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A Comparative Analysis of Modified Vogel's Approximation Methods for Solving Transportation Problems

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***Abstract*—In recent years, global economic challenges have compelled organizations to adopt effective cost-reduction strategies, with transportation cost optimization emerging as a key area of focus. Xlife Group has also been affected by these economic pressures, prompting this study to investigate methods for minimizing its transportation costs. A comparative analysis was conducted between the classical Vogel's Approximation Method (VAM) and its modified variants in solving transportation problems involving the distribution of manufactured goods from Xlife Group's supply locations to various destinations. The Modified Vogel's Approximation Methods, incorporating standard deviation, distribution indicators, and average cost, were employed to obtain initial basic feasible solutions and the corresponding optimal transportation plans. The results indicate that all the methods generally produced the same minimum transportation cost, with only one instance showing a difference in the obtained solution.**

***Index Terms*—Optimization; Vogel's Approximation Method; Standard Deviation; Transportation Cost.**

I. INTRODUCTION

Transportation plays a crucial role in the movement of goods and services for companies, organizations, institutions, and individuals. In periods of economic uncertainty, minimizing transportation costs becomes essential for improving operational efficiency and profitability. Transportation problems, a fundamental application of operations research, provide effective mathematical models for determining the most economical distribution of goods from multiple sources to multiple destinations.

The transportation problem was first introduced by Hitchcock [1], who established its fundamental mathematical framework. Subsequently, Gass [2] investigated solution procedures based on the simplex method. Juman et al. [3] applied Vogel's Approximation Method (VAM) to solve unbalanced transportation problems, while Sarbjit Singh [4] examined degenerate transportation problems using the North-West Corner Method, Least Cost Method, and VAM. Edokpia and Amiolemhen [5] employed genetic algorithms to minimize transportation costs for a beverage manufacturing company in Nigeria. Similarly, Lakshimi [6] compared various techniques for obtaining preliminary feasible solutions that lead to minimum transportation costs. Using data from the BUA Group and CCCNN Transport, Aliyu et al. [7] demonstrated that the North-West Corner Method, Least Cost Method, and VAM all produced optimal solutions with reduced transportation costs. Rafi et al. [8] compared several existing methods and concluded that VAM consistently provided superior initial feasible solutions. Likewise, Victor-Edema and Akehwe [9] successfully applied four classical transportation methods to reduce the transportation costs incurred by Dangote Cement, Port Harcourt.

To enhance the performance of classical transportation algorithms, several modified versions have been proposed. Das et al. [10] introduced a modified VAM by redefining the penalty cost calculation. Akpan et al. [11] proposed a hybrid VAM based on standard deviation, while Ullah et al. [12] developed a distribution indicator-based VAM that demonstrated improved performance over the conventional approach. Agaie et al. [13] presented an improved VAM by replacing the traditional penalty difference with average cost values. Karagu and Sahin [14] introduced the Karagu–Sahin Approximation Method, which showed competitive performance compared with the North-West Corner Method and VAM. Prasad and Singh [15] proposed a modified Least Cost Method, and Nopiyana et al. [16] developed an enhanced version of the ASM method for solving transportation problems. Furthermore, Crankson et al. [17] applied Akpan's modified VAM to a mining company's transportation problem and reported significant cost savings of GH¢996,315 over an eight-month period, demonstrating the practical effectiveness of the modified approach.

These studies indicate that although the classical Vogel’s Approximation Method remains one of the most efficient techniques for obtaining high-quality initial basic feasible solutions, its modified versions often improve computational efficiency, solution quality, and practical applicability across different transportation scenarios.

II. METHODOLOGY

Before solving a transportation problem, it is essential to determine whether it is balanced or unbalanced. A balanced transportation problem is characterized by equality between the total supply and total demand, whereas an unbalanced problem requires the introduction of a dummy source or dummy destination to compensate for the imbalance. Once the problem has been identified as balanced, an appropriate solution technique can be selected to obtain an efficient initial basic feasible solution. In this study, all the transportation problems considered are balanced; therefore, no dummy variables are required. The comparative analysis focuses on the conventional Vogel’s Approximation Method (VAM) and two of its modified variants to evaluate their effectiveness and computational performance in solving balanced transportation problems.

Conventional Vogel’s Approximation Method (VAM) [18]

1. Verify that the transportation problem is balanced, i.e., the total supply equals the total demand. If the problem is unbalanced, introduce a dummy source or destination to restore balance before proceeding.
2. Compute the penalty for each row and column by determining the difference between the two smallest transportation costs.
3. Identify the row or column with the highest penalty value. In the selected row or column, allocate the maximum feasible quantity, equal to the minimum of the available supply and demand, to the cell having the lowest transportation cost.
4. Update the remaining supply and demand values, and eliminate any satisfied row or column from further consideration.
5. Repeat Steps 2–4 until all supplies and demands have been completely satisfied.

Distribution Indicator-Based Modified Vogel’s Approximation Method (DIVAM) [12]

1. Ensure that the transportation problem is balanced. If necessary, introduce a dummy source or destination to balance the problem before continuing.
2. For each row, calculate the positive difference between the largest cost element and every other cost element, and record these values.
3. Similarly, for each column, calculate the positive difference between the largest cost element and every other cost element, and record these values.
4. Construct a new matrix by summing the corresponding row and column difference values obtained in Steps 2 and 3.
5. Compute the distribution indicator for each row and column by finding the difference between the two largest values in the newly formed matrix.
6. Select the row or column with the highest distribution indicator. Allocate the maximum feasible quantity, equal to the minimum of the corresponding supply and demand, to the cell having the largest value within the selected row or column.
7. Update the transportation table by eliminating the exhausted row or column.
8. Repeat Steps 5–7 until all supply and demand requirements have been fulfilled.

Standard Deviation-Based Modified Vogel’s Approximation Method (SDVAM) [11]

1. Confirm that the transportation problem is balanced. If not, add an appropriate dummy source or destination to achieve balance.
2. Calculate the standard deviation of the transportation costs for each row and column using $\left(\sigma = \sqrt{(x_i - \mu)^2/n}\right)$

where (x_i) denotes the transportation cost, μ is the mean transportation cost, and (n) is the number of cost elements.

3. Identify the row or column with the highest standard deviation. In the selected row or column, allocate the

maximum feasible quantity to the cell with the minimum transportation cost.
4. Update the remaining supply and demand values and remove any satisfied row or column from further consideration.
5. Repeat Steps 2–4 until all transportation requirements have been satisfied.

Average Cost-Based Modified Vogel's Approximation Method (ACVAM) [13]
1. Verify that the transportation problem is balanced. If necessary, introduce a dummy source or destination before proceeding.
2. Compute the average cost for each row and column using
$$\left(\bar{\bar{x}} = \frac{highest\ cost + lower\ cost}{2}\right)$$
3. Select the row or column having the lowest average cost. Allocate the maximum feasible quantity, equal to the minimum of the available supply and demand, to the cell with the lowest transportation cost in that row or column. If multiple rows or columns have the same minimum average cost, choose the one with the next most favorable average value.
4. Update the transportation table by eliminating any row or column whose supply or demand has been completely satisfied.
5. Repeat Steps 2–4 until all supplies and demands have been fully allocated.

III. DATA ANALYSIS

Xlife Company operates three manufacturing facilities, namely A, B, and C, with monthly production capacities of 280, 320, and 400 (hundreds of units), respectively. These products are to be transported to four destination points—Ile-Ife, Ede, Osogbo, and Imesi-Ile—which have corresponding monthly demands of 200, 240, 360, and 200 units, respectively. The unit transportation costs (in Nigerian Naira) from each production facility to the respective destinations are provided in the transportation cost matrix below.

Table 1: Transportation Tableau from Xlife Company

Source	Ile Ife	Ede	Osogbo	Imesi Ile	Supply
A	5	12	17	15	280
B	16	18	14	10	320
C	22	25	13	9	400
Demand	200	240	360	200	1000

- Model Formulation and Solution

Let x_{ij} = units of product to be moved from production i (i = 1,2,3) to destinations j (j = 1,2,3,4).
The transportation problem as a Linear Programming model takes the form:

Minimize (total transportation cost)
$Z = 5x_{11} + 12x_{12} + 17x_{13} + 15x_{14} + 16x_{21} + 18x_{22} + 14x_{23} + 10x_{24} + 22x_{31} + 25x_{32} + 13x_{33} + 9x_{34}$

Subject to the constraints
$x_{11} + x_{12} + x_{13} + x_{14} = 280,$
$x_{21} + x_{22} + x_{23} + x_{24} = 320,$
$x_{31} + x_{32} + x_{33} + x_{34} = 400$ (supply)
$x_{11} + x_{21} + x_{31} = 200,$
$x_{12} + x_{22} + x_{32} = 240,$
$x_{13} + x_{23} + x_{33} = 360,$
$x_{14} + x_{24} + x_{34} = 200$ (demand)
and $x_{ij} \geq 0$ for i = 1,2,3 and j = 1,2,3,4.

Table 2: Tables Showing Procedures for the Basic Feasible Solution to Problem 1 Using the Vogel's Approximation Method

Source	Ile Ife	Ede	Osogbo	Imesi Ile	Supply	RD	RD	RD	RD
A	5	12	17	15	280	7	3		
B	16	18	14	10	320	4	4	4	4
C	22	25	13	9	400	4	4	4	4
Demand	200	240	360	200					
CD	11	6	1	1					
CD		6	1	1					
CD		7	1	1					

Initial basic feasible solution by Vogel = 5 × 200 + 12 × 80 + 18 × 160 + 9 × 200 + 13 × 200 + 14 × 160 = 11480

Table 3: Tables Showing Procedures for the Basic Feasible Solution to Problem 1 Using the Distribution Indicator Based Modified Vogel's Approximation Method

Source	Ile Ife	Ede	Osogbo	Imesi Ile	Supply
A	5	12	17	15	280
B	16	18	14	10	320
C	22	25	13	9	400
Demand	200	240	360	200	

Adding up the numbers in subscripts and superscripts gives a reduced matrix below. The row and column distribution indicators are obtainable by subtracting the two biggest elements in each row and column. The min (demand, supply) is then allocated to the largest cell in the row or column with the highest distribution indicator.

Iteration 2

Source	Ile Ife	Ede	Osogbo	Imesi Ile	Supply	RD	RD	RD	RD
A	29	18	0	2	280	11	16		
B	8	7	7	13	320	5	6	6	0
C	3	0	16	22	400	6	6	6	16
Demand	200	240	360	200					
CD	21	11	9	9					
CD		11	9	9					
CD		7	9	9					
		7	9						
		7	9						

The basic feasible solution based on this table according to the Distribution Indicator Based Modified Vogel's Approximation = 5 × 200 + 12 × 80 + 9 × 200 + 13 × 200 + 18 × 160 + 14 × 160 = 11480

Table 4: Table for Obtaining the Initial Basic Feasible Solution to Problem 1 Via the Standard Deviation Based Modified Vogel's Approximation Method

Source	Ile Ife	Ede	Osogbo	Imesi Ile	Supply	SD	SD	SD	SD
A	5	12	17	15	280	4.6	2.1	2.5	2.5
B	16	18	14	10	320	3.0	3.3	2	2
C	22	25	13	9	400	6.5	6.8	6	
Demand	200	240	360	200					
	7.0	5.3	1.7	2.6					
	5.3	1.9							
	3.0	1.5							

Based on this above table on standard deviation-based Vogel modification, the initial basic feasible solution = 5 × 200 + 9 × 200 + 13 × 200 + 12 × 80 + 18 × 160 + 14 × 160 = 11480

Table 5: Table for Obtaining the Initial Basic Feasible Solution to Problem 1 Via the Average Cost Based Modified Vogel's Approximation Method

Source	Ile Ife	Ede	Osogbo	Imesi Ile	Supply	RA	RA	RA	RA	RA
A	5	12	17	15	280	11	14.5	14.5		
B	16	18	14	10	320	14	14	16	16	16
C	22	25	13	9	400	17	17	19	19	

Demand	200	240	360	200						
CA	13.5	18.5	15	12						
CA		18.5	15	12						
CA		18.5	15							
CA		18	14							

Based on this above table on the average cost-based Vogel modification, the initial basic feasible solution = 5 × 200 + 9 × 200 + 12 × 80 + 13 × 200 + 14 × 160 + 18 × 160 = 11480.

- Problem 2: The attempt is to minimize the cost of transportation from Xlife business based on data given in Table 6 using the methods explained earlier.

Table 6: Transportation Tableau from Xlife Business

Source	P	Q	R	T	Supply
A	2	2	2	4	1000
B	4	6	4	3	700
C	3	2	1	7	800
Demand	900	700	500	400	

The overview to the basic feasible solution for Problem 2 above is drafted in Tables 7, 8, 9, and 10 using the aforementioned techniques.

Table 7: Framework for the Basic Feasible Solution to Problem 2 Via the Vogel's Approximation Method

Source	P	Q	R	T	Suppl y	R D	R D
A	2	2	2	4	1000	0	1
B	4	6	4	3	700	1	1
C	3	2	1	7	800		
Deman d	90 0	70 0	50 0	40 0			
CD	1	0	1	1			
CD	1	0		1			

The basic feasible solution based on Table 6 via the Vogel's Approximation Method = 1 × 500 + 2 × 300 + 2 × 600 + 4 × 300 + 3 × 400 + 2 × 400 = 5500

Table 8: Outline for the Basic Feasible Solution to Problem 2 Using the Distribution Indicator Based Modified Vogel's Approximation Method

Iteration 1

Source	P	Q	R	T	Supply
A	2	2	2	4	1000
B	4	6	4	3	700
C	3	2	1	7	800
Demand	900	700	500	400	

Iteration 2

Source	P	Q	R	T	Supply	RD	RD	RD	RD
A	4	6	4	3	1000	2	2	2	2
B	2	0	2	7	700	5	5	2	2
C	5	9	9	0	800				
Demand	900	700	500	400					
	1	3	5	4					
	1	3		4					
	1	3							
	2	6							

The basic feasible solution based on this problem based on Table 8 according to the Distribution Indicator based Modified Vogel's Approximation = 1 × 500 + 3 × 400 + 2 × 300 + 2 × 400 + 2 × 600 + 4 × 300 = 5500

Table 9 and Table 10

Source	P	Q	R	T	Supply	SD	SD	SD	SD	SD
A	2	2	2	4	1000	0.9	0.9	1.0	2.0	
B	4	6	4	3	700	1.1	1.3	1.3	0.5	0.5

C	5	2	1	7	800	2.3	2.2			
Demand	900	700	500	400						
	0.8	1.9	1.3	1.7						
	0.8	1.9		1.7						
	1.0	2.0		0.5						
	1.0			0.5						

Based on Table 9 = 5500

Table 10

Source	P	Q	R	T	Supply	RA	RA	RA	RA	RA
A	2	2	2	4	1000	3	3	3		
B	4	6	4	3	700	4.5	4.5	4.5	4.5	3
C	5	2	1	7	800	4	4.5	4.5	4.5	
Demand	900	700	500	400						
	3	4	2.5	5						
	3	4		5						
		4		5						
		4		5						

Based on Table 10 = 6100

- Problem 3. Obtain the initial basic feasible solution for the transportation problem sourced from Xlife Exports put on Table 11 using aforementioned methods.

The overview to the basic feasible solution for Problem 3 above is drafted in Tables 12, 13, 14 and 15 through earlier introduced procedures

Table 11: Transportation Tableau from Xlife Exports

Source	P	Q	R	T	Supply
A	14	19	6	24	200
B	3	17	10	18	450
C	11	7	11	12	350
Demand	150	350	80	420	

Table 12: Framework for the Basic Feasible Solution to Problem 3 Via the Vogel's Approximation Method

Source	P	Q	R	T	Supply	RD	RD	RD
A	14	19	6	24	200	8	8	18
B	3	17	10	18	450	7	7	8
C	11	7	11	12	350	4		
Demand	150	350	80	420				
CD	8	10	4	6				
CD	11		4	6				
CD			4	6				

The basic feasible solution based on Table 12 via the Vogel's = $7 \times 350 + 3 \times 150 + 6 \times 80 + 24 \times 120 + 18 \times 300 =$ 11,660.

Tables 13 and 14

Table 13: Distribution Indicator Based Modified Vogel's Approximation

Iteration 1

Source	P	Q	R	T	Supply
A	14	19	6	24	200
B	3	17	10	18	450
C	11	7	11	12	350
Demand	150	350	80	420	

Iteration 2

Source	P	Q	R	T	Supply	RD	RD	RD	RD
A	10_{14}	5_{19}	23_{6}	0_{24}	200	13	18	5	0
B	26_{3}	3_{17}	9_{11}	6_{18}	450	17	3	3	6
C	4_{11}	17_{7}	1_{11}	12_{12}	350	5	5	5	12
Demand	150	350	80	420					
CD	16	12	14	6					
CD		12	4	6					
CD		12		6					
CDType equation here.				6					

The basic feasible solution ... = 11660

Table 14: Standard Deviation Based Modified Vogel's Approximation

Source	P	Q	R	T	Supply	SD	SD	SD
A	14	19	6	24	200	6.7	4.1	2.5
B	3	17	10	18	450	6.0	6.9	0.5
C	11	7	11	12	350	1.92	2.2	2.5
Demand	150	350	80	420				
	4.6	5.3	2.2	4.9				
	4.6	5.3		4.9				
		5.3		4.9				

Based on Table 14 ... = 11660

Table 15: Table for Obtaining the Initial Basic Feasible Solution to Problem 3 Via the Average Cost Based Modified Vogel's Approximation Method

Source	P	Q	R	T	Supply	RA	RA	RA
A	14	19	6	24	200	15	15	21.5
B	3	17	10	18	450	10.5	14	17.5
C	11	7	11	12	350	9.5	9.5	9.5
Demand	150	350	80	420				
CA	8.5	13	8.5	18				
CA		13	8.5	18				
CA		13		18				

Based on Table 15, on average cost-based Vogel modification, the initial basic feasible solution = 3 × 150 + 6 × 80 + 7 × 350 + 18 × 300 + 24 × 120 = 11660.

IV. RESULT AND DISCUSSION

The same procedure was applied to the second transportation problem to obtain the initial basic feasible solution. Subsequently, each initial basic feasible solution was tested for optimality using the Modified Distribution (MODI) method. The optimality test was carried out by computing the opportunity cost for each unallocated cell using the expression $E_{ij} = C_{ij} - (u_i + v_j)$, where C_{ij} represents the transportation cost associated with the (i, j) cell, and u_i and v_j denote the row and column potentials determined from the occupied cells. An extract from the data presented in Table 2 is used below to illustrate the application of the MODI method.

	$v_1 = 5$	$v_2 = 12$	$v_3 = 8$	$v_4 = 4$
$u_1 = 0$	5	12	17	15
$u_2 = 6$	16	18	14	10
$u_3 = 5$	22	25	13	9

Assume, without loss of generality, that $u_1 = 0$. Using the relationship $C_{ij} = u_i + v_j$ for all allocated cells, the row and column potentials are determined sequentially. For instance, since $C_{11} = 5$, we have

$5 = u_1 + v_1 = 0 + v_1$,
which gives $v_1 = 5$. Similarly, from $C_{12} = 12$,
$12 = u_1 + v_2 = 0 + v_2$,
yielding $v_2 = 12$. Applying the same procedure to the remaining allocated cells, all other row and column potential values are obtained.

After determining the potential values, the optimality test is carried out using the Modified Distribution (MODI) criterion,
$E_{ij} = C_{ij} - (u_i + v_j)$,
where E_{ij} denotes the opportunity cost associated with each unallocated cell.

The computed opportunity costs are as follows:
$E_{13} = 17 - (0 + 5) = 12 > 0$,
$E_{14} = 15 - (0 + 4) = 11 > 0$,
$E_{21} = 16 - (6 + 5) = 5 > 0$,
$E_{24} = 10 - (6 + 4) = 0 \geq 0$,
$E_{31} = 22 - (5 + 5) = 12 > 0$,
$E_{32} = 25 - (5 + 12) = 8 > 0$.
Since all the computed opportunity costs satisfy $E_{ij} \geq 0$, the obtained basic feasible solution is confirmed to be optimal. Similar optimality tests were performed for the remaining transportation problems, although the detailed computations are omitted for brevity. In every case, the opportunity costs were non-negative, confirming the optimality of the corresponding basic feasible solutions.

The only exception was the solution generated using the Average Cost approach, where the resulting transportation cost exceeded those obtained by the other methods. This indicates that the Average Cost method did not produce an optimal solution for that particular transportation problem.

Table 16 summarizes the optimal transportation costs obtained by each method for all the problems considered. The results identify the minimum and most practical transportation costs, thereby providing decision-makers with cost-effective allocation strategies. Overall, the findings demonstrate that nearly all the transportation solution techniques investigated in this study successfully produced optimal solutions, with the Average Cost method being the only notable exception in one instance.

Table 16: Optimal Solution to the Problems Considered

Problem	Methods	Transportation Cost
1	Vogel's, modified versions based on average cost, distribution indicator, standard deviation	11480
2	Vogel's, modified versions based on distribution indicator, standard deviation	5500
3	Vogel's, modified versions based on average cost, distribution indicator, standard deviation	11660

V. CONCLUSION

The following conclusions can be drawn from the results obtained:

- The classical Vogel's Approximation Method (VAM), together with its modified versions based on the distribution indicator and standard deviation, produced identical initial basic feasible solutions. These solutions successfully satisfied the Modified Distribution (MODI) optimality test, resulting in the same optimal transportation cost.
- The Average Cost-based modification did not consistently produce the optimal solution. Therefore, further refinement of this approach is required to ensure that it can reliably generate optimal solutions for all transportation problems.
- Among the proposed approaches, the Distribution Indicator-based method is computationally simpler and easier to implement. However, it generally requires more iterations and a longer computational time than the Standard Deviation-based method.

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