Contribution Of Algebraic Graph Theory and Its Application

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Abstract—**In the current paper we discussed graph theory, especially algebraic graph theory. graph theory plays an important role in the field of mathematics we discussed graph theory along with its properties with the properties of Algebra in the paper we saw different types of graph which include simple graph, empty graph, automorphism of graph, transitive type of graph and the most important part of this graph is Cayley graph there are two types of clayey graph we studied here which include graph of dihedral group and Cayley graph along with symmetric group one more type of graph here we studied known as Cayley graph along with frobenous group all type of graph theory studied with the help of theorem and proof of that theorem.**

Index Terms—**Graph theory, Automorphism, dihedral Group, Symmetric group, frobenous group Cayley graph.**

I. INTRODUCTION

The theory of Graph has been fascinated mathematician, research scholar, scientist from broad its disciplines with its spreading of different terminologies and notations there are some words which used by various ways of different theory of graph along with its communities the application of theory of graph are much more the graph can be used for representing the model of network like network of computer, network of road, electricity & network in other word we can say that graph is to be considered as one of the tool which play important role in making design & the communication network analysis.

The algebraic graph theory is prime field of mathematics in which graphs are studied with the help of properties of Algebra which are related with matrices and many more tools of algebra especially the theory of spectral graph gives the relation among different properties of graph with the adjacent matrix spectrum or we can say matrix of Laplace and the association of theory of schemes and the study of configuration coherence discussed by the algebra which originated by associated matrix for the representation of group the Cayley graph play an important role. every apex of this graph related with every part of the corresponding group let we can consider that group ‘g’ A is the subset of group g such that a is a component of A then its converse a^{-1} also belongs to the A. Two apexes of G_1 and G_2 are combined with a boundary if the component $G_2G_1^{-1}$ graph shown by cay (g, A)

1.1 Definition

A graph λ contains the set of vertex φ set (λ) and an border set e (λ), where an edge is unpaired ordered with different apexes of λ generally we apply p q instead of {p, q} for describing its edge or border if we consider p q is an border then we can asked p & q are its nearby or they are neighborhood of each other so we can write p ~ q a vertex is proceeding with its limit only if it is one among these two vertices for the representation of binary relationship with the objects in the domain we can used a loop like edge in which its two endpoints have similar end points.

1.2 Types of Graphs

- A graph is said to be a simple graph if it doesn’t have any loop or numerous edges. The edges which have similar end points are called multiple edges.
- The graph having P apexes and q edges is called (p, q) graph.
- The order of the graph is said to be when there are many numbers of vertices in.
- and a size of λ is decided from the edges of the graph.

- An Empty graph is that graph which includes only one vertex and edges are absent if it is also called a null or trivial graph.
- A graph is said to be a complete graph only when its vertices are nearby with each part.
- A connected graph is that in which for each u, v, ε, V, 7, u, v in a path of λ a connected graph having no cycle is known as a tree.
- A Regular graph is a graph having the same number of degrees with each vertex.

1.3 Automorphism of Graph λ

Definition - The isomorphism from a graph λ with itself is called automorphism.

It is denoted by Aut (λ) which is the set of all automorphisms (λ) generating a group. The graph of the automorphism group we can generally see many places in the form of the permutations group along with its vertices.

1.4 Transitive Graphs

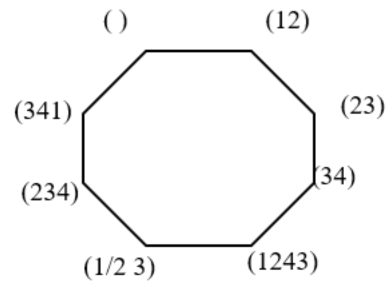
1. Vertex transitive graph: A graph is called a vertex transitive graph only when the group of automorphism acts as a login transitive.
2. Edge transitive graph: A graph is said to be edge transitive graph only when its group of automorphism acts along transitively on E (λ)

II. CAYLEY GRAPHS ALONG WITH DIHEDRAL GROUP

Group is the foundation of the construction of Cayley graph. Let A be a group which is abstract assume B $\underline{C}$ A such that $1_A \notin B$ then the Cayley graph of A correspondence with B in the graph λ = calyx (A, B) having the vertices along with its components factor of A and the set of its edges E (λ(A, B) {(l, m) | lm^{-1} ε A} the graph can be generated from this statement but for Cayley graph which was undirected then $A=A^{-1}$ if we consider that (B) is subgroup of A & l ε A, l ε (B) then the vertices 1_A,

l ε (U (A, B) = A is the part of its various components of that graph so that graph is said to be not connected. When the Cayley graph is connected at that A will create the whole group of A

Ex. The Cayley graph λ (A2 {1,2), (1,2,3), (1,4,3,2)}



Cay {A4 {(1 2) (2 3) (1 2 5) (1 2 3 4)}

D_{2n} is one type of dihedral group having the elements of 2n mean we can consider let it has an element m having order 2 and another element be l having n order with

$m^{-1}lm = l^{-1}$

Therefore D2n = <m, l | m^2 = ln = 1, $m^{-1}lm = l^{-1}$>

=<m, l | $m^2 = l^n = 1$, m^{-1} l m = l^{α}, α #1 (Mod n); α^2 = 1 (mod n)>

If we take the value of n = 2 then our dihedral group will be D4 which is nothing but abelian for h > 3, D2n is not abelian hence the components in the group of dihedral form can be written as

D2n = {1, m, ml, ml^2, …… ml^{n-1}, l: l^2 ……. L^{n-1}} in other way we can said that in are group the components can be given as $m^i l^i$ where I = 0, 1 & I = 0, 1 …. (n-1) so it was easy to explain clearly the multiplication product of any two components $m^i l^i$ $m^k l^x$ = $m^p l^q$ given as follows.

1. if K = 0 then p = I and q is equal to the i + 1 module n.
2. if K = 1 then p is the remainder of i + k having 2 modulo & q is the remainder of i α + x modulo n.

The component order in dihedral group D2n is

0 (1) = 1, 0 (ml^i) = 2, o<i<n-1 0 (l^i) = n where 0 < i < n -1 & if n is even than 0 $(l)^{n/2}$) 2

Theorem

If g is a finite group, then there is a group of Z m having an undirected and contrasting number of clayey graphs.

a) if n = 1, then the group Z m will be nonsexist & Cayley graph will be undirected.
b) If n = 2 or more than 2 then undirected Cayley graph number is one.
c) If n $\geq$ 4 & and if n is even then the undirected number of Cayley graphs is given as.

$$\begin{pmatrix} \frac{n}{2} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{n}{2} \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{n}{2} \\ 3 \end{pmatrix} + \ldots\ldots\ldots\ldots + \begin{pmatrix} \frac{n}{2} \\ \frac{n-2}{2} \end{pmatrix} + \begin{pmatrix} \frac{n}{2} \\ \frac{n}{2} \end{pmatrix}$$

d) if $n \geq 4$ and n is said to be odd then the undirected Cayley graph number are.

$$\begin{pmatrix} \frac{n-1}{2} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{n-1}{2} \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{n-1}{2} \\ 3 \end{pmatrix} + \ldots\ldots\ldots\ldots + \begin{pmatrix} \frac{n-1}{2} \\ \frac{n-3}{2} \end{pmatrix} + \begin{pmatrix} \frac{n-1}{2} \\ \frac{n-1}{2} \end{pmatrix}$$

Theorem =

Let $|A_S|$ be the Cayley graph number and λ = Cay (D_{2n}, A) with $|A|$ = S and n add then

$$|A_S| = \underset{L=0}{\overset{[\frac{s}{2}]}{©}} \begin{pmatrix} \frac{n}{S-2l} \end{pmatrix} \begin{pmatrix} \frac{n}{2} \\ l \end{pmatrix}$$

Where x be the integer which is greatest

Proof = let n is odd, then D2n – {1} = SUS

Where S = {p, pq, pq^2, ……. Pq^{n-1}} & {q, q^{n-1}, q^2, q^{n-2} …… $q^{\frac{n-1}{2}}$, $q^{\frac{n+1}{2}}$ | S | = n | S | = n-1 in the above set the element order in set S is 2 hence for each component of S is opposite to itself if q^i0 <i<n-1 arbitrary component of S then 0 (i/q) = n so the qn-1 be the inverse of means if s is component with a dihedral group of Dn having its subject A then q1 ε A, if q^{n-1} ε A, 0<i<n-1 since we have $\frac{2-1}{2} = \frac{n}{2}$ such pairs in S by this we can build the subset of Cayley group with S component by choosing 1 is parallel. Then the undirected Cayley graph λ = cay (D_{2n}, A) having an S component.

$$|A_S| = \underset{L=0}{\overset{[\frac{s}{2}]}{©}} \quad \frac{n}{S-2l} \quad \frac{\frac{n}{2}}{l}$$

III. CAYLEY GRAPH ALONG WITH SYMMETRIC GROUP BN

Here we developed some features of the Cayley group λ of the Bn symmetric group.

A set Ψ having its permutation with its bijection ϒ: Ψ Ψ if Ψ is finite set and | Ψ | = n then arrangement of Ψ is list all a_1, a_2, a_3 -------- a_n with no repetition of all the components of Ψ family the set Ψ along with its permutation is shown by BΨ is called symmetric group on Ψ if Ψ = {1, 2, ------ n} BΨ is generally shown by Bn which is known as symmetric group with 'n' letters.

If β ε Bn and i ε {1, 2, -------n} then β fixes I if β (i) = I & moves I if β (i) # let i_1, i_2, i_3, in be various integers M {1,2, ------- n} if βε Bn frees another integer and if. β (i_1) = i_2β (i_2) = i_3 ------- β (iq-1) = i_q, β (j_q) = i_1 then we can say β is known to be q – cycle where β will be the path distance q interchange of two cycle with I, & i_2__fixes everything here cycle 2 is known as transposition and cycle 1 is known for identity cycle having, I value for every component

∴ cycle 1 = (i) = 1 for ∀ value of I let we take two permutation σ ,ρ ε βn and there two permutations are said to be disjoint only if interchanged with 1 & fixed to the other if σ (i) I≠ then ρ (i) = I & if ρ (j) ≠ j then σ (j) = j a permutations along with its family $Ψ_1$, $Ψ_2$, $Ψ_3$, ------ $Ψ_t$ is said to be disjoint if each pair of component among them is disjoint hence they follow the properties of commutative and we can say that every permutation 6 ε βn is may be cycle or disjoint cycle along with its product means cycle 2 is a permutation with 2 product we can denote cycle 2 as a transposition even permutation is the permutation having product of even number of its transposition with ρ ε βn and odd permutation means that the permutations which is not even

Theorem

let | As| shows the number of Cayley graph λ = cay (An, A) with | A | = S then,

$$|A_k| = \underset{l=0}{\overset{[\frac{s}{2}]}{©}} \begin{pmatrix} \frac{|An|}{S-2l} \end{pmatrix} \begin{pmatrix} \frac{[n1-1-(An)]}{l2} \end{pmatrix}$$

here

$$(An) = \underset{s=1}{\overset{[\frac{n}{2}]}{©}} \frac{n1}{s!2s\,(n-2s)!}$$

Where x is the largest integer

Proof = let A*n = An \ {1_{An}} then A*n = BUB where B = {$σ_1$, $σ_2$, ------- $σ_c$} σ i'S are disjoint of cycle 2, 1 < C < [$\frac{n}{2}$]} & B = A*n / B because A is symmetric group therefore cycle S has order K and if αis cycle and φ is an p – cycle and those two cycle are S not joint then αφ = φ < having the component with order hence for every component of B with inverse oneself along with 1 < g < n there are $\frac{1}{g}$ [n (n-1) -------- (n – g + 1)] g- cycle in An and if Sg < n where 1<g<n then the number of σ εAn where σ is product of S disjoint

$$g - \text{cycle is } \frac{1}{s!gs} [n (n-1) ---------- (n-s\, g + 1)] \text{ so } |B| = © \sum_{s=1}^{\frac{n}{2}} \frac{ni}{s!2s (n-25)!}$$

Where we snow |S| by S (An) if A is Cayley subset of A with S element σ εA if σ^{-1}ε A because we have [$\frac{n!-1-S(An)}{2}$] such pair in B we can build a Cayley subset with S components by choosing l (o < l < [$\frac{5}{2}$] pairs B and S-2l elements of B this shows that the number of Cayley graph λ= Cay (An, A) is to be undirected and S be the elements is

$$| As | = \underset{l=0}{\overset{[\frac{s}{2}]}{©}} \quad \frac{|An|}{S-2l} \quad \frac{[n1-1-(An)]}{l2}$$

IV. CAYLEY GRAPH ALONG WITH FRESENIUS GROUP

Let A be group such that |A| = x y & x & y be the primes such that value of x is greater than y because 1, x, y, x y is the factor of |A| they may be a slow x of y subgroup as 1 + x m doesn't have y form because x > y.

That's why we can say that there is only one x – slow group with having X sub group with x order & if should be normal & let x = <a> then there are slow of y subgroup a) if y |x – 1| then we can't say that x = 1 + y m therefore there is identical slow q with Q subgroup with order q & it should be normal because X ∩ y both have subgroup x y so it must be factor of |x| = x & |y| = y therefore we can write

x ∩ y = (f) let

(|x y| = |x||y) = X z q~ = Z x y

Hence, we can say that group A is Abelian.

if y |x – 1| then we can write a slow p of q with the order of subgroup y such that y = (g) since X is normal $g^{-1}hg$ Ex. So, $g^{-1}hg = h^r$ for some 0<γ<x-1 hence we can write

$s\,h = g^{hr}\, g^{-1}$ and $h = g^{-y}\, h^{gy} = g^{1-y}\, (g^{-1}hg)\, gy^{-1} = g^{1-y}h^{r`}g^{y-1} = g^{2-y}h^{r2}g^{y-2} =$.

$g^{-y+y}h^{ry}g^{y-y} = h^{ry-1} = 1$ as | h | = x

∴ We can write γ^y = | (mod x) of γ =1 (mod x) then

$g^{-1}hg$ = h i.,e g h = g h = hg so A is abelian group & G ≅ Z x y if ϒ#! (mod x) then A = E x y = (g, h |g ^{y} = h ^{x} = 1, $g^{-1}hg$ = hrg#1 (mod x), r y = 1 (mod x)> such that every component a of E x y we can write in the form of a = g ^{I} h$^{i\ n}$ and (g ^{i} h^m) (g^m h^i) = g $^{I+j}$ h $^{x\ y}$- (m ++v^i) and a = g ^{i} h^m is identical for given x & y having x/y – 1 various value of r gives isomorphic groups such that γ &v^1 satisfied with γ1 = γI for some

i ε{1 ------ y-1} if E*x y <p: q |p ^{y}=q ^{x} = 1 $p^{-1}qp = q^{r1}$, r^1 # 1 (mod x)> so we can define

$$\rho: E\ x\ y - e^*x, y \text{ as } s\rho = \begin{cases} q - h \\ P - g^{\,i} \end{cases}$$

since ρ is isomorphism E x y ≅ E*x y

∴ the orders of component in the progenies group E $_{x\ y}$ are 0 (1) = 1

0 (g ^{i}) = y, 0< I < y - 1 & 0 < h m > = x where 0< m < x - 1

&0 (g ^{i} h^m) = y where 0<i<y-1 & 0<m<y-1

Theorem =

Vertex domination number of polynomial family graph

$$\gamma x\, (\gamma_{sn+1}) < [\frac{5n+1}{\sigma}] \text{ were } n> 6 \text{ \& it is even.}$$

Proof = let consider the graph of λu (Z_{sn+1}) Let U (λu (Z_{5n+1})) = {1, 2, 2, 3, -------5n}

& f (λu ($\lambda_{\rho n+1}$)) = {(p, q) εf (λu(λ_{5n+1})) / q = P + 1, Q = p + 5n; q = (5n) p+1, q = (5n) P + (5n), P = 1, 2, 3, 4, ------5n}

then each apex o λu (λ_{5n+1}) is of the form of 5m, 5m+1, 5m+2 & if n is even then n = 2l

Case Ist = if l is even

Assume set A = {5m / m ε {5, 11, 17, ------- n-1}

U {5m + s/mε{1, 6, 12, ------ n-5}} U

{5mlm = 0} to prove that A is an apex dominating set.

Sub case (a)

If P = 5m, M ε {0,6,12,18, ----- n-6} then by

q = p + 1, 5m is nearest to the 5m + 1, M ε {0, 6,12, -------- n-6}

when, M= n, p = 5m is dominated by {5k/k = 0}
∴ 5m, mε {0, 6, 12, ------ n-6} is dominated by A
If P = 5m, mε{1, 7, 13, ------ n-5} then by q = {(5n) p + (5n), mod (5n+1)
we get q = 5 (n-m) = $5m^1$ where m^1 = n-m & m^1ε{5, 11, 17, ----- n-1}
∴ 5 k, k ε{1, 7, 13 -------- n-5} is dominated by A
If P = 5m, m ε{2, 8, 14, ------- n-2} then by
q = p + (5n), mod (5m+1) we get q = 5(m-1) + 2
= 5m1 + 2 where m1 – m-1 & m1ε{1, 7, 13, --- n-5}
∴ 5m, m {1, 7, 13 -------- n-2} is dominated by A

Case II – If l is odd
Assume set A = {5k | k ε{5, 11, 17, ------- n-5}}
U {5m-1 / M ε{0, 6, 12, ------- n-2} by q = p+1,
∴ 5m is dominated by A
If p = 5m, mε{1, 7, 13, ----- n-1}
then by q = 5n) p+1, mod (5n+1) we get q = 5(n-m) + 2 = 3m`+2
5m, mε{2, 8, 14, -------n} dominated by A
If P = 5m + 2, εm {2, 8, 14, ----- n-6}
Then by q = (5n) p + 1 mod (5n + 1) we get
Q = 5 (n-m) = 5m1 where m1 = n-m & m1 ε{2,8,14, -----n-6} is adjacent with
5m1, m1ε{5, 11, 17, ---n-5} hence 5m+2, mε{2, 8, 14, -----n-6} is dominated by A
this A is vertex dominated set Now

$$|A| = \frac{n-2}{6} + \frac{n+2}{6} + \frac{n+2}{6} + \frac{n+2}{6} + \frac{n+2}{6} = \frac{5n+2}{6}$$

$$= \frac{5n+1}{6} + 1 \ \{\square\ n = 2l \ \&\ l \text{ is odd}\}$$

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