Advanced Approaches to Solving the Transportation Problem

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***Abstract*—The transportation problem is one of the most significant areas of Operations Research. It is a specialized linear programming problem that focuses on minimizing the total cost of transporting a single commodity from multiple sources to multiple destinations while satisfying supply and demand constraints. Transportation problems are extensively applied as decision-making tools in various fields, including engineering, business management, logistics, supply chain management, and industrial planning. These problems can be effectively solved using a variety of network optimization techniques. In this paper, an Advanced Method for Solving the Transportation Problem (AMSTP) is proposed to obtain both the initial basic feasible solution and the optimal solution efficiently. The performance of the proposed method is evaluated by comparing its results with those of several existing solution techniques, demonstrating its effectiveness and computational efficiency.**

***Index Terms*—Transportation problem; Linear programming; Optimal solution; Initial basic feasible solution.**

I. INTRODUCTION

The transportation problem is generally defined as an optimization model that determines the most economical way of distributing goods from a number of supply centers, such as factories or manufacturing units (called sources), to a number of receiving centers, such as warehouses or distribution centers (called destinations). The primary objective of this model is to minimize the total transportation cost while ensuring that all supply and demand requirements are satisfied. In a transportation problem, each source has a fixed quantity of goods available for shipment, and each destination has a predetermined demand that must be fulfilled [2].

Transportation models are also widely used by organizations when making strategic decisions regarding the establishment of new facilities. Companies utilize these models to identify the most suitable locations for production plants, warehouses, or distribution canters. By minimizing the combined production and transportation costs across the entire system, transportation models contribute significantly to cost-effective facility location planning and improved operational efficiency [8].

The transportation problem was first formulated by Hitchcock [3] and was later refined and extended by Koopmans and Dantzig [4, 1]. Since then, numerous modifications, extensions, and solution methods have been developed to address different types of transportation problems. The classical transportation problem in Operations Research focuses on determining the minimum-cost shipping schedule for transporting a single commodity from several sources to multiple destinations. Each source is characterized by a fixed supply, commonly referred to as availability, while each destination has a specified demand, known as the requirement. Owing to its ability to determine the most efficient movement of goods, the transportation problem has become an essential optimization tool in logistics and supply chain management. It is extensively applied in manufacturing industries, transportation systems, and business organizations to improve distribution efficiency and minimize overall transportation costs [6].

II. CATEGORIES OF TRANSPORTATION PROBLEMS

Transportation problems are broadly classified into two categories: Balanced Transportation Problems and

Unbalanced Transportation Problems [5]. A transportation problem is considered balanced when the total available supply from all sources is exactly equal to the total demand at all destinations. In such cases, all supply and demand constraints are satisfied without requiring any adjustments to the model. Conversely, a transportation problem is classified as unbalanced when the total available supply does not equal the total demand. To obtain a feasible solution for an unbalanced transportation problem, it is generally necessary to balance the model by introducing a dummy source or a dummy destination with zero transportation cost [5].

2.1 Mathematical Formulation of the Transportation Problem

A solution X_{ij} is called a feasible solution of a transportation problem if it satisfies all the supply and demand constraints. A feasible solution is considered a basic feasible solution (BFS) when the number of positive allocations is exactly ((m+n-1)), where (m) is the number of sources and (n) is the number of destinations, provided that all supply and demand requirements are fully satisfied. In other words, a basic feasible solution must satisfy both the availability constraints of the sources and the demand constraints of the destinations. Several methods have been developed to obtain an initial basic feasible solution, among which the three most commonly used methods are the North-West Corner Method (NWCM), the Least Cost Method (LCM), and Vogel's Approximation Method (VAM).

- North West Corner Rule (NWCM).
- Least Cost Method (LCM).
- Vogel’s Approximation Method or Regret Method (VAM).

A balanced transportation problem always possesses at least one optimal solution. The solution process begins by obtaining an initial basic feasible solution (IBFS) using one of the standard methods, such as the North-West Corner Method, the Least Cost Method, or Vogel's Approximation Method. After obtaining the initial solution, it is verified whether the number of occupied (allocated) cells is exactly equal to ((m+n-1)), where (m) and (n) denote the number of sources (rows) and destinations (columns), respectively. If this condition is not satisfied, the solution is considered degenerate, and an appropriate procedure is applied to remove the degeneracy while maintaining feasibility. The optimization process is based on the principle that if the current basic feasible solution is not optimal, an improving closed loop can be identified to enhance the objective function. The optimality of the transportation problem is commonly tested using the Modified Distribution (MODI) Method, also known as the (u)-(v) Method, or the Stepping Stone Method, both of which systematically evaluate the opportunity costs of unoccupied cells to determine the optimal solution.

The mathematical formulation of the problem is as follows:

Using the following notations;

Where:

- X_{ij} = quantity of goods transported from source (i) to destination (j),
  where (i = 1, 2, m) and (j = 1, 2, n).
- s_i = total supply available at source (i).
- d_j = total demand required at destination (j).
- c_{ij} = unit transportation cost of shipping goods from source (i) to destination (j).

The transportation problem can be mathematically formulated as a linear programming model and is typically represented in the form of a transportation matrix. This matrix provides a systematic representation of the transportation costs, supply capacities, and demand requirements, thereby facilitating the determination of an optimal transportation plan.

The data associated with a transportation problem are typically organized in a tabular format known as the transportation cost matrix (Table 2.1). This matrix provides a structured representation of the problem by listing the supply points (sources), the demand points (destinations), the available supply at each source, the required demand at each destination, and the corresponding unit transportation cost for shipping goods from each source to each destination.

Table 1 Transportation problem costs

Source	Destination				Supply
	1	2	....	n	
1	C_{11}	C_{12}	…	C_{1n}	S_1
2	C_{21}	C_{22}	…	C_{2n}	S_2
⋮	⋮	⋮	⋱	⋮	⋮
m	C_{m1}	C_{m2}	…	C_{mn}	S_m
Demand	d_1	d_2	…	d_n	

In mathematical form this expressed as

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \ldots\ldots\ldots\ldots(1)$$

Total supply = Total demand. This is known as a balanced problem.

The problem is modeled as a linear programming problem

$$Minimise\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \quad \ldots..(2)$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = s_i, \text{ for i=1,2,..m…..(3)}$$

$$\sum_{i=1}^{n} x_{ij} = d_j, \text{ for j=1,2,..n…..(4)}$$

$$x_{ij} \geq 0, for\ all\ i\ and\ j\ .$$

This mathematical formulation is applicable to all linear programming transportation problems, irrespective of their practical or industrial context. In most real-world applications, the supply and demand quantities are expressed as integer values, requiring the shipment allocations to be integral as well. The unit coefficients associated with the decision variables in the constraint equations ensure that, whenever the supply and demand values are integers, the optimal solution obtained is also composed of integer allocations.

2.2. Step-by-Step Algorithm of the MODI Method

Step 1: Begin with an initial basic feasible solution (IBFS) containing exactly $(m+n-1)$occupied (basic) cells. Determine the row potentials $(u_i)$and column potentials $(v_j)$by applying the relation

$$c_{ij} = u_i + v_j,$$

for all occupied cells only. To initiate the calculations, assign a value of zero to any one of the row or column potentials (either a selected $u_i$or v_j), and then compute the remaining potentials accordingly.

Step 2: For each unoccupied (non-basic) cell, compute the cell evaluation (also known as the opportunity cost or net evaluation) using the following expression:

$$\Delta_{ij} = c_{ij} - (u_i + v_j).$$

The computed values of $\Delta_{ij}$are used to assess the optimality of the current basic feasible solution and to identify whether further improvement in the total transportation cost is possible.

Step 3: Examine the opportunity costs $\Delta_{ij}$for all unoccupied (non-basic) cells and determine the optimality of the current solution as follows:

(a) If all $\Delta_{ij} \geq 0$(or equivalently, all opportunity costs are non-negative for a minimization problem), the current basic feasible solution is optimal. If every $\Delta_{ij} > 0$, the optimal solution is unique.

(b) If one or more $\Delta_{ij} = 0$, the current solution remains optimal, but alternative optimal solution(s) exist.

(c) If any $\Delta_{ij} < 0$, the current solution is not optimal, and a better solution can be obtained. Select the unoccupied cell with the most negative opportunity cost to enter the basis, construct a closed loop, and revise the allocations by replacing one basic cell with a non-basic cell. Then proceed to Step 4.

Step 4: Select the unoccupied cell with the most negative opportunity cost (cell evaluation) as the entering cell. Next, construct a closed loop (or closed path) that begins and ends at the entering cell while passing through occupied (basic) cells. The loop should be formed by moving only horizontally and vertically, with each change in direction occurring at a basic cell. Right-angle turns are permitted, and every corner of the loop must correspond to a basic cell except for the entering cell.

Step 5: Assign a positive (+) sign to the entering cell and label the remaining cells in the closed loop alternately with negative (−) and positive (+) signs, ensuring that the entering cell always carries the positive sign. Determine the maximum number of units that can be reallocated by identifying the smallest allocation among the cells marked with a negative sign. This minimum allocation represents the maximum quantity that can be transferred to the entering cell without violating the feasibility conditions. Add this quantity to the allocations in the cells marked with a positive sign and subtract it from the allocations in the cells marked with a negative sign. Consequently, the entering cell becomes a basic (occupied) cell, while the basic cell whose allocation becomes zero leaves the basis and becomes a non-basic (unoccupied) cell. Repeat Steps 1–5 until all opportunity costs $\Delta_{ij}$are non-negative, indicating that the optimal solution has been reached. Finally, compute the minimum total transportation cost corresponding to the optimal allocation [7].

2.3. Algorithm of (AMSTP) method
Advanced Method for Solving the Transportation Problem)

Step 1: Construct the transportation matrix for the given transportation problem by arranging the sources as rows and the destinations as columns. Next, prepare two auxiliary columns: the first column lists all the sources, while the second column lists the corresponding destinations. These columns are used to facilitate the implementation of the proposed solution procedure.

Step 2: List all the sources (e.g., $X_1, X_2, X_3, \dots, X_m$) in the first auxiliary column. For each source (row), identify the minimum transportation cost among all the destinations. Record the destination corresponding to this minimum cost in the second auxiliary column. Repeat this process for every source until the minimum-cost destination for each row has been identified.

Column 1	Column 2
X_1	Y_{min}
X_2	Y_{min}
X_3	Y_{min}
$\vdots$	$\vdots$
X_n	Y_{min}

Step 3: Repeat the procedure for each source until the minimum transportation cost has been identified for every row. If a source has two or more destinations with the same minimum transportation cost, record all such destinations in Column 2. Among these alternatives, select the destination corresponding to the minimum supply or demand value, as appropriate, and choose that cell for allocation.

Step 4: Identify the rows in Column 1 that correspond to a unique destination listed in Column 2. For each such row, allocate the minimum of the available supply and the corresponding destination demand. After the allocation, update the remaining supply and demand by subtracting the allocated quantity from both values. If the supply of a source or the demand of a destination is completely satisfied, eliminate the corresponding row or column from further consideration. In the event of a tie between the minimum supply and demand values, make the allocation in the cell associated with the maximum available supply or demand, as applicable. However, if destinations are not unique then go to step 4.

Step 5: If a destination listed in Column 2 is not unique, identify all the sources that correspond to the same destination. For the common destination column, compute the difference between the maximum and minimum transportation costs for each of these sources. Select the source with the largest cost difference as the preferred allocation. If two or more sources have the same cost difference, resolve the tie by calculating the difference between the minimum and the next higher transportation cost in the corresponding row. If no such higher cost exists, use the difference between the minimum transportation cost and zero as the tie-breaking criterion.

Step 6: Identify the source with the largest cost difference obtained in the previous step. Allocate the minimum of the available supply and the corresponding destination demand to the selected cell. After the allocation, update the supply and demand values by subtracting the allocated quantity. If the supply of a source or the demand of a destination is completely satisfied, eliminate the corresponding row or column from further consideration. Repeat the procedure until all supply and demand requirements have been fulfilled.

Step 7: Repeat Steps 4–6 for the remaining sources and destinations, updating the transportation matrix after each allocation. Continue the allocation process until exactly $(m + n - 1)$basic cells have been occupied, thereby obtaining a complete initial basic feasible solution (IBFS) for the transportation problem.
Total cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand. That is,

III. NUMERICAL EXAMPLES

3.1 Example

	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Solution: Form two auxiliary columns in which the first column represents the sources, and the second column records the destination(s) corresponding to the minimum transportation cost for each source. This format facilitates the identification of the initial allocation in the proposed algorithm.

Column 1 (Sources)	Column 2 (Destinations)	
A	F	
B	D	unique
C	F	

B has a unique destination at D directly allocate the minimum of demand/supply

	D	E	F	Supply
A	6	4	1	50
B	20 3	8	7	40-20=20
C	4	4	2	60
Demand	20-20=0	95	35	150

After satisfying the demand for destination D, remove the corresponding destination column (D) from the transportation matrix. Then, for the remaining sources, identify the destination with the minimum transportation cost and update the auxiliary columns accordingly before proceeding with the next allocation.

Column 1	Column 2	Difference
A	F	7 - 1 = 6
B	F	7 - 7 = 0
C	F	7 - 2 = 5

Since Source A has the maximum cost difference, select it for allocation and assign the minimum of the available supply and the corresponding demand to the selected cell.

	E	F	Supply
A	4	35 1	50-35=15
B	8	7	20
C	4	2	60
Demand	95	35-35=0	150

After satisfying the demand for destination F, eliminate the corresponding F column from the transportation matrix. Subsequently, identify the minimum-cost destination for each of the remaining sources and continue the allocation process.

Column 1	Column 2
A	E
B	E
C	E

	E	Supply
A	15 4	15
B	20 8	20
C	60 4	60
Demand	95	150

Number of allocations = (m + n - 1)
= 3 + 3 - 1
= 5
Optimal solution = 20 x 3 + 35 x 1 + 15 x 4 + 20 x 8 + 60 x 4 = 555
MODI = 555

3.2. Example

	1	2	3	Supply
A	8	6	9	20
B	6	3	8	30
C	10	7	9	70
Demand	90	20	10	

Solution: Construct two auxiliary columns in which the first column lists all the sources, and the second column records the destination corresponding to the minimum transportation cost for each source.

Column 1	Column 2	Deference
A	2	7 – 6 = 1
B	2	7 – 3 = 4
C	2	7 – 7 = 0

Since Source B has the maximum cost difference, select it for allocation and assign the minimum of the available supply and the corresponding destination demand to the selected cell. After the allocation, eliminate Destination 2 from the transportation matrix if its demand has been completely satisfied. Then, re-evaluate the remaining matrix by identifying the minimum-cost destination for each of the remaining sources before proceeding with the next allocation.

	1	2	3	Supply
A	8	6	9	20
B	6	20 3	8	30-20=10
C	10	7	9	70
Demand	90	20-20=0	10	

Column 1	Column 2
A	1
B	1
C	3 unique

Since Source C has a unique minimum-cost destination (Destination 3), allocate the minimum of the available supply and the corresponding destination demand to the selected cell.

	1	3	Supply
A	8	9	20
B	6	8	10
C	10 10 9	70-10=60	
Demand	90	10-10=0	

After satisfying the demand for Destination 3, remove the corresponding Column 3 from the transportation matrix. Then, determine the minimum-cost destination for each of the remaining sources and continue the allocation procedure.

Column 1	Column 2
A	1
B	1
C	1

	1	Supply
A	20 8	20
B	10 6	10
C	60 10	60
Demand	90	

Number of allocations = (m + n - 1) = 3 + 3 - 1 = 5
Optimal solution = 20 x 3 +10 x 9 + 20 x 8 + 10 x 6 + 60 x 10 = 970
MODI = 970

3.3. Example

	D_1	D_2	D_3	D_4	Supply
F_1	3	1	7	4	300
F_2	2	6	5	9	400
F_3	8	3	3	2	500
Demand	250	350	400	200	

Solution:

Column 1	Column 2	
F_1	D_2	unique
F_2	D_1	unique
F_3	D_4	unique

Since F_1, F_2, and F_3each have a unique destination at D_2, D_1, and D_4, respectively, allocate the minimum of the corresponding supply and demand directly to these cells. After the allocation, delete $F_1$from the transportation table because its supply has been exhausted (reduced to zero), along with the satisfied demand.

	D_1	D_2	D_3	D_4	Supply
F_1	3	300 1	7	4	300-300=0
F_2	250 2	6	5	9	400-250=150
F_3	8	3	3	200 2	500-200=300
Demand	250-250=0	350-300=50	400	200-200=0	

After deleting row $F_1$and columns $D_1$and D_4, determine the unique destination(s) for the remaining sources and allocate the minimum of the corresponding supply and demand.

Column 1	Column 2
F_2	D_3
F_3	D_2, D_3

	D_2	D_3	Supply
F_2	6	5	150
F_3	50 3	3	300 – 50 = 250
Demand	50 – 50 = 0	400	

Delete column D_2, then identify the unique destination(s) for the remaining sources and allocate the minimum of the corresponding supply and demand.

Column 1	Column 2
F_2	D_3
F_3	D_3

	D_3	Supply
F_2	150 5	150
F_3	250 3	250
Demand	350	

Number of allocations = (m + n - 1)
= 3 + 4 – 1
= 6
Optimal solution = 300 x 1 + 250 x 2 + 200 x 2 + 50 x 3 +150 x 5 + 250 x 3
= 2850
MODI = 2850

IV. RESULT AND DISCUSSION

The comparison between the results obtained using the MODI Method and the Advanced Method for solving the transportation problem is presented in the following table.

Table 2. Results of Balanced and Unbalanced Transportation Problems Using the MODI Method and AMSTP.

Problems	Problem Dimension	MODI Method	(AMSTP)
1	3X3	555	555
2	3X3	970	970
3	3X4	2850	2850
4	3X4	3500	3500
5	3X3	3900	3900
6	3X3	1320	1200

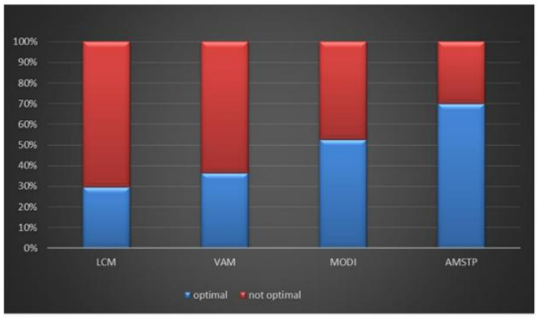

Figure 1 Comparison of Optimal and Non-Optimal Solution Percentages for Existing Methods and the Advanced Method for Solving the Transportation Problem (AMSTP).

V. CONCLUSION

In this study, the proposed Advanced Method for Solving the Transportation Problem (AMSTP) was successfully applied to various balanced and unbalanced transportation problem instances. The results demonstrate that AMSTP is more efficient than the existing methods in obtaining optimal solutions.

A comparison between the MODI Method and the proposed Advanced Method for Solving the Transportation Problem (AMSTP) reveals that the MODI Method requires more iterations and computational time to obtain the optimal solution. To overcome these limitations, the AMSTP was developed to reduce the number of iterations, minimize computation time, and simplify the solution procedure. Unlike the MODI Method, the proposed approach directly determines the optimal solution without first obtaining an Initial Basic Feasible Solution (IBFS). Consequently, AMSTP provides an efficient, straightforward, and computationally effective technique for solving both balanced and unbalanced transportation problems.

The proposed AMSTP is a faster and more efficient approach for solving transportation problems. It produces the same optimal solutions as the MODI Method while outperforming other existing methods in terms of computational efficiency, as illustrated in Figure 4.1.

The proposed Advanced Method for Solving the Transportation Problem (AMSTP) can be effectively applied to optimize the allocation and distribution of limited resources in combinatorial optimization problems. Owing to its computational efficiency, the method has significant potential for solving large-scale real-world transportation and logistics problems.

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