# Subdivision Of Super Geometric Mean Labeling for Some Graphs

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***Abstract*—Let f: V(G) →{1,2,…,p+q} an injective function. For a vertex labeling "f" the induced edge labeling f* (e=uv) is defined by**
**f*(e) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$. Then f is called a Super Geometric mean labeling if {f(V(G))}∪ {f(e): e∈E(G)}= {1,2,…p+q}. A graph which admits Super Geometric Mean labeling is called Super Geometric mean graph.**
**In this paper we prove that the Subdivision of Super Geometric mean labeling for some graphs.**

***Index Terms*—Graph, Subdivision graph, Geometric mean graph, Super Geometric mean graph.**

## I. INTRODUCTION

Throughout this paper, we consider only finite, undirected, simple graphs. Let G be a graph with p vertices and q edges. For all detailed survey of graph labeling of we refer to Gallian [1]. For all other Standard terminology and notations, we follow Harary [2]. We investigate the Super Geometric mean labeling behaviour of S(G) for some standard graphs. The definitions and other information which are useful for our present investigation.

Definition 1.1:
A graph G = (V, E) with p vertices and q edges is called a Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labeled with,
f(e=uv) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$, then the edge labels are distinct. In this case "f" is called Geometric mean labeling of G.

Definition 1.2:
If e=uv is an edge G and w is not a vertex of G, then e is said to be subdivided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the Subdivision graph of G and is denoted by S(G).

## II. MAIN RESULTS

Theorem: 2.1
$S(P_nAK_1)$ is a Super Geometric mean graph.
Proof:
Let G be the Comb graph.
Let $P_n$ be the Path $u_1, u_2 \ldots u_n$ and $v_i$ be the pendant vertices attach to each vertex of $P_n$
Let $w_i$ be the vertices which subdivide $u_i$ and $u_{i+1}$ and $z_i$ be the vertices which subdivide $u_i$ and $v_i$.
Let $G_1=S(P_nAK_1)$ be the graph obtained by subdividing each edge of a Path in G.
Define a function, f: $V(G_1) \to \{1,2,\ldots,p+q\}$ by,
$f(u_i) = 8i-3, \quad 1 \le i \le n$
$f(w_i) = 8i+1, \quad 1 \le i \le n-1$
$f(z_i) = 8i-5, \quad 1 \le i \le n$
$f(v_1) = 1$
$f(v_i) = 8i-9, \quad 2 \le i \le n$

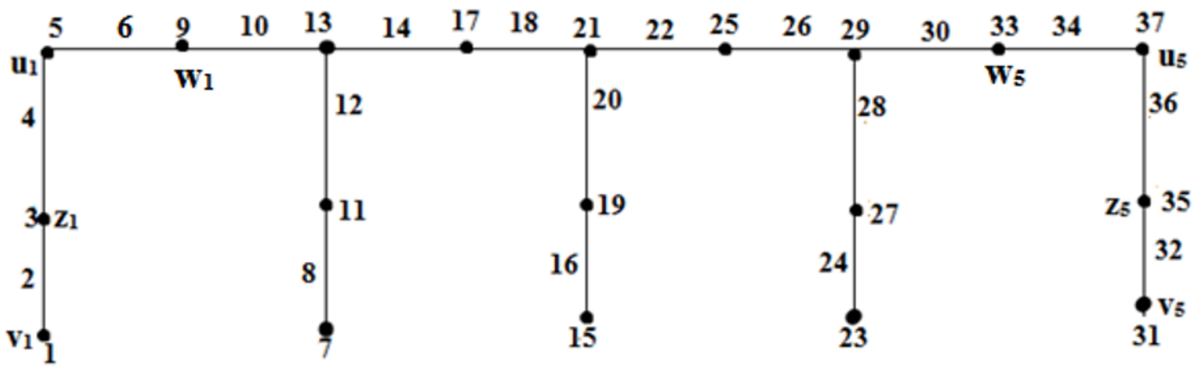


Figure: 1

From the above labeling pattern, we get distinct edge labels

Hence $G_1$ is a Super Geometric mean graph.

Theorem :2.2

Let G be the Comb graph and G′ be the graph obtained by attaching $K_{1,2}$ at each pendant vertex of G. Let $G_1$ be the graph obtained by subdividing each of a Path in G′. Then $G_1=S[(P_nAK_1)\ AK_{1,2}$is a Super Geometric mean graph.

Proof:

Let G be the Comb graph and G′ be the graph obtained attaching $K_{1,2}$ at each pendant vertex of G.

Let its vertices be $u_i$, $v_i$, $x_i$ and $y_i$, $1\le i\le n$.

Let $G_1$ be the graph obtained by subdividing each edge of a Path in G′.

Let $w_i$, $1\le i\le n-1$ be the vertices which subdivide $u_i$ and $u_{i+1}$.

Define a function $f: V(G_1) \to \{1,2,\dots,p+q\}$ by,

$f(u_1) = 5$

$f(u_i) = 10i-3,\ 2\le i\le n$

$f(w_i) = 10i+3,\ 1\le i\le n-1$

$f(v_1) = 7$

$f(v_i) = 10i-5,\ 2\le i\le n$

$f(x_1) =1$

$f(x_i) = 10i-2,\ 2\le i\le n$

$f(y_1) =3$

$f(y_i) = 10i-10,\ 2\le i\le n$

All the edge labels are distinct.

Thus, both vertices and edges together get distinct labels from $\{1,2,3,\dots,p+q\}$

Hence $G_1= S[(P_nAK_1)AK_{1,2})$ is a Super Geometric mean graph.

Example 2.3: $S[(P_5AK_1)\ AK_{1,2})$ is shown below.

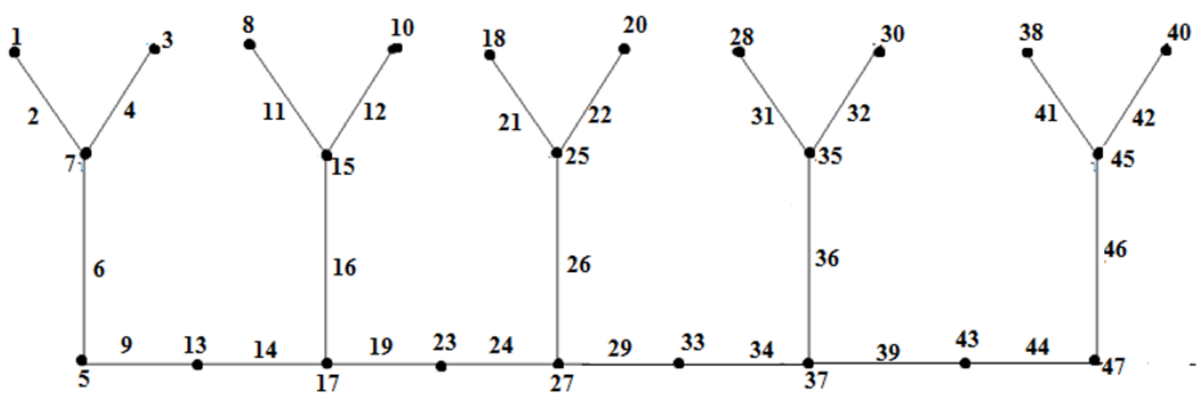


Figure:2

Theorem :2.4

Let G be the Comb graph and G′ be the graph obtained by attaching $K_3$ at each pendant vertex of G. Let $G_1$ be the graph obtained by subdividing each edge of a Path in G′. The $G_1= [(P_nAK_1)\ AK_3)$ is a Super Geometric mean graph.

Proof:
Let G be the Comb graph and G′ be the graph obtained by attaching $K_3$ at each pendant vertex of G.
Let its vertices be $u_i$, $v_i$, $x_i$ and $y_i$, $1 \le i \le n$.
Let $G_1$ be the graph obtained by subdividing each edge of a Path in G′.
Let $w_i$, $1 \le i \le n-1$ be the vertices which subdivide $u_i$ and $u_{i+1}$
Define a function f: $V(G_1) \rightarrow \{1,2,\ldots,p+q\}$ by,
$f(u_i) = 11i-3, 1 \le i \le n$
$f(w_i) = 11i+4, 1 \le i \le n-1$
$f(v_i) = 11i-5, \quad 1 \le i \le n$
$f(x_1) = 1$
$f(x_i) = 11i-13, 2 \le i \le n$
$f(y_1) = 4$
$f(y_i) = 11i-9, \quad 2 \le i \le n$
The edge labels are distinct.
Thus, f provides a Super Geometric mean labeling of G.

Example 2.5: $S[(P_5 A K_1) A K_3]$ is displayed below.

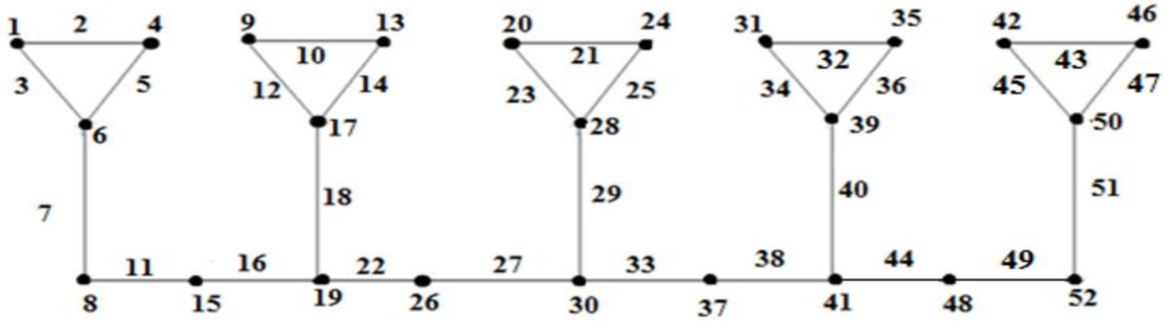


Figure :3

Theorem :2.6
Let $L_n$ $n \ge 2$, be the Ladder graph. Let G be the graph obtained by subdividing each edge of a Path in $L_n$. Then G is a Super Geometric mean graph.
Proof:
Let $L_n = P_n x P_2$ be a Ladder
$L_n$ has 2n vertices and 3n-2 edges.
Let $u_i$ and $v_i$ be the vertices of $L_n$, $1 \le i \le n$
Let G be the graph obtained by subdividing each edge of a Path in $L_n$.
Let $x_i$, $1 \le i \le n-1$ be the vertices which subdivide $u_i$ and $u_{i+1}$.
Let $y_i$, $1 \le i \le n-1$ be the vertices which subdivide $v_i$ and $v_{i+1}$
Define a function f: $V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,
$f(u_1) = 1$
$f(u_i) = 9i-6, 2 \le i \le n$
$f(v_1) = 4$
$f(v_i) = 9i-8, 2 \le i \le n$
$f(x_i) = 9i-1, 1 \le i \le n-1$
$f(y_i) = 9i-3, 1 \le i \le n-1$
we get distinct edge labels
In view of the above labeling pattern, f provides a Super Geometric mean labeling of G.
Hence G is a Super Geometric mean graph.

Example 2.7: A Subdivision of each edge of a Path in $L_5$ is given below.

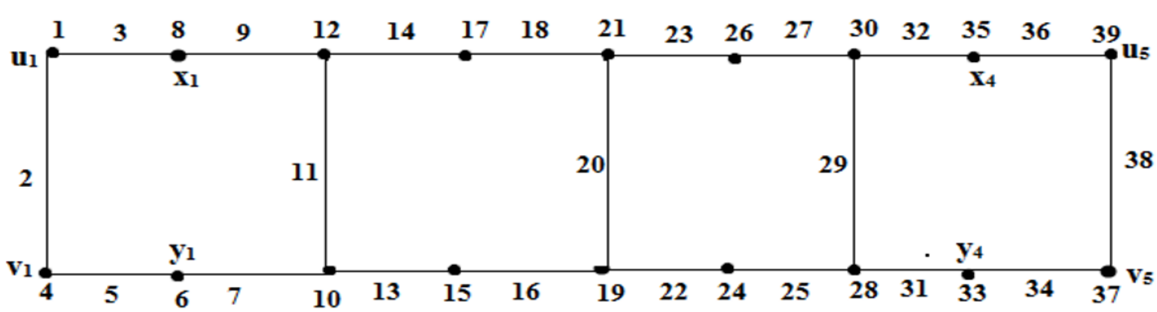


Figure: 4

Theorem :2.8

Let $P_n$ be the Path $u_1u_2\ldots u_n$. Let G be the graph obtained by attaching pendant edges to both sides of each vertex of a Path $P_n$. Let $G_1$ be the graph obtained by subdividing the Path $P_n$. Then $G_1$ is a Super Geometric mean graph.

Proof:

Let $P_n$ be the Path $u_1\ u_2\ldots u_n$

Let G be the graph obtained by attaching pendant edges to both sides of each vertex of a Path $P_n$.

Let $u_i$, $v_i$, $w_i$, $1\leq i\leq n$ be the vertices of G.

Let $G_1$ be the graph obtained by subdividing the Path $P_n$.

Let $t_i$, $1\leq i\leq n-1$ be the vertices which subdivide the Path $P_n$

Define a function f: $V(G_1) \rightarrow \{1,2,\ldots,p+q)$ by,

$f(v_1) = 1$

$f(v_2) = 9$

$f(v_i) = 8i-9,\ 3\leq i\leq n$

$f(u_1) = 5$

$f(u_i) = 8i-5,\ 2\leq i\leq n$

$f(w_1) = 3$

$f(w_i) = 8i-3,\ 2\leq i\leq n$

$f(t_1) = 7$

$f(t_i) = 8i+1,\ 2\leq i\leq n-1$

We get distinct edge labels.

Thus, f provides a Super Geometric mean labeling of $G_1$.

Example 2.9: A subdivision of each edge of a Path $P_6$ of Caterpillar is shown below.

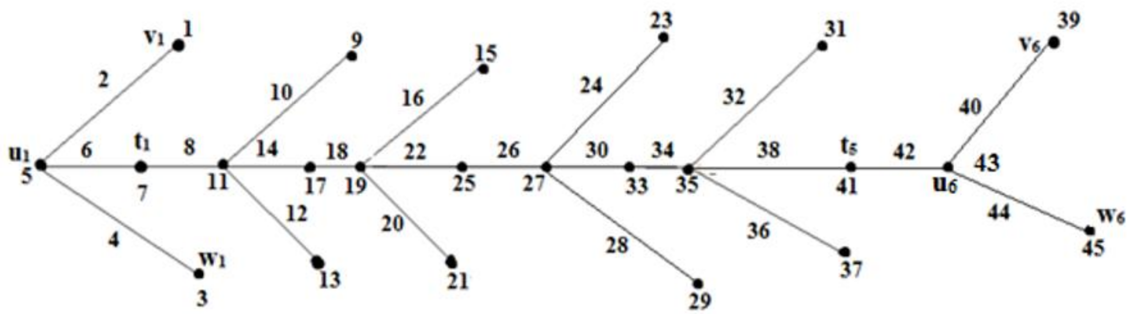


Figure:5

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