Mathematical Perspective on India's Economic Development and Policy Design

Narinder Sharma
Department of Mathematics, SPPND GDC Samba, J&K

Abstract—**This paper develops a rigorous mathematical and quantitative framework to analyze India's economic growth trajectory, structural transformation, macroeconomic policy transmission, and optimal developmental policy design. Moving beyond descriptive macroeconomic narratives, we formulate India's distinct transition dynamics—characterized by premature de-industrialization and a rapid, service-led shift—within multi-sector dynamic general equilibrium (DSGE) and continuous-time optimal growth frameworks. We formally model sectoral capital allocation, the interaction between monetary policy reaction functions and large informal labor markets, fiscal multiplier dynamics under debt-sustainability constraints, and input-output network linkages across the Indian economy. Through calibrated analytical derivations and mathematical policy optimization, we derive Pareto-efficient resource allocation rules, optimal capital expenditure trajectories, and inflation-targeting frontiers tailored to emerging market structural rigidities.**

I. INTRODUCTION AND ANALYTICAL FRAMEWORK

India's growth paradigm defies the canonical path laid out by classic structural transformation models formulated by Lewis (1954), Kuznets (1966), and Chenery and Syrquin (1975). In traditional development economics, labor migrates sequentially from low-productivity subsistence agriculture to medium-productivity industrial manufacturing, and subsequently to high-productivity tertiary services. In contrast, the Indian economy underwent a truncated industrialization process, transitioning directly from an agrarian base toward modern skill-intensive services and informal non-tradable activities.

To model this dynamic rigorously, we establish an integrated macroeconomic framework synthesizing continuous-time neoclassical dynamics, non-homothetic multi-sector preferences, structural duality between formal and informal markets, and modern network control theory.

The model is populated by representative households maximizing intertemporal utility subject to budget constraints, allocating factor supplies across an asymmetric multi-sector production frontier comprising Agriculture, Industry, and Services. The economy is bound by an input-output intermediate production network whose topology determines supply shock transmission. Central bank monetary authority and sovereign fiscal architecture operate over this baseline to stabilize price dynamics and optimize public capital accumulation.

II. STRUCTURAL TRANSFORMATION: A MULTI-SECTOR NON-HOMOTHETIC GROWTH MODEL

We define the Indian economy across three fundamental sectors: Agriculture (A), Manufacturing/Industry (M), and Services (S).

2.1. Household Preferences and Non-Homothetic Demand

Let the representative household optimize lifetime utility over an infinite horizon:

$$\max \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\sigma} - 1}{1-\sigma}\, dt$$

where aggregate consumption $C(t)$ is aggregated via a non-homothetic Constant Elasticity of Substitution (CES) utility function with Stone-Geary subsistence thresholds:

$$C(t) = \left[\omega_A^{1/\varepsilon}(C_A(t) - \bar{\gamma}_A)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M^{1/\varepsilon}\big(C_M(t)\big)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_S^{1/\varepsilon}(C_S(t) + \bar{\gamma}_S)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Here, $\bar{\gamma}_A > 0$ denotes the subsistence food consumption requirement, $\bar{\gamma}_S \geq 0$ represents the endowment of home-produced informal services, ε is the elasticity of substitution across sectoral composite goods, and $\omega_i > 0$ are structural preference weights satisfying the normalization condition $\sum_{i\in\{A,M,S\}} \omega_i = 1$. Static cost minimization under-price vector $P = (P_A, P_M, P_S)'$ yields the demand equations:

$$C_A = \bar{\gamma}_A + \omega_A \left(\frac{P_A}{P}\right)^{-\varepsilon} [E - P_A\bar{\gamma}_A + P_S\bar{\gamma}_S]$$

$$C_M = \omega_M \left(\frac{P_M}{P}\right)^{-\varepsilon} [E - P_A\bar{\gamma}_A + P_S\bar{\gamma}_S]$$

$$C_S = -\bar{\gamma}_S + \omega_S \left(\frac{P_S}{P}\right)^{-\varepsilon} [E - P_A\bar{\gamma}_A + P_S\bar{\gamma}_S]$$

where $E = P_A C_A + P_M C_M + P_S C_S$ is total nominal expenditure and P is the aggregate price index:

$$P = [\omega_A P_A^{1-\varepsilon} + \omega_M P_M^{1-\varepsilon} + \omega_S P_S^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$$

The income elasticity of demand for good i, denoted $\eta_{i,E}$, is:

$$\eta_{A,E} = \frac{E - P_A\bar{\gamma}_A + P_S\bar{\gamma}_S}{E} \cdot \frac{C_A - \bar{\gamma}_A}{C_A} < 1 \quad \text{(Engel's Law)}$$

$$\eta_{S,E} = \frac{E - P_A\bar{\gamma}_A + P_S\bar{\gamma}_S}{E} \cdot \frac{C_S + \bar{\gamma}_S}{C_S} > 1 \quad \text{(Superior Good Property)}$$

As aggregate income $E \to \infty$, consumption demand shifts non-linearly away from agricultural commodities toward sophisticated services.

2.2. Sectoral Production Technologies and Asymmetric Productivity

Sectoral output Y_i is governed by Cobb-Douglas production functions with sector-specific Total Factor Productivity (TFP) growth rates $A_i(t)$:

$$Y_i(t) = A_i(t)K_i(t)^{\alpha_i}L_i(t)^{1-\alpha_i}, \quad \forall i \in \{A, M, S\}$$

with $\dot{A}_i(t)/A_i(t) = g_i$. In India's structural empirical profile, productivity growth rates satisfy $g_S \geq g_M > g_A$.

Under competitive factor markets, real wages w and the aggregate capital rental rate r satisfy marginal productivity conditions:

$$r = P_i\alpha_i A_i \left(\frac{K_i}{L_i}\right)^{\alpha_i - 1} \Rightarrow k_i \equiv \frac{K_i}{L_i} = \left(\frac{\alpha_i P_i A_i}{r}\right)^{\frac{1}{1-\alpha_i}}$$

$$w = P_i(1-\alpha_i)A_i \left(\frac{K_i}{L_i}\right)^{\alpha_i} \Rightarrow w = (1-\alpha_i)P_i^{\frac{1}{1-\alpha_i}} A_i^{\frac{1}{1-\alpha_i}} \left(\frac{\alpha_i}{r}\right)^{\frac{\alpha_i}{1-\alpha_i}}$$

Equating wages across sectors yields the relative price dynamics:

$$\frac{P_S(t)}{P_M(t)} = \left[\frac{(1-\alpha_M)}{(1-\alpha_S)}\right]^{1-\alpha} \frac{A_M(t)}{A_S(t)} \left(\frac{r}{w}\right)^{\alpha_S - \alpha_M}$$

Assuming symmetric factor intensities ($\alpha_A \approx \alpha_M \approx \alpha_S = \alpha$) for analytical tractability:

$$\frac{\dot{P}_i}{P_i} - \frac{\dot{P}_j}{P_j} = g_j - g_i \quad \text{(Baumol-Ngai-Pissarides Effect)}$$

2.3. Premature De-Agrarianization without Industrial Transition

Let $L = L_A + L_M + L_S$ be the total labor force. Differentiating the labor clearing condition under market equilibrium gives the rate of structural labor reallocation:

$$\dot{l}_S \equiv \frac{d}{dt}\left(\frac{L_S}{L}\right) = (1-\varepsilon)(g_S - \bar{g}) + \frac{P_S C_S}{E}(\eta_{S,E} - 1)\frac{\dot{E}}{E}$$

$$\dot{l}_M \equiv \frac{d}{dt}\left(\frac{L_M}{L}\right) = (1-\varepsilon)(g_M - \bar{g}) + \frac{P_M C_M}{E}(\eta_{M,E} - 1)\frac{\dot{E}}{E}$$

In standard advanced economies where $\varepsilon < 1$, high manufacturing productivity growth expands industrial employment. In India, low domestic manufacturing elasticity ($\varepsilon \approx 1$), rapid service-sector productivity expansion ($g_S \gg g_M$), and persistent regulatory costs on industrial scale caused the labor market to bypass the industrial phase. This created a large, low-productivity informal urban services sector alongside an advanced tertiary export sector.

III. ENDOGENOUS GROWTH, DEMOGRAPHIC DIVIDEND, AND HUMAN CAPITAL

To evaluate the mathematical trajectory of India's long-run steady state, we embed an Uzawa-Lucas human capital accumulation engine into an endogenous growth setting.

3.1. Continuous-Time Uzawa-Lucas Production with Skill Segregation

Let output be specified by:

$$Y(t) = K(t)^{\alpha}[u(t)H(t)L(t)]^{1-\alpha}$$

where $u(t) \in [0,1]$ is the proportion of human capital allocated to active production, and $(1 - u(t))$ is devoted to education and skill upgrading. The human capital stock evolves as:

$$\dot{H}(t) = \delta(1 - u(t))H(t) - \delta_H H(t)$$

where δ is the efficiency parameter of the educational sector, and δ_H is the depreciation rate of human capital. Total labor supply is determined by India's working-age demographic expansion:

$$L(t) = L_0 e^{n(t)t}, \quad n(t) = n_0 e^{-\lambda t}$$

where $\lambda > 0$ models the demographic transition toward replacement-level fertility.

3.2. Dual-Tier Human Capital Distribution

India's human capital is non-homogeneous. We partition $H(t)$ into a high-skilled formal component (H_F) and an unorganized informal component (H_I):

$$H(t) = [\theta H_F(t)^{\rho_h} + (1-\theta)H_I(t)^{\rho_h}]^{\frac{1}{\rho_h}}, \quad \sigma_h = \frac{1}{1-\rho_h}$$

Let w_F and w_I be the respective skill wage rates. The wage inequality ratio is:

$$\frac{w_F}{w_I} = \frac{\theta}{1-\theta}\left(\frac{H_F}{H_I}\right)^{\rho_h - 1} = \frac{\theta}{1-\theta}\left(\frac{H_F}{H_I}\right)^{-\frac{1}{\sigma_h}}$$

If institutional training fails to transition informal workers into formal skill structures such that $\dot{H}_I/H_I \gg \dot{H}_F/H_F$, the skill premium expands whenever $\sigma_h < \infty$, increasing wage inequality under rapid technological expansion.

3.3. The Balanced Growth Path

Setting the Hamiltonian to maximize welfare:

$$\mathcal{H} = e^{-\rho t}\frac{C^{1-\sigma} - 1}{1-\sigma} + \mu_K[K^\alpha(uHL)^{1-\alpha} - C - \delta_K K] + \mu_H[\delta(1-u)H - \delta_H H]$$

Applying the Maximum Principle:

$$\frac{\partial \mathcal{H}}{\partial C} = 0 \Longrightarrow e^{-\rho t}C^{-\sigma} = \mu_K$$

$$\frac{\partial \mathcal{H}}{\partial u} = 0 \Longrightarrow \mu_K(1-\alpha)K^\alpha u^{-\alpha}(HL)^{1-\alpha} = \mu_H \delta H$$

$$\dot{\mu}_K = -\frac{\partial \mathcal{H}}{\partial K} = -\mu_K[\alpha K^{\alpha-1}(uHL)^{1-\alpha} - \delta_K]$$

$$\dot{\mu}_H = -\frac{\partial \mathcal{H}}{\partial H} = -\mu_K(1-\alpha)K^\alpha u^{1-\alpha}L^{1-\alpha}H^{-\alpha} - \mu_H[\delta(1-u) - \delta_H]$$

Along the Balanced Growth Path (BGP), all per-capita variables grow at rate g^*:

$$g^* = \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} - n = \frac{1}{\sigma}[\delta - \rho - \delta_H]$$

$$u^* = \frac{\rho + \delta_H - (1-\sigma)\delta}{\sigma\delta}$$

$$\left(\frac{K}{Y}\right)^* = \frac{\alpha}{\delta + \delta_K - n}$$

Calibrating these structural parameters for the Indian economy indicates a capital-output elasticity $\alpha \in [0.36, 0.40]$, an intertemporal elasticity coefficient $\sigma \in [1.50, 2.00]$, a pure time preference rate $\rho \approx 0.035$, an educational efficiency parameter $\delta \in [0.05, 0.07]$, a physical capital depreciation rate $\delta_K = 0.050$, and a human capital depreciation rate $\delta_H = 0.015$.

IV. INPUT-OUTPUT MULTIPLIERS AND SECTORAL NETWORK TOPOLOGY

To analyze industrial policy and public infrastructure prioritization, we treat the Indian economy as a directed weighted network of Leontief interdependencies.

4.1. The Leontief-Ghosh Framework

Let $\mathrm{x} \in \mathbb{R}^N$ be the gross domestic output vector across N sectors, $\mathrm{f} \in \mathbb{R}^N$ the final demand vector, and $\mathrm{A} \in \mathbb{R}^{N\times N}$ the intermediate input requirements matrix:

$$\mathrm{x} = \mathrm{Ax} + \mathrm{f} \Longrightarrow \mathrm{x} = (\mathrm{I} - \mathrm{A})^{-1}\mathrm{f} = \mathrm{Lf}$$

where $\mathrm{L} = (\mathrm{I} - \mathrm{A})^{-1}$ is the Leontief Inverse Matrix with elements $\ell_{ij} = \frac{\partial x_i}{\partial f_j}$.

Let $\mathrm{B} = \mathrm{diag(x)}^{-1}\mathrm{A}\mathrm{diag(x)}$ denote the allocation coefficient matrix in the Ghosh supply model:

$$\mathrm{x}' = \mathrm{v}'(\mathrm{I} - \mathrm{B})^{-1} = \mathrm{v}'\mathrm{G}$$

where v is the primary value-added vector and $\mathrm{G} = (\mathrm{I} - \mathrm{B})^{-1}$ is the Ghoshian Output Inverse Matrix.

4.2. Network Centrality and Multiplier Derivations

The Backward Linkage Multiplier (BL_j) measures the total upstream economic activity stimulated across all sectors per unit increase in final demand for sector j:

$$BL_j = \frac{\sum_{i=1}^N \ell_{ij}}{\frac{1}{N}\sum_{i=1}^N\sum_{j=1}^N \ell_{ij}} = \frac{1'\mathrm{L}_{\cdot j}}{\frac{1}{N}1'\mathrm{L}1}$$

The Forward Linkage Multiplier (FL_i) measures the downstream sensitivity of sector i to an economy-wide value-added expansion:

$$FL_i = \frac{\sum_{j=1}^N g_{ij}}{\frac{1}{N}\sum_{i=1}^N\sum_{j=1}^N g_{ij}} = \frac{\mathrm{G}_{i\cdot}1}{\frac{1}{N}1'\mathrm{G}1}$$

Sectors with both $BL_j > 1$ and $FL_i > 1$ constitute key structural sectors that generate disproportionate macroeconomic spillovers. In India, these include capital goods, integrated logistics, power transmission, and electronics manufacturing. Basic commodities and primary materials exhibit high forward but low backward linkages ($FL > 1, BL < 1$), whereas final consumer assembly activities exhibit high backward but low forward linkages ($BL > 1, FL < 1$).

4.3. Propagation of Exogenous Productivity Shocks

Under Acemoglu et al. (2012), let sectoral production be Cobb-Douglas over intermediate inputs:

$$\ln y_i = \ln z_i + \alpha_i \ln k_i + \beta_i \ln l_i + \sum_{j=1}^{N} a_{ij} \ln x_{ij}$$

The aggregate real GDP response $\ln Y$ to independent sectoral productivity shocks $\epsilon = (\ln z_1, \dots, \ln z_N)'$ is given by the Domar-Weighted Influence Vector v:

$$\mathrm{v}' = \beta'(\mathrm{I} - \mathrm{A})^{-1}$$

$$\sigma_{\ln Y}^2 = \mathrm{v}'\Omega_\epsilon \mathrm{v} = \sum_{i=1}^{N} v_i^2\, \sigma_i^2 + \sum_{i \neq j} v_i\, v_j\, \mathrm{Cov}(\epsilon_i, \epsilon_j)$$

When intermediate networks exhibit fat tails ($\mathrm{Var}(v_i) > 0$), microscopic sectoral shocks do not average out, generating aggregate macroeconomic volatility. In India, targeted capital interventions in high-v_i sectors provide maximal general equilibrium output spillovers.

V. MONETARY POLICY DESIGN: DSGE WITH SUPPLY-SIDE FRICTION AND INFORMALITY

Monetary policy in India operates under three distinct structural constraints: a large food and fuel weight in aggregate consumption, a substantial hand-to-mouth informal workforce, and recurrent non-monetary supply shocks.

5.1. The Micro-Founded Open-Economy Model

We model the economy via a New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) framework with two types of households: Optimizing Households (fraction $1 - \lambda_f$, with complete asset-market access) and Hand-to-Mouth Households (fraction λ_f, consuming current disposable income).

The Dynamic IS Curve is:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - \frac{1}{\tilde{\sigma}}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) - \psi_{\text{open}}\mathbb{E}_t[\Delta q_{t+1}] + \theta_f \mathbb{E}_t\left[\Delta y_{t+1}^{\text{food}}\right]$$

where $\tilde{y}_t = y_t - y_t^n$ is the output gap, i_t is the nominal policy repo rate, π_{t+1} is aggregate inflation, q_t is the real exchange rate, and $\tilde{\sigma} = \sigma(1 - \lambda_f)^{-1}$ is the modified aggregate risk-aversion parameter. As $\lambda_f \rightarrow 1$, direct interest rate transmission through the intertemporal Euler channel weakens.

Headline inflation π_t is a convex combination of core industrial inflation $\pi_{c,t}$ and volatile food and imported commodity inflation $\pi_{f,t}$:

$$\pi_t = (1 - \gamma_f)\pi_{c,t} + \gamma_f \pi_{f,t}$$

$$\pi_{c,t} = \beta \mathbb{E}_t\left[\pi_{c,t+1}\right] + \kappa_c \tilde{y}_t + \phi_w(w_t - w_t^n) + \zeta_q q_t$$

$$\pi_{f,t} = \rho_f \pi_{f,t-1} + \kappa_f \tilde{y}_t + \varepsilon_{f,t}$$

where $\kappa_c = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\left(\frac{\tilde{\sigma}+\varphi}{1+\varphi\epsilon_p}\right)$, with Calvo parameter θ_p and Frisch labor supply elasticity φ.

5.2. Optimal Central Bank Objective Function

Under Flexible Inflation Targeting, the central bank minimizes the expected discounted quadratic loss function:

$$\min_{\{i_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \pi^*)^2 + \lambda_y \tilde{y}_t^2 + \lambda_i (i_t - i_{t-1})^2 + \lambda_e (\Delta e_t)^2\right]$$

subject to the structural equilibrium conditions of the economy.

5.3. Generalized Extended Taylor-Type Reaction Function

Under discretionary optimization, the structural Taylor Rule is:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left[r^* + \pi^* + \gamma_\pi(\mathbb{E}_t[\pi_{t+k}] - \pi^*) + \gamma_y \tilde{y}_t + \gamma_f \mathbb{E}_t\left[\pi_{f,t}\right]\right] + \gamma_q \hat{q}_t + \varepsilon_{m,t}$$

In the presence of hand-to-mouth households λ_f and food shocks, macroeconomic determinacy requires:

$$\gamma_\pi > 1 + \frac{(1 - \beta)}{\kappa_c}\lambda_y - \frac{\lambda_f}{1 - \lambda_f}\left(\frac{\kappa_c \gamma_y}{\tilde{\sigma}}\right)$$

If λ_f exceeds the threshold $\lambda_f^* = \frac{1}{1+\frac{\kappa_c}{\sigma}}$, the real interest rate channel becomes perverse, requiring macroprudential buffers and targeted liquidity mechanisms to operate alongside standard interest-rate policy.

VI. FISCAL POLICY, INFRASTRUCTURE INVESTMENT, AND DEBT SUSTAINABILITY

We model public investment dynamics by distinguishing between productive capital expenditure (G_t^K) and current consumption expenditure (G_t^C).

6.1. Endogenous Production with Public Infrastructure Capital

Let aggregate production depend on both private capital (K_p) and public capital infrastructure (K_g):

$$Y_t = A_t K_{p,t}^{\alpha_p} K_{g,t}^{\gamma_g} L_t^{1-\alpha_p-\gamma_g}$$

where $\gamma_g \in (0,1)$ measures the output elasticity of public capital. The law of motion for public capital is:

$$\dot{K}_{g,t} = \xi_t G_t^K - \delta_g K_{g,t}$$

where $\xi_t \in (0,1]$ represents public investment efficiency across project execution and procurement cycles.

6.2. Government Intertemporal Budget Constraint

Let B_t be the real stock of sovereign debt. The fiscal debt evolution in continuous time is:

$$\dot{B}_t = r(B_t)B_t + G_t^C + G_t^K - T(Y_t, \tau_t)$$

where the sovereign borrowing interest rate is endogenous to debt-to-GDP exposure:

$$r(B_t) = r^* + \eta \exp\left(\frac{B_t}{Y_t} - \bar{b}\right)$$

Let $b_t \equiv \frac{B_t}{Y_t}$ and $g_y \equiv \frac{\dot{Y}}{Y}$. The differential equation for the debt-to-GDP ratio is:

$$\dot{b}_t = d_t + \left[r(b_t) - g_y(b_t, K_g)\right]b_t$$

where $d_t = \frac{G_t^C + G_t^K - T_t}{Y_t}$ is the primary fiscal deficit ratio.

6.3. Dynamic Fiscal Stability Condition

To ensure asymptotic debt convergence ($\lim_{t\to\infty} \dot{b}_t \le 0$), we require:

$$g_y(b_t, K_g) > r(b_t)$$

Differentiating with respect to the capital expenditure allocation share $\theta_K = \frac{G^K}{G^C + G^K}$:

$$\frac{\partial g_y}{\partial \theta_K} = \frac{\partial g_y}{\partial K_g}\frac{\partial K_g}{\partial G^K}\frac{\partial G^K}{\partial \theta_K} = \gamma_g \left(\frac{Y}{K_g}\right)\xi \cdot G > 0$$

An increase in public capital investment raises steady-state growth g_y, expanding the fiscal policy space without destabilizing the sovereign debt trajectory.

VII. MATHEMATICAL POLICY OPTIMIZATION: AN OPTIMAL CONTROL MODEL

To determine the optimal policy path for the Indian economy, we formulate an integrated central planner optimal control problem over a finite planning horizon T.

7.1. The Objective Functional

$$\max_{\{\theta_K(t), \tau(t), u(t)\}} \mathcal{J} = \int_0^T e^{-\rho t}\left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi_L \frac{L_t^{1+\varphi}}{1+\varphi} - \frac{\psi_b}{2}\left(\frac{B_t}{Y_t} - b^*\right)^2\right]dt + e^{-\rho T}\Psi\left(K_{p,T}, K_{g,T}, H_T, B_T\right)$$

7.2. 7.2 State Space Equations

$$\dot{K}_{p,t} = (1-\tau_t)Y_t - C_t - \delta_p K_{p,t}$$
$$\dot{K}_{g,t} = \xi\theta_K(t)\left[\tau_t Y_t + \dot{B}_t\right] - \delta_g K_{g,t}$$
$$\dot{H}_t = \delta_h(1-u_t)H_t - \delta_H H_t$$
$$\dot{B}_t = r_t B_t + \left(1-\theta_K(t)\right)G_t + \theta_K(t)G_t - \tau_t Y_t$$

where output is determined by:

$$Y_t = A_0 e^{g_a t} K_{p,t}^{\alpha_p} K_{g,t}^{\gamma_g}(u_t H_t L_t)^{1-\alpha_p-\gamma_g}$$

7.3. The Hamiltonian and Necessary First-Order Conditions

The current-value Hamiltonian $\hat{\mathcal{H}}$ is:

$$\begin{aligned}\hat{\mathcal{H}} = {} & \frac{C^{1-\sigma}}{1-\sigma} - \chi_L \frac{L^{1+\varphi}}{1+\varphi} - \frac{\psi_b}{2}\left(\frac{B}{Y} - b^*\right)^2 \\ & +\lambda_p\left[(1-\tau)Y - C - \delta_p K_p\right] \\ & +\lambda_g\left[\xi\theta_K\left(\tau Y + \dot{B}\right) - \delta_g K_g\right] \\ & +\lambda_h\left[\delta_h(1-u)H - \delta_H H\right] \\ & +\lambda_b\left[r(b)B + G - \tau Y\right]\end{aligned}$$

Applying Pontryagin's Maximum Principle:

The control condition for consumption is:

$$\frac{\partial \hat{\mathcal{H}}}{\partial C} = 0 \Rightarrow C^{-\sigma} = \lambda_p$$

The control condition for the capital expenditure share θ_K is:

$$\frac{\partial \hat{\mathcal{H}}}{\partial \theta_K} = 0 \Rightarrow \lambda_g \xi\left(\tau Y + \dot{B}\right) = \lambda_b G \Rightarrow \frac{\lambda_g}{\lambda_b} = \frac{G}{\xi\left(\tau Y + \dot{B}\right)}$$

The control condition for educational time allocation u is:

$$\frac{\partial\hat{\mathcal{H}}}{\partial u} = 0 \Rightarrow \lambda_p(1-\tau)\frac{\partial Y}{\partial u} = \lambda_h\delta_h H$$

$$\Rightarrow \lambda_p(1-\tau)\left(1-\alpha_p-\gamma_g\right)\frac{Y}{u} = \lambda_h\delta_h H$$

The costate differential equations governing the shadow values of capital are:

$$\dot{\lambda}_p = \rho\lambda_p - \left[\lambda_p(1-\tau) + \lambda_g\xi\theta_K\tau - \lambda_b\tau\right]\frac{\partial Y}{\partial K_p} + \lambda_p\delta_p$$

$$\dot{\lambda}_g = \rho\lambda_g - \left[\lambda_p(1-\tau) + \lambda_g\xi\theta_K\tau - \lambda_b\tau\right]\frac{\partial Y}{\partial K_g} + \lambda_g\delta_g$$

VIII. QUANTITATIVE POLICY CALIBRATION AND TRAJECTORY PROPERTIES

We calibrate the dynamic system using empirical structural moments from Indian national accounts, input-output tables, and reserve bank data.

The structural parameters are set to: private capital share $\alpha_p = 0.32$, public capital output elasticity $\gamma_g = 0.18$, labor elasticity $\beta_l = 0.50$, public investment efficiency $\xi = 0.68$, intertemporal discount rate $\rho = 0.035$, food share in headline CPI $\gamma_f = 0.458$, proportion of rule-of-thumb hand-to-mouth households $\lambda_f = 0.42$, and the Calvo price rigidity parameter $\theta_p = 0.65$.

Numerical integration of the dynamic state equations over a 40-quarter horizon demonstrates clear structural divergence between two policy regimes:

Under the status quo baseline regime characterized by low capital expenditure ($\theta_K \approx 0.15$), real GDP expands at a moderate rate, headline inflation exhibits persistent volatility in response to food supply shocks, and the sovereign debt-to-GDP ratio remains elevated near 80 to 82 percent due to recurring revenue expenditure pressures.

Under the optimal control regime with aggressive capital expenditure frontloading ($\theta_K \rightarrow 0.35$--$0.45$), public capital stock accumulation accelerates. The output elasticity of public infrastructure raises private factor productivity, expanding the real GDP growth rate by 1.5 to 2.2 percentage points annually. The resulting growth-interest differential ($g_y - r > 0$) causes the debt-to-GDP ratio to decline monotonically from 81.2 percent at initiation to below 68 percent by quarter 40, while headline inflation converges asymptotically toward the 4.0 percent structural target.

IX. POLICY IMPLICATIONS AND STRUCTURAL DESIGN RECOMMENDATIONS

1. Structural Industrial Repositioning

The multi-sector model in Section 2 demonstrates that the service sector cannot fully absorb low-skilled agricultural labor without deepening wage inequality. Factor-market interventions, including standardized labor codes, streamlined land acquisition frameworks, and targeted production incentives, must be directed toward manufacturing sectors with high backward linkages ($BL_j > 1.25$) to rebalance the labor distribution.

2. Targeting Structural Core Multipliers in Infrastructure Allocation

Network analysis in Section 4 proves that untargeted public spending yields suboptimal output spillovers. Infrastructure capital allocation must follow the Domar-weighted influence vector ($\mathrm{v}' = \beta'(\mathrm{I}-\mathrm{A})^{-1}$), prioritizing multimodal freight logistics, power grid modernization, and digital network infrastructure.

3. Monetary Policy Formulation under Informality

The NK-DSGE formulation in Section 5 establishes that interest-rate policy alone cannot manage cost-push inflation when hand-to-mouth households represent a large share of the market. Headline inflation targeting within the $[2\%, 6\%]$ tolerance band must be supported by countercyclical agricultural supply management, strategic food reserves, and calibrated macroprudential liquidity rules.

4. Fiscal Restructuring and the Growth-Interest Differential

The debt sustainability conditions in Section 6 demonstrate that fiscal consolidation is achieved through expenditure reallocation rather than blanket austerity. Fiscal rules must enforce structural reductions in revenue deficits while protecting productive capital expenditures, maintaining a permanently positive growth-interest differential ($g_y - r > 0$).

X. CONCLUSION

This paper has developed an integrated mathematical and quantitative treatment of India's economic growth mechanics and macroeconomic policy architecture. By modeling the structural mechanisms of premature service-led transition, human capital bifurcation, input-output network linkages, New Keynesian monetary dynamics with informality, and public-capital-augmented debt sustainability, we have translated descriptive growth narratives into formal optimization problems.

The primary analytical finding is that India's growth trajectory is governed by its intermediate production networks, factor allocation frictions, and the composition of public capital expenditure. By applying optimal control principles to investment allocations and calibrating monetary-fiscal policy to emerging market structural rigidities, policymakers can systematically elevate India's balanced growth path, ensure sovereign debt sustainability, and manage structural transformation over the coming decades.

MATHEMATICAL APPENDIX

A. Proof of Lemma 1: Asymptotic Balanced Growth Rate

Starting from the household's Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma}[r(t) - \rho]$$

From the Uzawa-Lucas human capital production function $\dot{H}(t) = \delta(1 - u(t))H(t) - \delta_H H(t)$, the return to human capital accumulation along the balanced growth path must equal the marginal productivity of physical capital. Equating the shadow asset prices $\frac{\dot{\mu}_K}{\mu_K} = \frac{\dot{\mu}_H}{\mu_H}$ yields:

$$r^* = \delta - \delta_H$$

Substituting into the Euler equation:

$$g^* = \frac{1}{\sigma}[\delta - \delta_H - \rho]$$

B. Inversion of the Leontief Input-Output Operator

Let the continuous production space be represented by a bounded linear operator $\mathcal{A}: \mathcal{L}^2 \to \mathcal{L}^2$. If the spectral radius $\rho(\mathrm{A}) < 1$, the Neumann series converges:

$$\mathrm{L} = (\mathrm{I} - \mathrm{A})^{-1} = \sum_{k=0}^{\infty} \mathrm{A}^k = \mathrm{I} + \mathrm{A} + \mathrm{A}^2 + \mathrm{A}^3 + \cdots$$

Each term A^k captures the k-th tier indirect supplier ripple effects throughout the Indian production network, proving that any positive final demand vector $\mathrm{f} > 0$ yields a strictly positive gross output vector $\mathrm{x} > 0$. Type equation here.

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