Topological evolution of magnetic field in ideal magnetohydrodynamics

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***Abstract*—Under the frozen-in condition of ideal magnetohydrodynamics (MHD), magnetic field lines are tied to the plasma fluid elements and move together with them. When unbalanced forces compress two distinct, magnetized plasma domains toward each other, their respective field lines are brought into close contact. Because the ideal frozen-in constraint prevents the fluid elements and their embedded field lines from intermixing, a sharp magnetic discontinuity forms along their boundary of interaction. According to Ampere's law, this steep gradient in the magnetic field vector generates an intense electric current density confined to a narrow, two-dimensional plane, forming what is known as a current sheet. When finite electrical resistivity or non-ideal plasma effects are present, the ideal frozen-in condition breaks down within these thin current sheets. The current sheet decays through Ohmic dissipation and magnetic diffusion, allowing opposing field lines to break, rejoin and relax into a simpler geometric configuration. This fundamental process, called magnetic reconnection, converts stored magnetic energy into kinetic energy, thermal energy, and particle acceleration. The formation and dissipative decay of current sheets provide a key physical mechanism for explaining diverse plasma phenomena across the universe, including the million-degree temperature of the solar corona, the triggering of solar flares and coronal mass ejections. In this work, simulations are carried out by solving the MHD equations numerically with suitable initial conditions of magnetic and velocity fields in Cartesian coordinates. The simulation results show the expansion of magnetic field lines and triggering of magnetic reconnection, leading to changes in magnetic field topologies.**

***Index Terms*—Current sheet, Magnetic topology, Magnetic discontinuity, Magnetic reconnection, Solar corona**

I. INTRODUCTION

The dynamical evolution of magnetic field in a fluid with infinite electrical conductivity is governed by ideal magnetohydrodynamics (MHD) equations [1]. Under the assumption of zero electrical resistivity, the magnetic field lines are permanently tied to the motion of the fluid elements. This important property of ideal MHD is termed as the frozen-in condition [2]. As the plasma flows and deforms, embedded magnetic field lines exactly follow the fluid's velocity field. Consequently, two distinct plasma volumes can never mix, and the global geometric connectivity, or topology, of the magnetic field remains strictly preserved over time. While flux freezing forces the magnetic field to maintain its topological structure, it also acts as a primary mechanism for building up extreme spatial stresses within the fluid. When macroscopic fluid motions drive two separate magnetic domains toward each other, the plasma trapped between them cannot simply escape across field lines. Instead, the intervening plasma is squeezed out along the boundary, forcing opposing magnetic fields into extremely close proximity. This compression narrows the spatial scale between opposing field orientations, creating steep spatial gradients in the magnetic field. These intense field gradients directly give rise to concentrated electrical current layers. According to Ampère's law, any region characterized by a rapid variation in magnetic field direction or magnitude across a short distance must carry a proportional electric current density. As the boundary layer thins down to very small scales, localized current density increases. This process leads directly to the formation of a current sheet [3]: a thin, planar region of intense current density that physically separates two distinct, oppositely directed magnetic flux systems.

The formation of a thin current sheet introduces a severe physical limitation to the ideal MHD framework. As current sheet thickness approaches to very small scales, localized non-ideal effects— such as finite electrical resistivity— can no longer be ignored. Even a tiny amount of resistivity inside a thin current sheet becomes dominant due to the enormous local current density. This localized breakdown of the frozen-in condition allows magnetic field lines to diffuse through the plasma, severing the strict topological constraint that previously bound them. This localized topological breakdown triggers the dynamic process known as magnetic reconnection [1]. Within the current sheet, opposing magnetic field lines from different domains meet at a central magnetic null point, break their original connections, and instantly rebind into a newly reconfigured field geometry. This topological transition allows the system to relax to a much lower magnetic energy state. The vast amount of magnetic energy stored in compressed fields is explosively liberated, converting into plasma kinetic energy, bulk high-speed outflows, intense localized heating, and particle acceleration. When applied to space plasmas, these fundamental principles provide the framework for understanding the violent dynamic behavior of the solar corona. The corona is an immense, highly conducting plasma environment where magnetic fields dominate energetic dynamics. Photospheric convective flows continuously churn, shear, and twist the anchor points of magnetic loops. Because coronal plasma is virtually collisionless and highly conductive, it initially obeys the frozen-in condition, causing immense amounts of free magnetic energy to accumulate within distorted coronal archways over days or weeks. This continuous photospheric driving pushes adjacent, misaligned magnetic loops against one another, forming an intricate network of current sheets throughout the coronal atmosphere. At these compressed interfaces, the ideal MHD description breaks down, and magnetic reconnection ignites. This process of magnetic reconnection then provides a way to explain the the long-standing coronal heating problem [1, 3], explaining why the corona reaches multi-million-degree temperatures despite sitting directly above a much cooler surface.

On global scales, magnetic reconnection within major coronal current sheets acts as the trigger for the most explosive events in the solar system, such as solar flares and coronal mass ejections [1]. When a massive coronal flux rope or sheared magnetic arcade becomes unstable, an elongated current sheet develops beneath the erupting structure. Fast reconnection cascades through this sheet, unleashing stored magnetic energy in mere minutes. This process hurls billion-ton plasma clouds into interplanetary space and accelerates charged particles to near-light speeds, driving space weather phenomena that impact satellite operations, communications, and power grids on Earth. Also, the formation of current sheets and its dissipation through magnetic reconnection play an important role in laboratory plasmas [4].

II. PARKER THEORY OF CURRENT SHEET FORMATION

The idea of the formation of current sheet in a fluid with high electrical conductivity was first proposed by E.N. Parker [3]. For demonstrating the formation of current sheet, Parker considers magnetic field embedded in an infinitely conducting fluid in between two conducting plates. All the magnetic field lines anchored at one conducting plate are held fixed while at the other conducting plate fluid motion is generated. Because of the motion of the fluid, the field lines will also move with the fluid following the frozen-in condition. As time passes the magnetic field lines wrap around each other and form a complex magnetic topology. The fluid motion is stopped when the field lines achieve such a complex topological configuration. The two ends of each magnetic field lines are tied and the whole system is allowed to relax towards the static equilibrium. Under the action of Maxwell stresses, the field lines achieve the terminal state of static equilibrium. In this equilibrium state the arbitrary topology of magnetic field lines is not the same as the topology of static equilibrium. The contradiction between the two sets of field topologies will result in the formation of magnetic discontinuity or current sheet. In this way by the formation of current sheets, the arbitrary field topology gets accommodated in the terminal state of static equilibrium.

An arbitrary interlaced continuous magnetic field embedded in a highly conducting fluid develops current sheets while relaxing towards the static equilibrium [3, 5]. An important property of static equilibrium is that the twist per unit length is constant along the magnetic field line [6]. For an arbitrary magnetic field, the twist per unit length may not have constant value along the field lines. Consequently, an arbitrary field topology will develop magnetic discontinuity or current sheets in the static equilibrium in order to accommodate the constant value of twist per unit length along the field lines. The formation of current sheets in a highly conducting fluid can also be understood mathematically by studying the nonlinear character of the magnetostatic equilibrium equation. The solution of the magnetostatic equation has real characteristics along with two families of imaginary characteristics [3]. The real characteristic of the magnetostatic equilibrium equation is nothing but the magnetic field line. Because of the inherent nonlinear character of this equilibrium equation, any two real characteristic curves may intersect at any point and the magnetic field will become discontinuous at that point. The discontinuous magnetic field then generates a high value of electric current density or current sheet.

III. FORMATION OF CURRENT SHEETS IN THE SOLAR CORONA

Observation of the Sun in X-ray regime reveals that the temperature of outermost layer of the Sun, called the solar corona, is of the order of million-degree Kelvin [1, 3, 7]. The temperature of the surface of the Sun, called the photosphere, is of the order of few thousand-degree Kelvin. The relatively high temperature of the solar corona is in general contradiction to the second law of thermodynamics that heat must flow from high temperature to a low temperature region. In solar atmosphere, the heat flows from the corona to the photosphere. It indicates that there must be some heat input to the solar corona in order to balance the corresponding heat loss from the corona. It is believed that the formation of magnetic discontinuity or the current sheet is responsible to to generate heat via joule dissipation in order to maintain the corona at its million-degree Kelvin temperature. The Sun has its own magnetic field which is generated in the interior of the Sun through the dynamo mechanism. Because of the very high temperature of the solar corona, the gaseous medium of corona becomes ionized and is in plasma state. Thus, coronal medium has a high value of electrical conductivity. Therefore, the coronal medium can be modeled as a magnetofluid which is well described by the magnetohydrodynamic equations. The coronal magnetic field lines are rooted to the photosphere. These field lines are shuffled by the convective motion of the fluid and the field lines become interlaced to generate complex structure of magnetic field lines in the solar corona. Thus, energy from the photosphere to the solar corona is transferred through the magnetic field lines. Under the action of unbalanced forces, any two portions of the magnetofluid may come into a direct contact with each other and form magnetic discontinuity wherever they meet. Following the Ampere's law, at the site of magnetic discontinuity, intense value of electric current density is generated in the form of current sheet. This high value of current density is then dissipated through the otherwise small but finite electrical resistivity of the coronal medium. In this process huge amount of energy is liberated in the form of heat and energetic particles along with the change in field topology. This is the mechanism which delivers enough heat to the solar corona to keep its temperature hovering in the range of million-degree Kelvin. Also, topological changes in the coronal magnetic field lines lead to the various eruptive phenomena in the solar corona such as solar flares, coronal mass ejections, etc.

IV. GOVERNING EQUATIONS

The dynamics of a conducting fluid in presence of magnetic field is governed by the magnetohydrodynamic (MHD) equations. The full sets of ideal MHD equations can be written as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \qquad ..(1)$$

$$\rho\left(\frac{\partial u}{\partial t} + (u \cdot \nabla)u\right) = -\nabla p + \frac{1}{\mu_0}(\nabla \times B) \times B + \mu \nabla^2 u \qquad ...(2)$$

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) \qquad ...(3)$$

$$\frac{\partial}{\partial t}(p\rho^{-\gamma}) + u \cdot \nabla(p\rho^{-\gamma}) = 0 \qquad ...(4)$$

$$\nabla \cdot B = 0 \qquad ...(5)$$

In the above sets of equations, ρ is the mass density, **u** is the fluid velocity, **B** is the magnetic field, p is the fluid pressure and μ is the coefficient of viscosity.

Equations (1)-(5) represent the continuity equation, momentum balance equation, induction equation, energy equation and the solenoidality condition for the magnetic field, respectively.

V. SIMULATION RESULTS

Here numerical simulations are carried out to study the topological evolution of magnetic field in ideal magnetohydrodynamics. For this purpose, the sets of ideal MHD equations are solved numerically using the well-established magnetohydrodynamic code Athena in Cartesian coordinates. For the initial condition of magnetic field, we have chosen a potential field. A Gaussian velocity field is applied at the bottom boundary (at z=0 plane). We employed periodic boundary conditions in the lateral directions and open boundary conditions in the vertical directions. The computational domain size is $[0, 2\pi]$ and the grid resolution is 128^3. All the MHD equations are casted in their dimensionless form.

The initial magnetic field lines are plotted in Panel a of Figure 1. This initial field topology consists of three flux systems--- the central magnetic loops and the two half loops sitting on both sides of the central loop. The dynamical evolution of the magnetic field is shown in panels (a) to (f) of Figure 1. The initial velocity profile is as such that it shears the magnetic field lines at the bottom boundary (z=0 plane) and as a result of this the magnetic field lines rapidly rises in the vertical direction along with expansion in the lateral directions. (from Panel (a) to (c) of Figure 1). The expanding magnetic flux systems press the neighboring flux systems and form current sheets at the common surface of contact. Further pressing of the flux systems below the grid resolution generates numerical dissipation because of the under-resolved scales. The current sheets get dissipated via the numerical dissipation along with change in field topologies. The changing field topology is evident from the figure 1(d) to 1(f), where the field lines associated with the central flux system is getting reconnected with the field lines of the neighboring flux systems. Our simulation results are qualitatively in agreement with the observed rising and expanding magnetic loops in the solar corona [3, 8]. Also, similar kind of field lines restructuring due to magnetic reconnection occur in the corona [8].

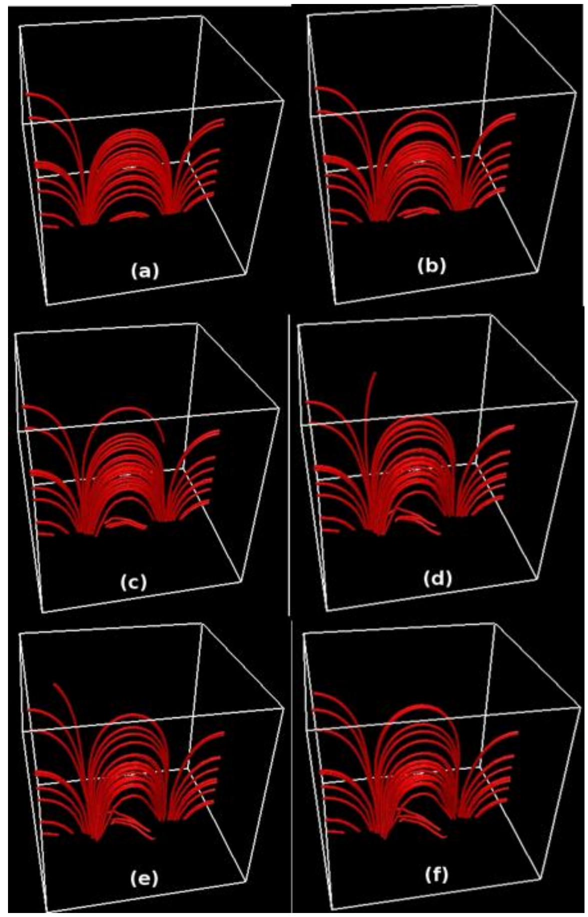

Figure 1: Panels (a) to (f) depict the time evolution of magnetic field from t=0 to t=10 with time step Δt=2 in a Cartesian box of size $[0, 2\pi]$, resolved with 128^3 grid resolution. The magnetic field lines are getting sheared due to the fluid motion at the bottom boundary. The expansion of the field lines in both lateral and vertical directions is evident from panels (a) to (c). The reconnecting field lines are shown in panels (d) to (f).

VI. CONCLUSION

In a highly conducting fluid, magnetic field lines are frozen into the fluid elements on large length scales. When two fluid elements carrying magnetic field lines are driven together by unbalanced forces and brought into direct contact, a sharp discontinuity in the magnetic field forms at their interface because the frozen-in condition prevents the distinct fluid elements from immediately mixing. This narrow contact layer develops a steep magnetic field gradient, which generates an intense electric current density—effectively forming a current sheet. Although the fluid's electrical resistivity is normally small, it becomes crucial within these thin sheets, dissipating the current and converting magnetic energy into thermal and kinetic energy. This process, accompanied by a fundamental restructuring of the magnetic field topology, is known as magnetic reconnection. Magnetic reconnection and the formation of current sheets explain many high-energy astrophysical and space plasma phenomena, including the million-degree heating of the solar corona, solar flares, and coronal mass ejections.

The topological evolution of magnetic field is studied in a compressible viscous fluid with infinite electrical conductivity. A simple potential magnetic field is evolved by applying the Gaussian velocity field at the footpoints (z=0 plane) of the magnetic field lines. The promising results of our simulation is that the field lines rise in the vertical direction and expand in the lateral directions. Because of the rise and expansion of the field lines the neighboring flux systems come close to each other and form current sheets at their common surface of contact. Once the field lines are pushed below the model resolution then numerical dissipation becomes effective and field lines get reconnected, leading to a change in magnetic field topologies. These results are qualitatively in general agreement with the observed rise and expansion of magnetic loops in the solar corona.

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