: q: r: s means that numerically we can investigate pairs of (p, q) and (r, s) which can develop similar ratio for example (q – p = s – r) or q/p = s/r this feature makes it possessive property of arithmetic and geometric series. Analogical proportion is the foundation of conclusive mechanism which shows us to find fourth term from other terms. here we focus attention on case-based reasoning and decision.**

***Index Terms*—Analogical proportion, reasoning algebra, comparison.**

## I. INTRODUCTION

The term Analogical proportion is also called as analogy. Which was nothing but one way of describing the reasoning which was going on throughout the history of philosophy which was broadly used in the field of artificial intelligence and linguistic for problem solving analogical proportion is consider to identify for long time as useful appeal for finding the solution of problems in mathematics there was a special popular book having name ‘How to solve it’ in 1945. This book is supposed to be supported for great search for analogical proportion useful for finding solutions of mathematical problems. This is helpful for showing the conditions of between both conditions allows us to predict that which condition is right in first phase or in second phase. Analogical proportion gives relation between two pair of terms for example (p, q) and (r, s) are two terms so it is related to analogies among these two terms also giving correspondence between them at the end of 1990’s there was new movement seen in the research study of analogical proportion. The Basic foundation of analogy is depends on the word equality or similarity even though it was also focus on when quality also when we describing any terms along with it similarity and dissimilarity then it is said that it was study of analogy means that for describing it multiple attributes are required actually it is comprehensive based but in particular case if there are two variable like p & S which was differentiate at least two of them is put down among them one should be some pairs ( l , m ) such that p : l :: m : s applicable for four different variables. The reasoning concept in analogical proportion is deals with the scheme of creative thinking. Some formal models are also included between them.

Assumptions
Let M be a set of some items analogical proportions give relation among at least three of them.
$\forall$, p, q, r $\varepsilon$ M

(1) Reflexivity property
p : s : q :: p : q

(2) Symmetry property
$p : q :: r : s \rightarrow r : s :: p : q$

(3) Property of central permutation
$p : q :: r : s \rightarrow r : r :: q : s$

(4) Property of identity
p : p : : q : q

(5) Interval reversal property

$p : q : : r : s \rightarrow q : p :: s : r$

(6) Complete reversal property

$p : q : : r : s \rightarrow s : r :: q : p$

(7) Property of extreme permutation

$p : q : : r : s \rightarrow s : q :: r : p$

## II. REPRESENTATION OF ANALOGICAL PROPORTION

The Analogical proportion can be represented by three different way they are following

(1) Logical way

(2) Algebraic way

(3) Complexity – Based way

1). Logical way –

We can represent analogical proportion by logical way through the corresponding quaternary logical connectivity's if we consider S be the set of Boolean Algebra of A = {0, 1} then

$p : q :: r : s = [ (p \wedge - q) \equiv (r \wedge -d )] \wedge [(-p \wedge q) \equiv (-r \wedge s)]$

It seen clearly that p is not similar with q as r is dissimilar from s it is not hard to examine the formula is applicable only for G value which was given in following table.

| | |
|---|---|
| 0 : 0 : : 0 : 0 | 1 : 1 : : 1 : 1 |
| 0 : 1 : : 0 : 1 | 1 : 0 : : 1 : 0 |
| 0 : 0 : : 1 : 1 | 1 : 1 : : 0 : 0 |

It is the model having minimal Boolean number which was following 3 assumptions of analogical proportions p : q :: r : s can be similarly written in the form of Numerical proportion proportions containing (p . s = q. r and p + s = q + r)

$p : q :: r : s = (p \wedge s \equiv q \wedge r) \wedge (p \vee s \equiv q \vee r)$

Above equation snows that here analogy fogous on similarity between two variables but not dissimilarity in this table we can see that the number 0 and 1 play important role. In the modeling of Boolean algebra there is a relation between Analogical proportion and its set-based view which contain (0, 1, 1, 0) and (1, 0, 0, 1) and it will remain in the highly database having consequence of

$p : q :: r : s \rightarrow q : p : : s : r$

Whenever that p : q, r, s hold its value in the finite set of X which have the value greater than it's mod of 2 if we consider X has value between 0 & 1 which belongs to R then we can express it by two ways through which we got the definition of p : q :: r : s having multiple expression and in this they are seen to be independent on code such that l , m ε [ 0 , 1] Which is equal to p : q : : r : s = 1 if p – q = r – s. If we think about analogical proportion in continuous form with the form p : q :: r : s

Are significant in its numerical form this can be denoted by vectors from X to $X^n$ such that p : q :: r : s if and only if for all

i ε [ 1, n], $p_i : q_i : : r_i : s_i$

we can use arithmetic proportion also such that p – q = r – s in the model of R

2). Algebraic Way –

Depends on assumptions of factorization there are some another definition of Analogical proportion stated by stropped and Yvon.

(p, q, r, a) ε $y^4$ is an AP (p : q :: r : a) if

1)Either (q, r) ε {(p, a)}, {(a, p)}

2)Then there exist ($p_1$, $p_2$, $a_1$, $a_2$) ε $y^4$ such that

$P = p_1 \times p_2$, $q = p_1 \times a_2$, $v = a_1 \times p_2$ and $r = a_1 \times a_2$

Above equations fulfilled the assumptions of analogical proportions. In the second equation of above expression gives that p & q are equal in $p_1$ while p and q as r and a different from $p_2$ to $a_2$ such as l m n: l m p :: bn : bp which seen to be occur simultaneously through the view of logical way which enable as to explain Analogical proportion only when X is showing its different structure.

3). A complexity – Based way –

This is the way of analogical proportion in which we can express it in the form of machine learning oriented based view. Its main theme is to count the size of least program lm (s, t) to provide s into t or we can say for the definition of complexity Kolmogorov complexity is one way through which we can express it.

4). Some other Views –

If we take the proportion in the form of p : f (p) : : q : f (q) which seen to be based form of Analogical proportion having the meaning of P is to f (p) and Y is to f (y) from this we can obtain the value of f (x) and f

(y) but it is also note that we can't apply this view for our original form of arithmetic proportion. We can arrange this in the form of p : q : : f (p) : f (q) because of its central permutations so there exist a l in such a way that p : l (p) : : f (p) : f (l (p)) where l and f are commutative.

There are some others view which are as simple as p : q : : r : s which include the relation based view and the view of generalizing the function which based on the function R C S x S such that R ( p , q ) = R ( r, s )

Some other proportion related to AP

There is one proportion which is the combination of two equivalence classes especially logical class known to be A logical proportion every class distributed between its measures one for ( p, q ) and other for ( r, s ) in this we can manifests the equality and inequality of the operators it was seen that about 110 to 120 different lexical expressions and meaningful different proportions which is logical each of them included at least 6 patterns which become true and in case of false pattern it was 10 in between them 8 is consider tube independent on cade means we can say.

A (p, q, r, s) = (-p, -q, -r, -s)

Above proportions are divided into four uniform proportions. Which are equivalent means we can interchange in between that proportion only like (p, q) with (r, s) and four different proportion which seen to be non – uniform here we refer homogeneity means all the expressions in the proportions are both have same variables have same kind but in case of heterogeneity the linked hearable show opposite nature beside that the homogenous proportion contains only 0 and 1 even numbers with only 6 true patterns. Instead of this heterogeneous giving six different patterns with the form of (l, m, m, m), (m, l, m, m), (m, m, l m), (m, m, m, l)

where, l and m belongs to 0 and 1 the above four proportions shows that it is an invader between the proportions of {p, q, r, s} which doesn't include, p, q, r and s respectively in homogenous proportion out of 4 code three has directly linked of analogical proportion as we know p : q : : r : s but away from this they are known to be reverse analogy and pathology.

Which are directly linked with analogical proportion with the permutation that is

Rev (p, q, r, s) = q : p : :r : s and

Par ( p, q, r, s) = r : q : : p : s

We can again extend its pharology like par (p, q, r, s) = $((p \wedge q) \equiv (r \wedge s))$

$((-p \wedge q) \equiv (-r \wedge -s))$ this shows that p and q are common and r & s are converse the appearance of these proportions with its equivalent proportions. Shows impartial points in $R^2$ so we can form a parallelogram by constructing three ways another homogenous proportion is said to be inverse pharology is defined by inv

$[(p, q, r, s) = ((-p \wedge q) \equiv (c \wedge d)) \equiv (c \wedge d)]$ which shows a type of equilateral among the couple of (p, q) and (r, s)

Theorem –

Let M = (M) be an algebra including only of its universe for any p, q, r, s ε M we have

$$(M) \mid = p : q : : r : s \longleftrightarrow (p = q \text{ and } r = s) \text{ or } (p \neq a \text{ and } r \neq s)$$

Proof - if p = q and r = s then (M) | = p : q : : r : s hold by its inner reflexivity.

If $p \neq q$ and $r \neq s$ then

M (p, q) U M (r, s) = M (q, p) U M (s, r) = φ

Which grant (M) = p: q : : r : s

By considering we have (M) = p $\rightarrow$ q : : r $\rightarrow$ s

We differentiate two cases

a) if M (p, q) U M (r, s) contains only of its insignificant justification then we must take $p \neq q$ and $r \neq s$ otherwise M $(p \leftrightarrow q :: r \rightarrow : s)$

include the only accessible significant justification if

n $\rightarrow$ n which shows that p = q and r = s.

First Isomorphism Theorem

For any isomorphism $\eta$: p ≅ q for any elements p, q, ε P.

(P, Q) | = p : q : : η G) : η (q)

Now consider the expansion P\` = ([p, q, r, s]), f) P given by

$$a \xrightarrow{f} b$$
c d

We have

$P^1 \mid\neq p : q :: r : s$

Analogical proportion among finite set

definition. - Let P, Q, R, S are consider as analogical proportion such that P: Q :: R: S if can be change into Q and R into S through the operation of addition and subtraction of the similar element to pond R.

Analogical proportion between sequences

suppose that l, m, n, o be the four sequences on $\sum$ which we can explain its analogy. So, we can conclude that these four sequences are in analogical proportion only if there exist its converse form means it becomes $l^1$, $m^1$, $n^1$, $o^1$ having similar measurement long with $\varepsilon^1$ and this satisfying the following conditions.

a) $l^1 = l$, $m^1 = m$, $n^1 = n$, $o^1 = o$
b) $\forall$ M$\varepsilon$ [ 1, n] the analogies are $l^1$m, $m^1$m, $n^1$n and $o^1$n in $\varepsilon^1$ in above sequence the variation is that they recite only insignificant analogy among the alphabets.

For example p: q: : q : p or

P: ~ : : p : ~

If we consider $\sum^1$ = {p, q, $\sigma$, $\rho$, R, S, $\lambda$] with its insignificant sequences

{p: q : : P : Q }, P : $\sigma$ : : q : $\rho$ and p : $\sigma$ : : Q : $\rho$ the following is the position of the sequence in which its analogical proportions are given on $\sum$

| | | | |
|---|---|---|---|
| P | $\lambda$ | Q | P |
| $\sigma$ | $\rho$ | Q | P |
| $\rho$ | $\lambda$ | p | $\lambda$ |
| $\rho$ | q | p | $\lambda$ |

Analogical Dissimilarity / inequality -

The analogical inequality among four sequences of binary numbers is shown in following table.

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| q | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| r | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| s | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| A U eq (p, q, r, s) | 0 | 1 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 2 | 0 | 1 | 0 | 1 | 1 | 0 |

In this part we studied as a relaxed analogy which showing bilateral variables such that P is to q almost r is to s to keep its coherence property with our analogical proportion, we used the term 'almost with the real positive value equivalent to zero in the care of increasing and true value of four variables consider to be in analogy we called that value is to be analogical inequalities showing properties with respect to analogy.

## III. APPLICATION OF ANALOGICAL PROPORTION

Processing of Image –

In the field of processing of image, it is relevant with its broad range by solving its problem related to image formation, filtering of image texture merging, tenure transformation and many more currently it was seen that the most benefited area of because of analogical proportion is the processing of image through the emission of deep neutral network containing operators which is to be subtle like convolution, pooling, etc. in this field. While taking the image formation the term AP is seen to be moray useful which pictures ASE means Analogy – preserving semantic embedding it act as me type of regularize for its image proportion.

Case – Based Reasoning –

It is also called similarity-based reasoning showing the reputation of problems with its solution it works for searching equivalent problems by adapting its solutions here we discussed inference of analogical proportion along with its pattern we can't use in this case because the problems are same in P with $P^1$ but solution is different and in analogical proportion problem and its solution is not given along with its description with its similar feature.

Transfer learning –

It is one type of analogical reasoning which performs on high level it is profitable when we need to increase our domain source in this, we can rearrange through reassign of existence of knowledge with variable tasks. Which is called as multi-task learning structure analogy is used for problems of transfer learning.

## IV. CONCLUSION

With the use of analogical proportion as a formal model linking with pairs for the formation new way for explaining learning patterns seen effectively as it is broody used in context form image processing, learning of multi-tasking. It doesn't work only for similarity but also for dissimilarity ultimately analogical proportion is helpful for used of high-level human precipitation affecting to flexible construction of representation to given context & growing the way of creativity.

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