Parameter Estimation of Shanker Distribution Under Type II Censored Sample

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***Abstract*—The study deals with estimating the unknown parameter and the reliability measures of Shanker distribution using data obtained from a Type-II censored sample. Various Bayesian estimators are derived under both symmetric and asymmetric loss functions, including squared error, LINEX, and general entropy losses. The Lindley approximation technique is applied to compute these Bayesian estimates. Through simulation experiments, the Bayesian estimators are compared with the maximum likelihood estimators based on their mean squared errors, and relevant observations are discussed. Finally, real data set are examined to demonstrate the practical application of the proposed methods.**

***Index Terms*—Bayesian estimates, Lindley approximation, Type II censoring, Reliability measures, Symmetric and Asymmetric loss functions.**

I. INTRODUCTION

Studies on lifetime and reliability are vital for analyzing time-to-event outcomes in areas such as health sciences, engineering, and social research. The notion of “lifetime” or “failure” can be interpreted in many ways, including human longevity, mechanical failure, the length of time a social event like marriage lasts, the survival period after diagnosis, time until tumor recurrence after treatment (Cordeiro et al., 2020). But sometimes we come across life time data which is incomplete i.e. the exact time of an event or failure cannot be recorded for certain participants or items, this rise to the occurrence of Censoring. A participant is considered censored if they are lost to follow-up or if the event has not occurred by the end of the study (Prinja et al., 2010). There are so many censoring scheme are available but here we will deal with type II censoring scheme. In Type II censoring, the life test stops as a fixed number of items failed and failure time of items is considered as random variable (Cohen,1965). Type II censoring is also called fixed item censoring.

Balakrishnan et al. (2007) develops point and interval estimation methods for a step-stress model with Type-II censoring, using maximum likelihood and bootstrap approaches to improve reliability analysis in accelerated testing. Wingo (1993) proposed maximum likelihood methods for estimating Burr XII distribution parameters under Type II censoring, ensuring valid and finite estimates. The study also provided asymptotic confidence intervals and applied the approach to reliability test data.

Survival analysis plays a central role in making accurate inferences. The use of flexible statistical models further strengthens this process, as they capture both the nature of the data and the patterns of its distribution, thereby enhancing practical applications (Cordeiro et al., 2020).

Shanker (2015) introduced a new one-parameter lifetime model, termed the Shanker distribution (mixture of exponential (θ) and gamma (2,θ) distribution), designed for modeling lifetime data more effectively than exponential and Lindley distributions. The study explored its key mathematical properties, including moments, hazard rate, mean residual life, stochastic ordering, order statistics, entropy, and stress–strength reliability. Conditions of over-dispersion, equi-dispersion, and under-dispersion were also discussed. Methods of parameter estimation, particularly maximum likelihood and the method of moments, were presented. The paper further demonstrated the practical applicability and superiority of the Shanker distribution over existing models through analyses of real lifetime data sets.

If X is random variable follow Shanker distribution then its pdf and c.d.f, respectively, are

$$f(x;\theta) = \frac{\theta^2}{(\theta^2+1)}(\theta + x)\, e^{-\theta x}\,,\ \ x > 0,\ \theta > 0, \tag{1}$$

$$F(x;\theta) = 1 - \frac{(\theta^2 + x\theta + 1)}{(\theta^2+1)}\, e^{-\theta x}\,,\ x > 0,\ \theta > 0, \tag{2}$$

Abushal (2019) develops Bayesian methodologies to estimate unknown parameters and reliability measures of the Shanker distribution, utilizing techniques such as maximum likelihood estimation, Lindley's approximation, Metropolis-Hastings algorithm, and bootstrap confidence intervals with application to real datasets for reliability modeling in lifetime data analysis.

Thus, this paper presents the approach on both Classical and Bayesian inference on Shanker distribution under type II censoring. We also apply Lindley's approximation under various loss functions such as square error, LINEX (linear exponential) and general entropy, to obtain Bayesian estimates for the unknown parameter and reliability function. All the Bayesian estimates and their corresponding MLE are numerically compared through simulation study in terms of their mean square error. Further, the effectiveness of the proposed method is further illustrated through the analysis of a real dataset.

In Bayesian inference, prior knowledge about a parameter is updated with new data to form the posterior distribution, which is then used for estimation. The aim of Bayesian estimation is not only to find parameter values but to identify the optimal estimate of the unknown parameter. For this purpose, loss function is introduced to measure the cost or penalty of choosing an estimate that deviates from the actual parameter. By selecting an appropriate loss function, the estimator is chosen so that the expected loss is minimized. Thus, Bayesian analysis relies on four fundamental components: the observed data, the prior knowledge or beliefs, the resulting posterior distribution, and the loss function used to assess decision accuracy (Sangal, 2025).

In statistics, various loss functions exist, classified as either symmetric or asymmetric. A symmetric loss function assigns the same penalty to both underestimation and overestimation, which often reflects real-world scenarios. In contrast, there are cases where the impact of overestimation is more critical than underestimation, or vice versa, making asymmetric loss functions more appropriate. A well-known symmetric loss function is the squared error loss function (SELF) and asymmetric loss functions, namely the linear exponential (LINEX) and the general entropy loss function (GELF) (Al-Jarallah et al., 2024; Nassar et al., 2022; Srivastava & Shah, 2019; Calabria & Pulcini 1996; Basu & Ebrahimi, 1991).

Several works in the literature have focused on Bayesian estimation under type II censoring. Abbas et al. 2020 presents Bayesian estimation for the Gumbel Type-II distribution under Type-II censoring using Lindley's approximation with various loss functions. Bayesian and maximum likelihood estimators are compared via simulation, and the method is illustrated with cancer patient survival datasets. Asgharzadeh et al. (2017) examines inference for the Lindley distribution under Type II censoring, employing moment-based, maximum likelihood, and Bayesian approaches. The study introduces bias-corrected and bootstrap estimators, proposes interval estimation methods, and applies the techniques to real data, highlighting their practical effectiveness. Okasha (2014) applies the E-Bayesian approach to estimate parameters and survival measures of the Lomax distribution under Type-II censoring. The Bayesian inference and prediction methods for the inverse Weibull distribution under Type-II censored data. Using MCMC and importance sampling, Bayes estimators with credible intervals are obtained and compared with MLE through simulations, including one- and two-sample prediction problems are proposed by Kundu & Howlader (2010).

II. MAXIMUM LIKELIHOOD ESTIMATION (MLE) UNDER TYPE II CENSORING

Suppose there are some items say (n) are put under life test whose life times have the Shanker distribution with parameter θ and the experiment stops when a fixed number say r ($r < n$) items get failed. The ordered failure time of these r items are $X_1, X_2, \ldots, X_r$, respectively. The failure time of remaining $(n-r)$ items are not known but survive till the experiment stops. Thus, $X_1 < X_2 < \ldots < X_r$ is a type II censored sample of r size.

The likelihood function can be expressed for r observed failures and $(n-r)$ censored observations as,

$$L \propto \prod_{i=1}^{r} f(x_i, \theta)\, [1 - F(x_r, \theta)]^{n-r}$$

$$L \propto \left(\prod_{i=1}^{r} \frac{\theta^2 (\theta + x_i)}{(\theta^2 + 1)}\, e^{-\theta x_i} \right) \left[\frac{(\theta^2 + x_r \theta + 1)}{(\theta^2 + 1)}\, e^{-\theta x_r} \right]^{n-r} \tag{3}$$

The log likelihood function is,

$$\log L \propto 2r \log \theta - r \log(\theta^2 + 1) + \sum_{i=1}^{r} \log(\theta + x_i) - \sum_{i=1}^{r} x_i \theta + (n-r)\,[\log(\theta^2 + x_r \theta + 1)$$

$$- \log(\theta^2 + 1) - x_r \theta\,] \tag{4}$$

In order to obtain MLE of unknown parameter θ, differentiate equation (4) with respect to θ and equate it zero,

$$\frac{d \log L}{d\theta} = \frac{2r}{\theta(\theta^2 + 1)} + \sum_{i=1}^{r} \frac{1}{(\theta + x_i)} - \sum_{i=1}^{r} x_i + (n-r) \left[\frac{(2\theta + x_r)}{(\theta^2 + x_r \theta + 1)} - \frac{2\theta}{(\theta^2 + 1)} - x_r \right] = 0$$

The above equation cannot be expressed in closed form. Thus there is required to apply numerical techniques with the help of R statistical software to obtain the MLE of unknown parameter θ.

The reliability, hazard and mean residual life functions of Shanker distribution are given as,

$$R(t) = \frac{(\theta^2 + t\theta + 1)}{(\theta^2 + 1)}\, e^{-\theta t}, \qquad t > 0 \tag{5}$$

$$h(t) = \frac{\theta^2 (\theta + t)}{(\theta^2 + t\theta + 1)}, \qquad t > 0 \tag{6}$$

$$m(t) = \frac{(\theta^2 + t\theta + 2)}{\theta(\theta^2 + t\theta + 1)}, \qquad t > 0 \tag{7}$$

The MLE of above functions can be obtained by using the invariance property of the MLE, as

$$\hat{R}(t) = \frac{(\hat{\theta}^2 + t\hat{\theta} + 1)}{(\hat{\theta}^2 + 1)}\, e^{-\hat{\theta} t}, \qquad t > 0 \tag{8}$$

$$\hat{h}(t) = \frac{\hat{\theta}^2 (\hat{\theta} + t)}{(\hat{\theta}^2 + t\hat{\theta} + 1)}, \qquad t > 0 \tag{9}$$

$$\hat{m}(t) = \frac{(\hat{\theta}^2 + t\hat{\theta} + 2)}{\hat{\theta}(\hat{\theta}^2 + t\hat{\theta} + 1)}, \qquad t > 0 \tag{10}$$

III. BAYESIAN ESTIMATION UNDER TYPE II CENSORING

We consider here the same sample observations as discussed in above section. Assuming Gamma (a,b) be the prior distribution of θ with probability density function as,

$$\pi(\theta) = \frac{b^a}{\Gamma a}\theta^{a-1}e^{-b\theta} \ ; \quad \theta > 0\,,\ a > 0\,,\ b > 0\,. \tag{11}$$

Then the posterior distribution of θ is obtained as,

$$\pi(\theta\,|\,\underline{x}) = \frac{1}{k}\ \frac{\theta^{2r+a-1}e^{-\theta(b+s)}}{(\theta^2+1)^r}\left[\frac{(\theta^2+x_r\theta+1)}{(\theta^2+1)}\,e^{-\theta x_r}\right]^{n-r}\prod_{i=1}^{r}(\theta+x_i)\ d\theta \tag{12}$$

Where s = $\sum_{i=1}^{r} x_i$ and

k = $\int_0^\infty \frac{\theta^{2r+a-1}e^{-\theta(b+s)}}{(\theta^2+1)^r}\left[\frac{(\theta^2+x_r\theta+1)}{(\theta^2+1)}\,e^{-\theta x_r}\right]^{n-r}\prod_{i=1}^{r}(\theta+x_i)$

Square Error Loss Function (SELF)

$$L(\theta,\tilde{\theta}) = (\tilde{\theta}-\theta)^2$$

Where $\tilde{\theta}$ is considered as an estimate of true parameter θ.

Bayesian estimates of θ based on SELF is,

$\tilde{\theta}$ = E[$\theta\,|\,\underline{x}$] = Posterior Mean

Thus, the Bayesian estimates of parameter θ, reliability, hazard and mean residual life functions of Shanker distribution under type II censoring based on SELF is given as follows:

$$\tilde{\theta}_S = \mathrm{E}[\,\theta\,|\,\underline{x}\,]$$
$$= \frac{1}{k}\int_0^\infty \theta\ \pi(\theta\,|\,\underline{x})d\theta \tag{13}$$

$$\tilde{R}_S(t) = \mathrm{E}[\,R(t)\,|\,\underline{x}\,]$$
$$= \frac{1}{k}\int_0^\infty R(t)\ \pi(\theta\,|\,\underline{x})d\theta \tag{14}$$

$$\tilde{h}_S(t) = \mathrm{E}[\,h(t)\,|\,\underline{x}\,]$$
$$= \frac{1}{k}\int_0^\infty h(t)\ \pi(\theta\,|\,\underline{x})d\theta \tag{15}$$

$$\tilde{m}_S(t) = \mathrm{E}[\,m(t)\,|\,\underline{x}\,]$$
$$= \frac{1}{k}\int_0^\infty m(t)\ \pi(\theta\,|\,\underline{x})d\theta \tag{16}$$

Linear Exponential Loss Function (LINEX)

$$L(\theta,\tilde{\theta}) = \exp[v(\tilde{\theta}-\theta)] - v(\tilde{\theta}-\theta) - 1, \qquad v \neq 0\,,$$

Bayesian estimates of θ based on LINEX is,

$$\tilde{\theta} = -\frac{1}{v}\log[\,E(e^{-v\theta}\,|\,\underline{x})]$$

Thus, the Bayesian estimates of parameter θ, reliability, hazard and mean residual life functions of Shanker distribution under type II censoring based on LINEX is given as follows:

$$\tilde{\theta}_L = -\frac{1}{v}\log[\,E(e^{-v\theta}\,|\,\underline{x})]$$

$$= -\frac{1}{v}\log\left[\int_0^\infty e^{-v\theta}\pi(\theta \mid \underline{x})d\theta\right] \tag{17}$$

$$\tilde{R}_L(t) = -\frac{1}{v}\log[E(e^{-vR(t)} \mid \underline{x})]$$

$$= -\frac{1}{v}\log\left[\int_0^\infty e^{-vR(t)}\pi(\theta \mid \underline{x})d\theta\right] \tag{18}$$

$$\tilde{h}_L(t) = -\frac{1}{v}\log[E(e^{-vh(t)} \mid \underline{x})]$$

$$= -\frac{1}{v}\log\left[\int_0^\infty e^{-vh(t)}\pi(\theta \mid \underline{x})d\theta\right] \tag{19}$$

$$\tilde{m}_L(t) = -\frac{1}{v}\log[E(e^{-vm(t)} \mid \underline{x})]$$

$$= -\frac{1}{v}\log\left[\int_0^\infty e^{-vm(t)}\pi(\theta \mid \underline{x})d\theta\right] \tag{20}$$

General Entropy Loss Function (GELF)

$$L(\theta,\tilde{\theta}) = \left(\frac{\tilde{\theta}}{\theta}\right)^q - q\log\left(\frac{\tilde{\theta}}{\theta}\right) - 1, \qquad q \neq 0.$$

Bayesian estimates of θ based on GELF is,

$$\tilde{\theta} = [E(\theta^{-q} \mid \underline{x})]^{-1/q}$$

Thus, the Bayesian estimates of parameter θ, reliability, hazard and mean residual life functions of Shanker distribution under type II censoring based on GELF is given as follows:

$$\tilde{\theta}_E = [E(\theta^{-q} \mid \underline{x})]^{-1/q}$$

$$= \left[\int_0^\infty \theta^{-q}\pi(\theta \mid \underline{x})d\theta\right]^{-1/q} \tag{21}$$

$$\tilde{R}_E(t) = [E((R(t))^{-q} \mid \underline{x})]^{-1/q}$$

$$= \left[\int_0^\infty (R(t))^{-q}\pi(\theta \mid \underline{x})d\theta\right]^{-1/q} \tag{22}$$

$$\tilde{h}_E(t) = [E((h(t))^{-q} \mid \underline{x})]^{-1/q}$$

$$= \left[\int_0^\infty (h(t))^{-q}\pi(\theta \mid \underline{x})d\theta\right]^{-1/q} \tag{23}$$

$$\tilde{m}_E(t) = [E((m(t))^{-q} \mid \underline{x})]^{-1/q}$$

$$= \left[\int_0^\infty (m(t))^{-q}\pi(\theta \mid \underline{x})d\theta\right]^{-1/q} \tag{24}$$

It is evident that the all the above Bayesian estimates from equation (11) to (22) cannot be expressed in simple closed form. Hence, the Lindley approximation method is utilized to obtain the approximate Bayes estimates in the following section.

IV. LINDLEY APPROXIMATION METHOD

The Lindley approximation method is an analytical technique introduced by D.V. Lindley (1980) to obtain approximate Bayesian estimates when integrals involved in finding estimators are complex and lack closed-form expressions. It involves the ratio of two integrals such that

$$I(\underline{x}) = \frac{\int_0^\infty u(\theta) e^{I(\theta)+\rho(\theta)} d\theta}{\int_0^\infty e^{I(\theta)+\rho(\theta)} d\theta}$$

Where $u(\theta)$ is the function of parameter θ only, $l(\theta)$ is the log likelihood and $\rho(\theta)$ is the log of prior. Under Lindley approximation technique, $I(\underline{x})$ can be evaluated as

$$I(\underline{x}) = u(\hat{\theta}) + 0.5\,[(\hat{u}_{\theta\theta} + 2\hat{u}_\theta \hat{\rho}_\theta)\hat{\sigma}_{\theta\theta} + \hat{u}_\theta \hat{\sigma}^2_{\theta\theta} \hat{l}_{\theta\theta\theta}]$$

Where $\hat{u}_\theta = \frac{\partial u(\hat{\theta})}{\partial \hat{\theta}}$, $\hat{u}_{\theta\theta} = \frac{\partial^2 u(\hat{\theta})}{\partial \hat{\theta}^2}$, $\hat{l}_{\theta\theta\theta} = \frac{\partial^3 l(\hat{\theta})}{\partial \hat{\theta}^3}$, $\hat{\rho}_\theta = \frac{\partial \rho(\hat{\theta})}{\partial \hat{\theta}}$ and

$$\hat{\sigma}_{\theta\theta} = \left[-\frac{\partial^2 l}{\partial \theta^2}\right]^{-1}$$

As per this paper, the above quantities are obtained as,

$$\hat{l}_{\theta\theta} = -\frac{2r(3\theta^2+1)}{\theta^2(\theta^2+1)^2} - \sum_{i=1}^{r} \frac{1}{(\theta+x_i)^2} - (n-r)\left[\frac{(2\theta^2+2x_r\theta+x_r^2-2)}{(\theta^2+x_r\theta+1)^2} - \frac{2(\theta^2-1)}{(\theta^2+1)^2}\right]$$

$$\hat{l}_{\theta\theta\theta} = \frac{4r(6\theta^4+3\theta^2+1)}{\theta^3(\theta^2+1)^3} + \sum_{i=1}^{r} \frac{2}{(\theta+x_i)^3} + 2(n-r)\left[\frac{(2\theta-x_r)(\theta^2+x_r\theta+x_r^2-3)}{(\theta^2+x_r\theta+1)^3} - \frac{2\theta(3-\theta^2)}{(\theta^2+1)^3}\right],$$

$$\hat{\rho}_\theta = \frac{(a-1)}{\hat{\theta}} - b$$

Bayesian estimate of parameter θ against SELF can be obtained as,

$$u(\hat{\theta}) = \hat{\theta}, \quad \hat{u}_\theta = 1, \quad \hat{u}_{\theta\theta} = 0,$$

$$\tilde{\theta}_S = u(\hat{\theta}) + 0.5\,[(\hat{u}_{\theta\theta} + 2\hat{u}_\theta \hat{\rho}_\theta)\hat{\sigma}_{\theta\theta} + \hat{u}_\theta \hat{\sigma}^2_{\theta\theta} \hat{l}_{\theta\theta\theta}]$$

Similarly, Bayesian estimate of reliability, hazard and mean residual life functions against SELF i.e. $\tilde{R}_S(t)$, $\tilde{h}_S(t)$, $\tilde{m}_S(t)$ can be obtained.

Bayesian estimate of parameter θ against LINEX can be obtained as,

$$u(\hat{\theta}) = e^{-v\hat{\theta}}, \quad \hat{u}_\theta = -ve^{-v\hat{\theta}}, \quad \hat{u}_{\theta\theta} = v^2 e^{-v\hat{\theta}},$$

$$E(e^{-v\theta} \mid \underline{x}) = u(\hat{\theta}) + 0.5\,[(\hat{u}_{\theta\theta} + 2\hat{u}_\theta \hat{\rho}_\theta)\hat{\sigma}_{\theta\theta} + \hat{u}_\theta \hat{\sigma}^2_{\theta\theta} \hat{l}_{\theta\theta\theta}]$$

$$\tilde{\theta}_L = -\frac{1}{v}\log[E(e^{-v\theta} \mid \underline{x})]$$

Similarly, Bayesian estimate of reliability, hazard and mean residual life functions against LINEX i.e. $\tilde{R}_L(t)$, $\tilde{h}_L(t)$, $\tilde{m}_L(t)$ can be obtained.

Bayesian estimate of parameter θ against GELF can be obtained as,

$$u(\hat{\theta}) = \hat{\theta}^{-q}, \quad \hat{u}_\theta = -q\hat{\theta}^{-(q+1)}, \quad \hat{u}_{\theta\theta} = q(q+1)\hat{\theta}^{-(q+2)},$$

$$E(\theta^{-q} \mid \underline{x}) = u(\hat{\theta}) + 0.5\,[(\hat{u}_{\theta\theta} + 2\hat{u}_\theta \hat{\rho}_\theta)\hat{\sigma}_{\theta\theta} + \hat{u}_\theta \hat{\sigma}^2_{\theta\theta} \hat{l}_{\theta\theta\theta}]$$

$$\tilde{\theta}_E = [E(\theta^{-q} \mid \underline{x})]^{-1/q}$$

Similarly, Bayesian estimate of reliability, hazard and mean residual life functions against GELF i.e. $\tilde{R}_E(t)$, $\tilde{h}_E(t)$, $\tilde{m}_E(t)$ can be obtained.

V. SIMULATION STUDY

This section presents numerical experiment based on simulated data from the shanker distribution under type II censoring with chosen parameter values to illustrate the effectiveness of different estimation method as discussed in previous sections. Here 5000 samples are generated of varying size (n = 40, 50, 60) and censored data (r = 30, 35, 40). The MLE and Bayesian estimates of parameter θ, $R(t)$, $h(t)$ and $m(t)$ under both symmetric and asymmetric loss functions along with their corresponding mean square error (MSE) are obtained. We also consider three different values of asymmetric parameter (v = − 0.25, 0.1, 0.5) and entropy parameter (q = −0.5, 0.5, 1) under LINEX and GELF respectively.

Conclusion:

For parameter θ,

- As clearly shown in the Fig. 1, 2 and 3 as the value of 'r' increases the MSE of parameter θ decreases for all estimation method consider in this paper.
- As sample size 'n' increases, we get smaller MSE at a certain value of 'r'. This means our estimate is more accurate as we increase in sample size for fixed no. of failure items.
- GELF performs better than others and MLE performs worst in all cases. For negative value of v, SELF is slight better than LINEX and for positive value of v, LINEX is slightly better than SELF. This implies overestimation is more severe than underestimation.

For reliability function $R(t)$,

- From fig. 4, 5 and 6 we observed same result for 'r' and 'n' as in above for parameter θ.
- Among all estimators, GELF consistently exhibits the weakest performance. LINEX and SELF perform equally in most cases while MLE yield better results than GELF but inferior to LINEX and SELF.

For hazard function $h(t)$,

- From fig. 7, 8 and 9 we get similar result for 'n' and 'r' as above.
- MLE shows the poorest performance in every case. For negative value of v, LINEX is marginally better than SELF and for positive value of v, SELF is seems slightly better than LINEX. Thus, for hazard function underestimation is more severe than overestimation. GELF gives better results compared to others.

For mean life function $m(t)$,

- From the fig. 10, 11 and 12 it is clearly seen that the results aree again similar to above for 'n' and 'r'.
- LINEX and SELF shows similar performance and better from MLE and GELF. GELF perform worst among all.

Table 1 : Estimates of parameter θ and their corresponding MSE (in parenthesis) under different loss function for $v = -0.25$ and $q = -0.5$

n	r	$\hat{\theta}$	$\hat{\theta}_S$	$\hat{\theta}_L$	$\hat{\theta}_E$
40	30	0.507098	0.506003	0.506422	0.504369
		(0.00351)	(0.003133)	(0.003148)	(0.003099)
	35	0.505909	0.505096	0.505474	0.503618
		(0.003038)	(0.002745)	(0.002757)	(0.002718)
	40	0.504713	0.504091	0.504438	0.502728
		(0.0028)	(0.002553)	(0.002563)	(0.002532)
50	30	0.505797	0.504724	0.505111	0.503212

		(0.003112)	(0.002803)	(0.002815)	(0.002777)
	35	0.503596	0.502837	0.503183	0.501476
		(0.002792)	(0.002546)	(0.002555)	(0.002528)
	40	0.504393	0.503744	0.504063	0.502495
		(0.002658)	(0.002442)	(0.00245)	(0.002423)
60	30	0.505987	0.504857	0.505223	0.50343
		(0.002916)	(0.002641)	(0.002652)	(0.002616)
	35	0.505522	0.504634	0.504962	0.503349
		(0.002643)	(0.002419)	(0.002428)	(0.002399)
	40	0.503777	0.503113	0.503411	0.501941
		(0.002421)	(0.002236)	(0.002243)	(0.002222)

Table 2 : Estimates of parameter θ and their corresponding MSE (in parenthesis) under different loss function for v = 0.1 and q = 0.5

n	r	$\hat{\theta}$	$\hat{\theta}_S$	$\hat{\theta}_L$	$\hat{\theta}_E$
40	30	0.507803	0.506674	0.506506	0.501809
		(0.003465)	(0.003087)	(0.003081)	(0.002999)
	35	0.506128	0.505298	0.505147	0.500894
		(0.003134)	(0.00283)	(0.002826)	(0.002763)
	40	0.504914	0.504282	0.504143	0.50022
		(0.002804)	(0.002557)	(0.002553)	(0.002504)
50	30	0.505099	0.504058	0.503903	0.499561
		(0.003134)	(0.002825)	(0.002821)	(0.002769)
	35	0.504597	0.503797	0.503658	0.499737
		(0.002759)	(0.002514)	(0.00251)	(0.002467)
	40	0.505309	0.504627	0.5045	0.500898
		(0.002619)	(0.002404)	(0.0024)	(0.002354)
60	30	0.506062	0.504923	0.504776	0.500673
		(0.003021)	(0.002735)	(0.002731)	(0.002675)
	35	0.505386	0.504497	0.504366	0.50067
		(0.002752)	(0.00252)	(0.002517)	(0.00247)
	40	0.504317	0.503632	0.503512	0.500135
		(0.002453)	(0.002266)	(0.002263)	(0.002227)

Table 3 : Estimates of parameter θ and their corresponding MSE (in parenthesis) under different loss function for v = 0.5 and q = 1

n	r	$\hat{\theta}$	$\hat{\theta}_S$	$\hat{\theta}_L$	$\hat{\theta}_E$
40	30	0.506325	0.50528	0.504445	0.49885
		(0.003366)	(0.003006)	(0.002979)	(0.002921)
	35	0.50546	0.504667	0.503913	0.498835
		(0.003045)	(0.002752)	(0.002729)	(0.002681)
	40	0.504722	0.504096	0.503402	0.49871
		(0.00284)	(0.002589)	(0.00257)	(0.002529)
50	30	0.506182	0.505083	0.504309	0.499114
		(0.003211)	(0.002888)	(0.002864)	(0.002811)

	35	0.505333	0.504489	0.503793	0.499101
		(0.002949)	(0.002686)	(0.002666)	(0.002622)
	40	0.504087	0.503459	0.502824	0.49852
		(0.002523)	(0.002318)	(0.002303)	(0.002272)
60	30	0.505227	0.504128	0.503401	0.498501
		(0.002955)	(0.002678)	(0.002658)	(0.002618)
	35	0.503955	0.503133	0.502481	0.498068
		(0.002607)	(0.00239)	(0.002375)	(0.002347)
	40	0.503169	0.50253	0.501936	0.497897
		(0.002369)	(0.00219)	(0.002178)	(0.002156)

Table 4 : Estimates of reliability function and their corresponding MSE (in parenthesis) under different loss function for $\nu = -0.25$, $q = -0.5$ and t = 1.5

n	r	$\hat{R}(t)$	$\tilde{R}_S(t)$	$\tilde{R}_L(t)$	$\tilde{R}_E(t)$
40	30	0.749704	0.7502	0.750462	0.747472
		(0.002199)	(0.001934)	(0.001926)	(0.002568)
	35	0.750699	0.75099	0.751226	0.748864
		(0.001905)	(0.001698)	(0.001692)	(0.00219)
	40	0.751664	0.751822	0.752039	0.750104
		(0.001754)	(0.00158)	(0.001574)	(0.001992)
50	30	0.750774	0.751262	0.751504	0.748655
		(0.001949)	(0.001731)	(0.001724)	(0.002249)
	35	0.752552	0.75281	0.753027	0.750861
		(0.001748)	(0.001573)	(0.001568)	(0.001984)
	40	0.751943	0.752143	0.752342	0.750437
		(0.001667)	(0.001514)	(0.001509)	(0.001874)
60	30	0.750651	0.751201	0.751429	0.748536
		(0.001827)	(0.001633)	(0.001627)	(0.002095)
	35	0.751054	0.751439	0.751645	0.749282
		(0.001659)	(0.001501)	(0.001496)	(0.001876)
	40	0.752459	0.752683	0.752869	0.750995
		(0.001518)	(0.001387)	(0.001383)	(0.001695)

Table 5 : Estimates of reliability function and their corresponding MSE (in parenthesis) under different loss function for $\nu = 0.1$, $q = 0.5$ and t = 1.5

n	r	$\hat{R}(t)$	$\tilde{R}_S(t)$	$\tilde{R}_L(t)$	$\tilde{R}_E(t)$
40	30	0.749175	0.749696	0.749591	0.744646
		(0.002173)	(0.001908)	(0.001911)	(0.002652)
	35	0.750518	0.750825	0.75073	0.746633
		(0.001966)	(0.001752)	(0.001755)	(0.002344)
	40	0.751504	0.751671	0.751584	0.748072
		(0.001758)	(0.001583)	(0.001585)	(0.002062)
50	30	0.751319	0.75178	0.751683	0.747168
		(0.001962)	(0.001743)	(0.001746)	(0.002344)
	35	0.751765	0.752059	0.751972	0.74817

		(0.001729)	(0.001556)	(0.001558)	(0.00203)
	40	0.751228	0.751457	0.751377	0.747976
		(0.001644)	(0.001493)	(0.001495)	(0.00191)
60	30	0.750583	0.751142	0.751051	0.7465
		(0.001894)	(0.001692)	(0.001695)	(0.002251)
	35	0.751141	0.75153	0.751447	0.747612
		(0.001726)	(0.001563)	(0.001565)	(0.002013)
	40	0.752021	0.752265	0.752191	0.748938
		(0.001538)	(0.001406)	(0.001407)	(0.001767)

Table 6 : Estimates of reliability function and their corresponding MSE (in parenthesis) under different loss function for ν = 0.5, q = 1 and t = 1.5

n	r	$\hat{R}(t)$	$\tilde{R}_S(t)$	$\tilde{R}_L(t)$	$\tilde{R}_E(t)$
40	30	0.75033	0.750777	0.750254	0.744804
		(0.002108)	(0.001855)	(0.001871)	(0.002609)
	35	0.75105	0.751323	0.750851	0.746215
		(0.001907)	(0.001701)	(0.001714)	(0.002312)
	40	0.751657	0.751817	0.751383	0.747312
		(0.00178)	(0.001603)	(0.001614)	(0.002123)
50	30	0.750476	0.750986	0.750502	0.745226
		(0.002014)	(0.001786)	(0.0018)	(0.002461)
	35	0.75116	0.751497	0.751061	0.746587
		(0.001848)	(0.001662)	(0.001674)	(0.002209)
	40	0.752198	0.752379	0.751981	0.748176
		(0.001582)	(0.001437)	(0.001446)	(0.00186)
60	30	0.751245	0.751767	0.751311	0.746251
		(0.001851)	(0.001655)	(0.001667)	(0.002234)
	35	0.752287	0.752613	0.752205	0.747983
		(0.001633)	(0.001479)	(0.001489)	(0.001928)
	40	0.752938	0.753139	0.752767	0.749128
		(0.001485)	(0.001357)	(0.001365)	(0.001725)

Table 7 : Estimates of hazard function and their corresponding MSE (in parenthesis) under different loss function for $\nu = -0.25$, $q = -0.5$ and t = 1

n	r	$\hat{h}(t)$	$\tilde{h}_S(t)$	$\tilde{h}_L(t)$	$\tilde{h}_E(t)$
40	30	0.221151	0.221597	0.221772	0.216291
		(0.002089)	(0.001881)	(0.001866)	(0.001768)
	35	0.220056	0.220614	0.220783	0.215829
		(0.001801)	(0.001641)	(0.001629)	(0.001551)
	40	0.219057	0.219678	0.219842	0.215281
		(0.001648)	(0.001514)	(0.001504)	(0.00144)
50	30	0.220004	0.220391	0.220539	0.215438
		(0.001841)	(0.00167)	(0.001658)	(0.001582)
	35	0.218209	0.218727	0.218876	0.214297
		(0.001644)	(0.001506)	(0.001496)	(0.001443)

	40	0.218751	0.219267	0.219408	0.21521
		(0.001569)	(0.00145)	(0.001441)	(0.001387)
60	30	0.220062	0.220347	0.220472	0.215637
		(0.001728)	(0.001575)	(0.001564)	(0.001494)
	35	0.219595	0.219958	0.220081	0.215738
		(0.001563)	(0.001441)	(0.001432)	(0.001372)
	40	0.218186	0.218636	0.218759	0.214807
		(0.001425)	(0.001323)	(0.001315)	(0.001271)

Table 8 : Estimates of hazard function and their corresponding MSE (in parenthesis) under different loss function for ν = 0.1, q = 0.5 and t = 1

n	r	$\hat{h}(t)$	$\tilde{h}_S(t)$	$\tilde{h}_L(t)$	$\tilde{h}_E(t)$
40	30	0.221653	0.222076	0.222006	0.209642
		(0.002083)	(0.001871)	(0.001876)	(0.001774)
	35	0.220259	0.220802	0.220733	0.209522
		(0.001863)	(0.001696)	(0.0017)	(0.001624)
	40	0.21921	0.219824	0.219758	0.209404
		(0.00165)	(0.001517)	(0.00152)	(0.001464)
50	30	0.21949	0.219898	0.219837	0.20833
		(0.001851)	(0.001679)	(0.001683)	(0.001632)
	35	0.21895	0.219439	0.21938	0.208981
		(0.001626)	(0.001491)	(0.001495)	(0.001451)
	40	0.219423	0.219916	0.21986	0.210309
		(0.00155)	(0.001433)	(0.001436)	(0.00138)
60	30	0.220162	0.220435	0.220382	0.209464
		(0.001794)	(0.001634)	(0.001638)	(0.001575)
	35	0.219542	0.219901	0.21985	0.21001
		(0.001623)	(0.001496)	(0.0015)	(0.001444)
	40	0.218609	0.219043	0.218993	0.209998
		(0.001439)	(0.001338)	(0.00134)	(0.0013)

Table 9 : Estimates of hazard function and their corresponding MSE (in parenthesis) under different loss function for ν = 0.5, q = 1 and t = 1

n	r	$\hat{h}(t)$	$\tilde{h}_S(t)$	$\tilde{h}_L(t)$	$\tilde{h}_E(t)$
40	30	0.220509	0.220999	0.220635	0.205388
		(0.001999)	(0.001799)	(0.00182)	(0.001777)
	35	0.219721	0.220294	0.219946	0.206079
		(0.001802)	(0.001641)	(0.001658)	(0.001623)
	40	0.219078	0.219695	0.219361	0.206525
		(0.001676)	(0.001539)	(0.001554)	(0.001524)
50	30	0.220328	0.220692	0.220379	0.20612
		(0.001917)	(0.001736)	(0.001754)	(0.001713)
	35	0.219585	0.220034	0.219735	0.206831
		(0.001742)	(0.001597)	(0.001613)	(0.001575)
	40	0.218464	0.219	0.21871	0.206857

		(0.001484)	(0.001373)	(0.001385)	(0.001369)
60	30	0.219507	0.219813	0.219538	0.206005
		(0.001747)	(0.001593)	(0.001609)	(0.001587)
	35	0.218403	0.218816	0.218549	0.20634
		(0.00153)	(0.001411)	(0.001424)	(0.001417)
	40	0.217709	0.21818	0.217921	0.206734
		(0.001386)	(0.001287)	(0.001299)	(0.001297)

Table 10 : Estimates of mean residual function and their corresponding MSE (in parenthesis) under different loss function for $\nu = -0.25$, $q = -0.5$ and t = 10

n	r	$\hat{m}(t)$	$\tilde{m}_S(t)$	$\tilde{m}_L(t)$	$\tilde{m}_E(t)$
40	30	2.32096	2.35968	2.35973	2.31948
		(0.089822)	(0.086068)	(0.086071)	(0.09174)
	35	2.32223	2.35684	2.3569	2.32108
		(0.078282)	(0.075346)	(0.075347)	(0.079786)
	40	2.32625	2.35778	2.35785	2.32531
		(0.07388)	(0.07147)	(0.07147)	(0.075159)
50	30	2.32377	2.36044	2.36049	2.32235
		(0.081112)	(0.078384)	(0.078385)	(0.082686)
	35	2.33198	2.36438	2.36445	2.33091
		(0.073726)	(0.071919)	(0.071919)	(0.07499)
	40	2.32618	2.35565	2.35572	2.32525
		(0.069192)	(0.067247)	(0.067245)	(0.070301)
60	30	2.32055	2.3559	2.35595	2.31909
		(0.075452)	(0.073017)	(0.073016)	(0.076847)
	35	2.31998	2.35134	2.3514	2.3188
		(0.067923)	(0.065833)	(0.065831)	(0.069061)
	40	2.3269	2.35506	2.35513	2.32598
		(0.063384)	(0.061897)	(0.061895)	(0.06433)

Table 11 : Estimates of mean residual function and their corresponding MSE (in parenthesis) under different loss function for $\nu = 0.1$, $q = 0.5$ and t = 10

n	r	$\hat{m}(t)$	$\tilde{m}_S(t)$	$\tilde{m}_L(t)$	$\tilde{m}_E(t)$
40	30	2.31618	2.35507	2.35505	2.31391
		(0.085847)	(0.08198)	(0.08198)	(0.087899)
	35	2.32198	2.3566	2.35657	2.32013
		(0.080111)	(0.077064)	(0.077064)	(0.081783)
	40	2.32521	2.35677	2.35674	2.32364
		(0.073655)	(0.071196)	(0.071196)	(0.075035)
50	30	2.32779	2.36435	2.36433	2.32571
		(0.08241)	(0.079895)	(0.079894)	(0.084108)
	35	2.32632	2.35887	2.35884	2.32459
		(0.072316)	(0.070223)	(0.070223)	(0.073667)
	40	2.32077	2.35037	2.35034	2.31924
		(0.067019)	(0.064872)	(0.064873)	(0.068204)

60	30	2.32115	2.35648	2.35645	2.31903
		(0.07742)	(0.074942)	(0.074942)	(0.078976)
	35	2.32205	2.35335	2.35332	2.32028
		(0.071623)	(0.069484)	(0.069485)	(0.072899)
	40	2.32455	2.35275	2.35272	2.32307
		(0.064629)	(0.062958)	(0.062959)	(0.065664)

Table 12 : Estimates of mean residual function and their corresponding MSE (in parenthesis) under different loss function for ν = 0.5, q = 1 and t = 10

n	r	$\hat{m}(t)$	$\tilde{m}_S(t)$	$\tilde{m}_L(t)$	$\tilde{m}_E(t)$
40	30	2.32361	2.36227	2.36216	2.32106
		(0.087076)	(0.083671)	(0.083665)	(0.089127)
	35	2.32477	2.35931	2.35917	2.32263
		(0.079231)	(0.076407)	(0.076406)	(0.08091)
	40	2.32649	2.35802	2.35787	2.32463
		(0.074216)	(0.071811)	(0.071812)	(0.075658)
50	30	2.32227	2.359	2.35888	2.31979
		(0.081017)	(0.078209)	(0.07821)	(0.082828)
	35	2.32447	2.35704	2.3569	2.32239
		(0.076963)	(0.074519)	(0.074522)	(0.078463)
	40	2.32641	2.35588	2.35573	2.32466
		(0.066077)	(0.064291)	(0.064294)	(0.067248)
60	30	2.32505	2.3603	2.36018	2.32266
		(0.077101)	(0.074892)	(0.074894)	(0.078668)
	35	2.32816	2.35932	2.35919	2.32619
		(0.069061)	(0.067412)	(0.067415)	(0.070296)
	40	2.32976	2.35785	2.35771	2.32807
		(0.063199)	(0.06187)	(0.061873)	(0.064222)

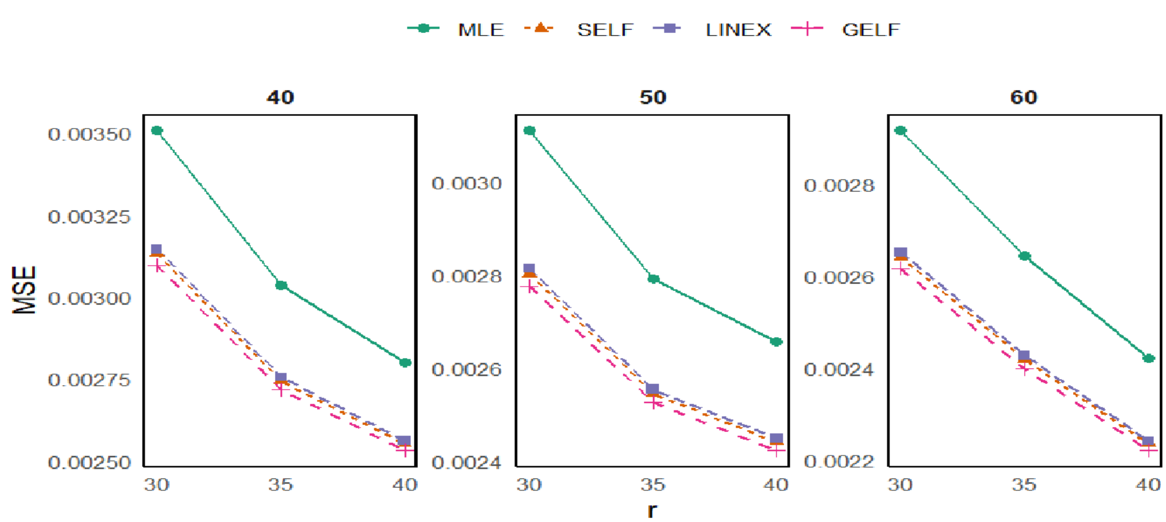

Fig. 1. Graph of MSE of parameter θ for $\nu = -0.25$ and $q = -0.5$

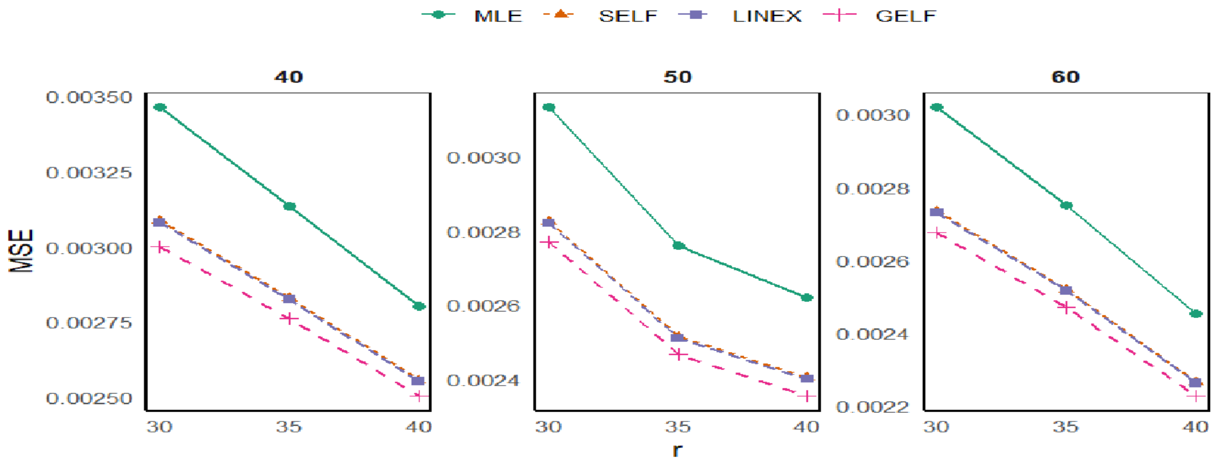

Fig. 2. Graph of MSE of parameter θ for $\nu = 0.1$ and $q = 0.5$

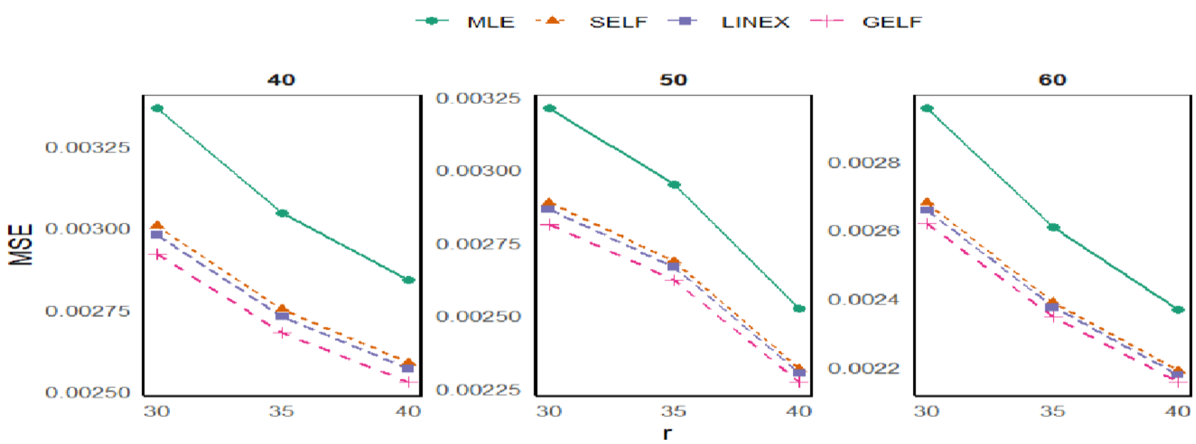

Fig. 3. Graph of MSE of parameter θ for $\nu = 0.5$ and $q = 1$

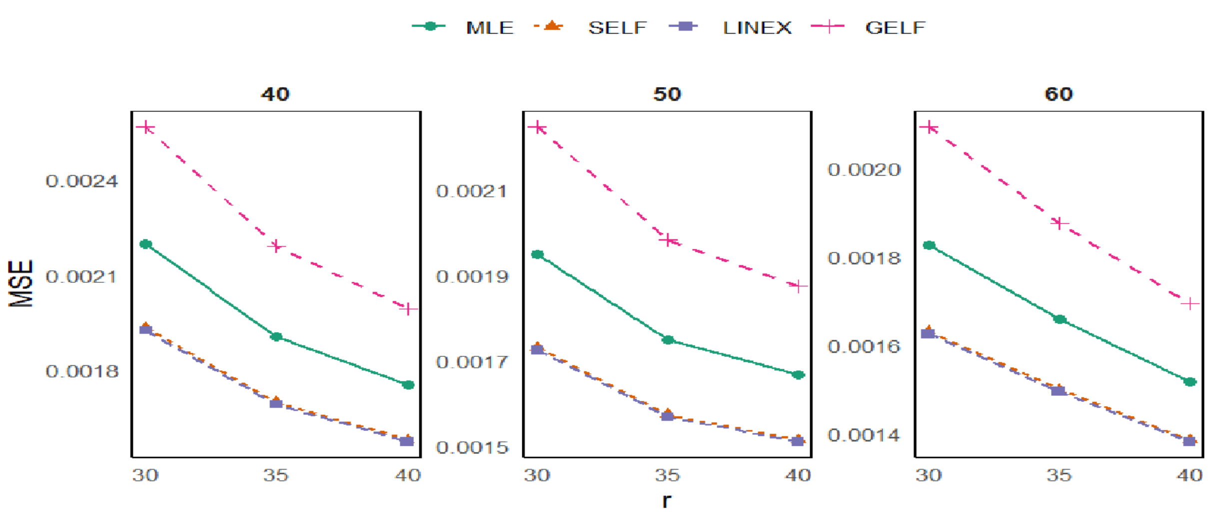

Fig. 4. Graph of MSE of reliability function for $\nu = -0.25$, $q = -0.5$ and t = 1.5

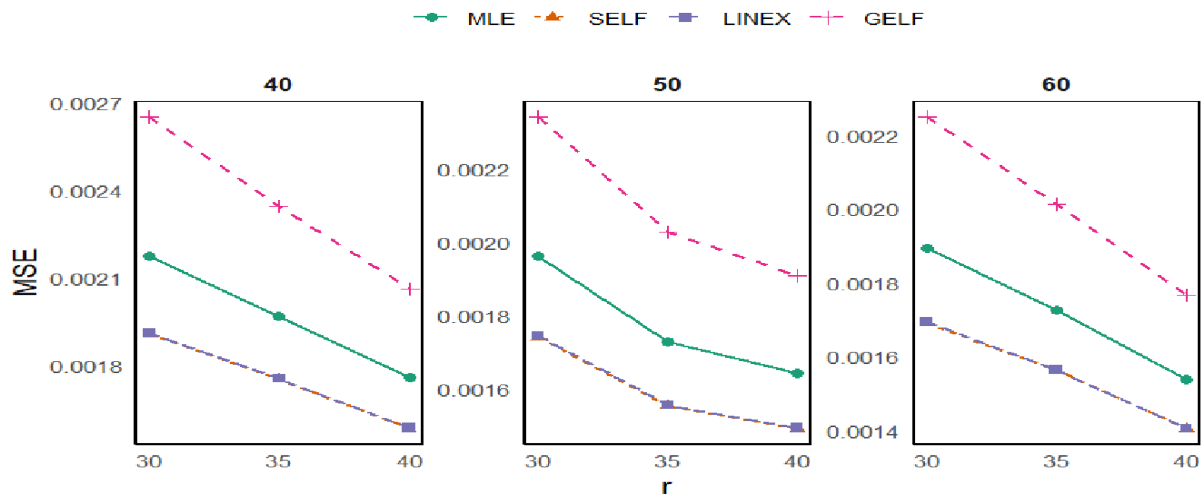

Fig. 5. Graph of MSE of reliability function for $\nu = 0.1$, $q = 0.5$ and t = 1.5

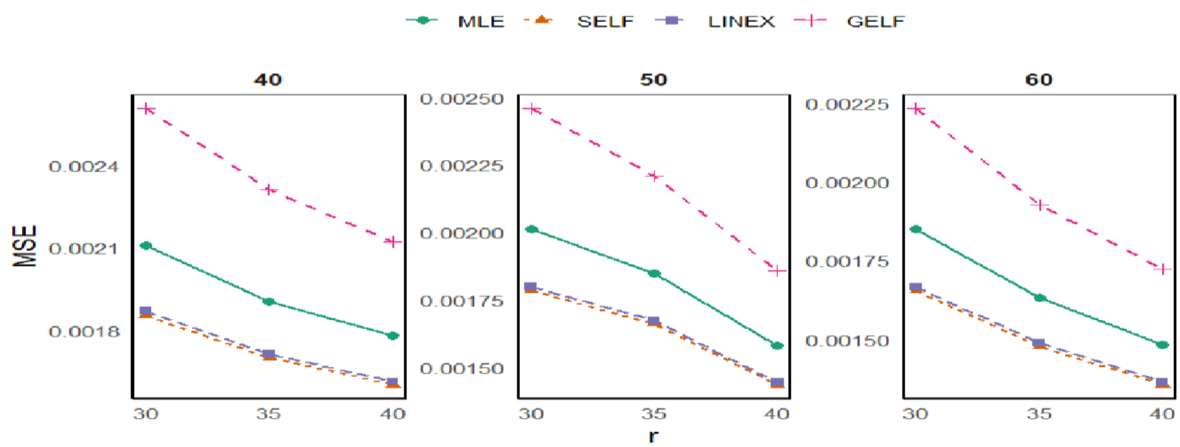

Fig. 6. Graph of MSE of reliability function for $\nu = 0.5$, $q = 1$ and t = 1.5

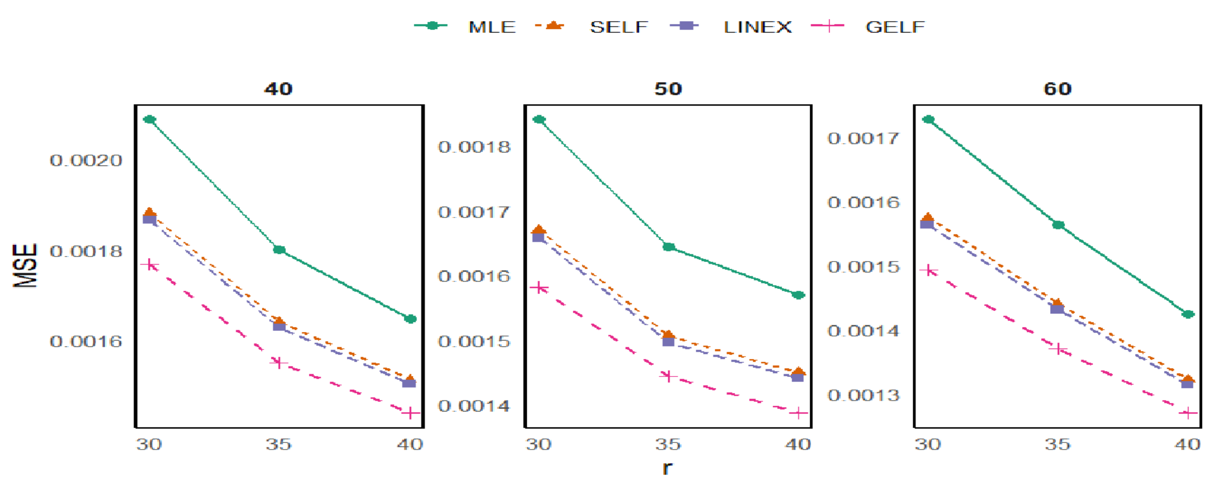

Fig. 7. Graph of MSE of hazard function for $\nu = -0.25$, $q = -0.5$ and t = 1

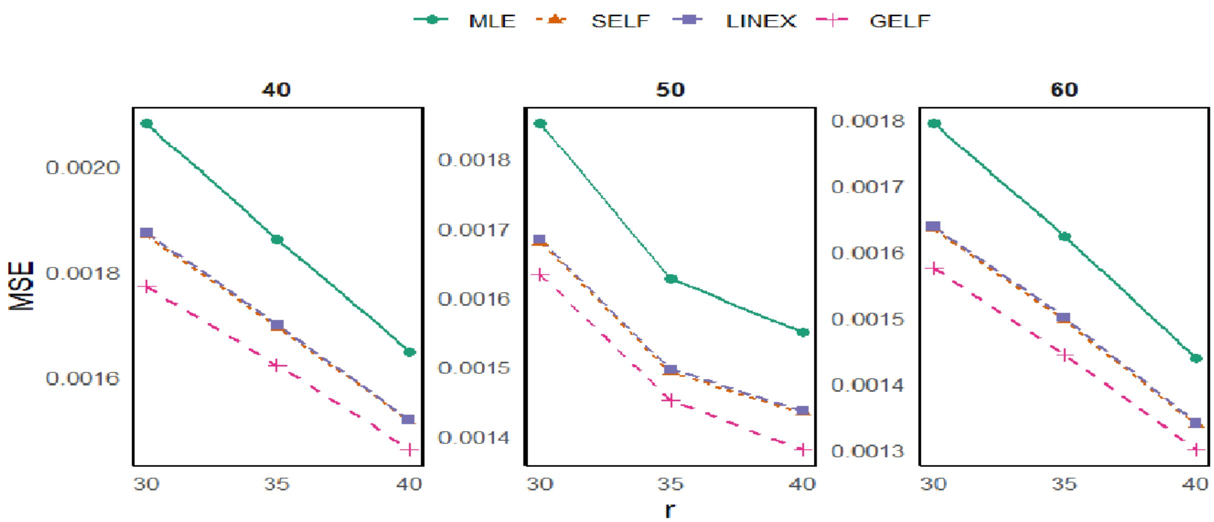

Fig. 8. Graph of MSE of hazard function for ν =0.1, q = 0.5 and t = 1

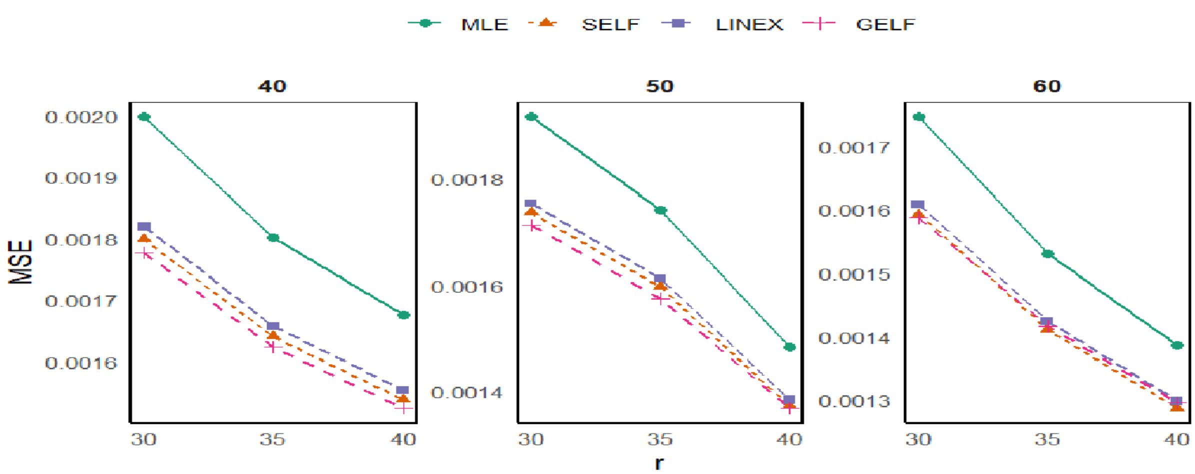

Fig. 9. Graph of MSE of hazard function for ν =0.5, q = 1 and t = 1

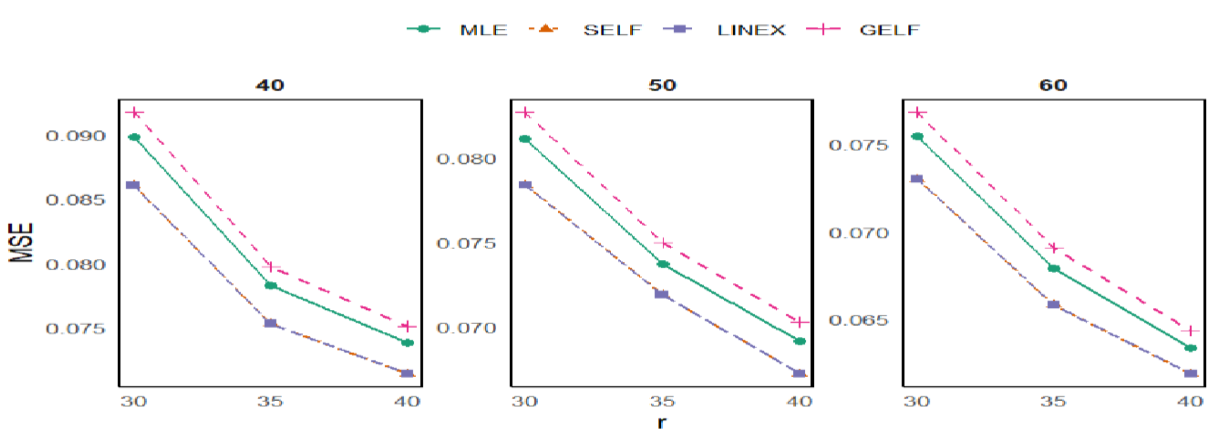

Fig. 10. Graph of MSE of mean residual function for $\nu = -0.25$, $q = -0.5$ and t = 10

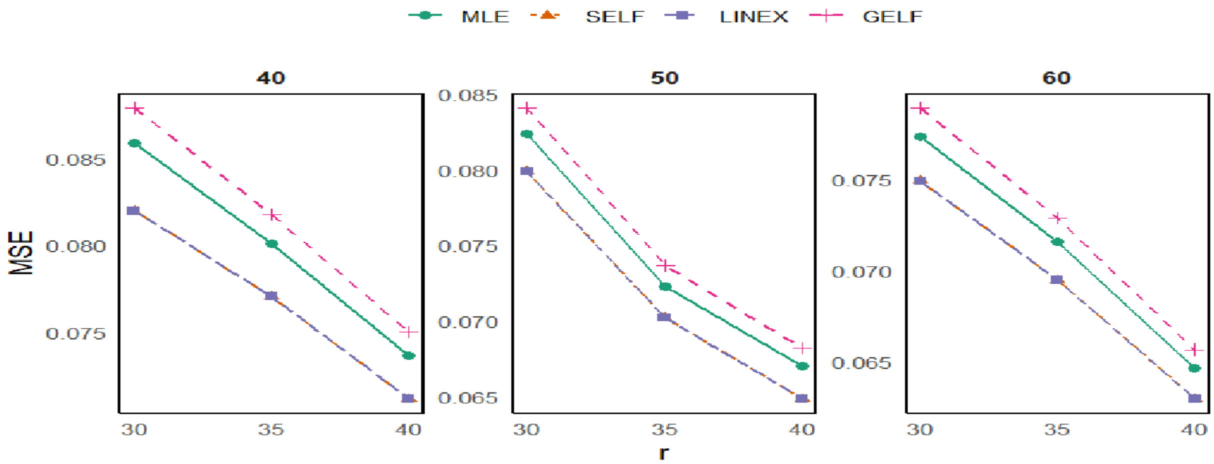

Fig. 11. Graph of MSE of mean residual function for ν =0.1, q = 0.5 and t = 10

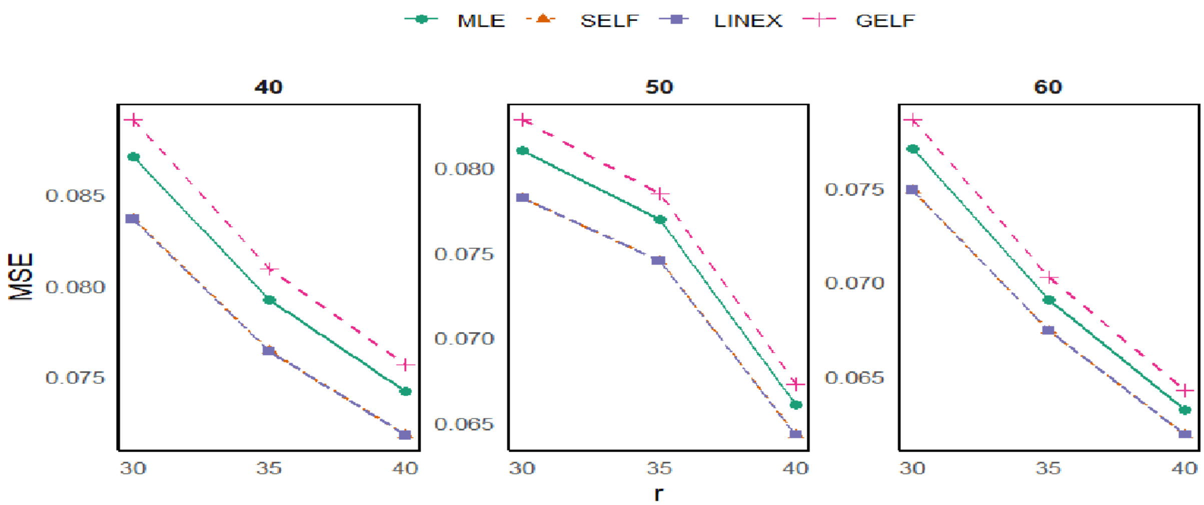

Fig. 12. Graph of MSE of mean residual function for ν =0.5, q = 1 and t = 10

VI. DATA ANALYSIS

In this section, we perform analysis on real data set. To illustrate the approach, we consider a real dataset that was earlier examined by Shanker (2015). In their study, they applied the Shanker distribution to this data and found that it captured the pattern of the data quite well.The data set represents the waiting times (in minutes) before service of 100 bank customers. The data are as follows:

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13,7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5

Using the above real dataset, we first obtained the maximum likelihood estimate (MLE) of the unknown parameter θ and reliability measures $R(t)$, $h(t)$ and $m(t)$. In addition, we derived Bayesian estimates under different loss functions—SELF, LINEX, and GELF—by applying Lindley's approximation method. For the LINEX and GELF cases, we used three distinct values of the asymmetry and entropy parameters, consistent with those chosen in the simulation study. The initial value of the parameter θ was selected close to its MLE, specifically 0.1983. We also incorporated prior information by setting a=4 and b=8. The resulting estimates for θ, along with measures such as reliability, hazard rate, and mean residual life, are presented in Tables 13 to 24.

Table 13 : Estimates of parameter under different loss function for $\nu = -0.25$ and $q = -0.5$ based on real data. (θ =0.1983)

n	r	$\hat{\theta}$	$\hat{\theta}_S$	$\hat{\theta}_L$	$\hat{\theta}_E$
100	40	0.204986	0.209264	0.209312	0.2088
	70	0.20302	0.205909	0.20594	0.205605
	100	0.198317	0.200597	0.20062	0.200363

Table 14 : Estimates of reliability function under different loss function for $\nu = -0.25$ and $q = -0.5$ and t = 1.5, based on real data. (R(t) = 0.95527)

n	r	$\hat{R}(t)$	$\tilde{R}_S(t)$	$\tilde{R}_L(t)$	$\tilde{R}_E(t)$
100	40	0.95227	0.949935	0.949945	0.956449
	70	0.953161	0.951607	0.951613	0.955986
	100	0.955263	0.954066	0.95407	0.957495

Table 15 : Estimates of hazard function under different loss function for $\nu = -0.25$ and $q = -0.5$ and t = 1 based on real data. (h(t) = 0.0380735)

n	r	$\hat{h}(t)$	$\tilde{h}_S(t)$	$\tilde{h}_L(t)$	$\tilde{h}_E(t)$
100	40	0.040603	0.042573	0.043	0.042469
	70	0.039852	0.041163	0.041449	0.041157
	100	0.03808	0.039089	0.039311	0.039121

Table 16 : Estimates of mean residual function under different loss function for $\nu = -0.25$ and $q = -0.5$ and t = 10 based on real data. (m(t) = 6.7114)

n	r	$\hat{m}(t)$	$\tilde{m}_S(t)$	$\tilde{m}_L(t)$	$\tilde{m}_E(t)$
100	40	6.45618	6.37653	6.37574	6.46044
	70	6.52931	6.47231	6.4719	6.53219
	100	6.71073	6.66201	6.66171	6.71301

Table 17 : Estimates of parameter under different loss function for ν = 0.1 and q = 0.5 based on real data. (θ =0.1983)

n	r	$\hat{\theta}$	$\hat{\theta}_S$	$\hat{\theta}_L$	$\hat{\theta}_E$
100	40	0.204986	0.209264	0.209246	0.207848
	70	0.20302	0.205909	0.205897	0.204985
	100	0.198317	0.200597	0.200588	0.199889

Table 18 : Estimates of reliability function under different loss function for ν = 0.1 and q = 0.5 and t = 1.5, based on real data. (R(t) = 0.95527)

n	r	$\hat{R}(t)$	$\tilde{R}_S(t)$	$\tilde{R}_L(t)$	$\tilde{R}_E(t)$
100	40	0.95227	0.949935	0.949931	0.956249

	70	0.953161	0.951607	0.951605	0.955856
	100	0.955263	0.954066	0.954064	0.957398

Table 19 : Estimates of hazard function under different loss function for ν = 0.1 and q = 0.5 and t = 1 based on real data. (h(t) = 0.0380735)

n	r	$\hat{h}(t)$	$\tilde{h}_S(t)$	$\tilde{h}_L(t)$	$\tilde{h}_E(t)$
100	40	0.040603	0.042573	0.042428	0.037743
	70	0.039852	0.041163	0.041065	0.038023
	100	0.03808	0.039089	0.039013	0.036663

Table 20 : Estimates of mean residual function under different loss function for ν = 0.1 and q = 0.5 and t = 10 based on real data. (m(t) = 6.7114)

n	r	$\hat{m}(t)$	$\tilde{m}_S(t)$	$\tilde{m}_L(t)$	$\tilde{m}_E(t)$
100	40	6.45618	6.37653	6.37685	6.46041
	70	6.52931	6.47231	6.47247	6.53217
	100	6.71073	6.66201	6.66212	6.71299

Table 21 : Estimates of parameter under different loss function for ν = 0.5 and q = 1 based on real data. (θ =0.1983)

n	r	$\hat{\theta}$	$\hat{\theta}_S$	$\hat{\theta}_L$	$\hat{\theta}_E$
100	40	0.204986	0.209264	0.20917	0.207364
	70	0.20302	0.205909	0.205848	0.204671
	100	0.198317	0.200597	0.200551	0.199649

Table 22 : Estimates of reliability function under different loss function for ν = 0.5 and q = 1 and t = 1.5, based on real data. (R(t) = 0.95527)

n	r	$\hat{R}(t)$	$\tilde{R}_S(t)$	$\tilde{R}_L(t)$	$\tilde{R}_E(t)$
100	40	0.95227	0.949935	0.949916	0.956149
	70	0.953161	0.951607	0.951595	0.955791
	100	0.955263	0.954066	0.954057	0.957349

Table 23 : Estimates of hazard function under different loss function for ν = 0.5 and q = 1 and t = 1 based on real data. (h(t) = 0.0380735)

n	r	$\hat{h}(t)$	$\tilde{h}_S(t)$	$\tilde{h}_L(t)$	$\tilde{h}_E(t)$
100	40	0.040603	0.042573	0.041958	0.035795
	70	0.039852	0.041163	0.040751	0.036644
	100	0.03808	0.039089	0.03877	0.035555

Table 24 : Estimates of mean residual function under different loss function for ν = 0.5 and q = 1 and t = 10 based on real data. (m(t) = 6.7114)

n	r	$\hat{m}(t)$	$\tilde{m}_S(t)$	$\tilde{m}_L(t)$	$\tilde{m}_E(t)$
100	40	6.45618	6.37653	6.37807	6.4604

	70	6.52931	6.47231	6.4731	6.53216
	100	6.71073	6.66201	6.66258	6.71298

VII. CONCLUSION

This paper focuses on estimating the unknown parameter and key reliability measures of the Shanker distribution in the presence of Type-II censoring, using both classical and Bayesian frameworks. We derived maximum likelihood estimates alongside Bayesian estimates under a variety of loss functions to capture different decision perspectives. To assess and compare the effectiveness of these methods, we carried out a detailed simulation study, and the findings have been thoroughly presented and discussed in this paper.

REFERENCES

[1] Cordeiro, G. M., Silva, R. B., & Nascimento, A. D. C. (2020). Recent advances in lifetime and reliability models. Bentham Science Publishers.

[2] Prinja, S., Gupta, N., & Verma, R. (2010). Censoring in clinical trials: Review of survival analysis techniques. Indian Journal of Community Medicine, 35(2), 217–221. https://doi.org/10.4103/0970-0218.66859

[3] Cohen, A. C. (1965). Maximum likelihood estimation in the Weibull distribution based on complete and on censored samples. Technometrics, 7(4), 579–588.

[4] Balakrishnan, N., Kundu, D., Ng, K. T., & Kannan, N. (2007). Point and interval estimation for a simple step-stress model with Type-II censoring. Journal of Quality Technology, 39(1), 35–47.

[5] Wingo, D. R. (1993). Maximum likelihood estimation of Burr XII distribution parameters under type II censoring. Microelectronics Reliability, 33(9), 1251–1257.

[6] Shanker, R. (2015). Shanker distribution and its applications. American Journal of Statistics and Applications, 5(6), 338–348.

[7] Abushal, T. A. (2019). Bayesian estimation of the reliability characteristic of Shanker distribution. Journal of the Egyptian Mathematical Society, 27(1), Article 30. https://doi.org/10.1186/s42787-019-0033-x

[8] Sangal, P. K. (2025). Unit-12 Loss Functions and Bayesian Estimation. Indira Gandhi National Open University. Retrieved from http://egyankosh.ac.in/handle/123456789/115093

[9] Al-Jarallah, R. A., Ghitany, M. E., & Kundu, D. (2024). Bayesian inference on Gompertz–Lindley distribution based on different loss functions. International Journal of Applied Mathematics, 37(2), 187–204.

[10] Nassar, M., Alotaibi, R., Okasha, H., & Wang, L. (2022). Bayesian estimation using expected LINEX loss function: A novel approach with applications. Mathematics, 10(3), 436. https://doi.org/10.3390/math10030436

[11] Calabria, R., & Pulcini, G. (1996). Point estimation under asymmetric loss functions for left-truncated exponential samples. Communications in Statistics: Theory and Methods, 25(3), 585–600.

[12] Basu, A. P., & Ebrahimi, N. (1991). Bayesian approach to life testing and reliability estimation using asymmetric loss function. Journal of Statistical Planning and Inference, 29(1), 21–31.

[13] Srivastava, R., & Shah, T. (2019). Shrinkage testimators of exponential scale parameter using general entropy loss function. Statistics and Applications, 17(1), 141–159.

[14] Abbas, K., Hussain, Z., Rashid, N., Ali, A., Tai, M., & Khan, S. A. (2020). Bayesian estimation of Gumbel Type-II distribution under Type-II censoring with medical applications. Computational and Mathematical Methods in Medicine, 2020, 1–14. https://doi.org/10.1155/2020/1876073

[15] Asgharzadeh, A., Ng, H. K. T., Valiollahi, R., & Azizpour, M. (2017). Statistical inference for Lindley model based on Type II censored data. Journal of Statistical Theory and Applications, 16(2), 178–197.

[16] Okasha, H. M. (2014). E-Bayesian estimation for the Lomax distribution based on Type-II censored data. Journal of the Egyptian Mathematical Society, 22(3), 489–495.

[17] Kundu, D., & Howlader, H. (2010). Bayesian inference and prediction of the inverse Weibull

distribution for Type-II censored data. Computational Statistics & Data Analysis, 54(6), 1547–1558.
[18] Lindley DV (1980). Approximate Bayesian method, Trabajos de Estadistica, 31, 223–237.