

Using Mathematics to Model Poverty and Mental Illness of Alcohol Drinkers Among Youths in the Society. A Case Study of Khana Local Government Area of Rivers State

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Abstract—Alcohol drinking is a menace in our society nowadays. It is a major concern in that it is the cause of social, economic and health related problems in the society. This calls for the study to analyse the effects of the aftermaths of this social activity. It then moved the writer to model the rate of drinking and the effects or consequences thereof. The mathematics model looked at how the intake caused poverty and mental disorder. This study focused on the dynamics of poverty and mental disorder (madness) among the youths in the society and the behaviour of the pattern of change of the mathematics model was evaluated qualitatively. For effective study the system has three stable points, mental free point, alcohol free point and the confined point or region. The existence of these points was explained using the following; no condition, a condition and conditions and two reproductive numbers were identified. The current model system used demonstrated theorems on local fixed points stability analysis. In the study, extensive numerical simulation was carried out using the MATLAB ODE solver Ode45. It was discovered from the analysis that the endemic equilibrium was locally asymptotically stable and both strains alcohol addiction, poverty and the mental disorder (madness) in youths persist, thus continues to exist among youths in the community. This revealed the validity of qualitative analysis of the dynamic system. Thus, the researcher advised the general public to desist from excessive consumption of alcohol and urged the government agencies to regulate the indiscriminate selling and consumption of alcohol in the society.

Index Terms—mathematics, addiction, alcohol drinking, stable points, mental disorder, numerical simulation, stability analysis, death.

I. INTRODUCTION

Drinking of alcohol is one of the social vices found among youths of this generation. This activity nowadays is associated with several personal, socio-economic and health problems in the society. Not only that drinking excessive alcohol create personal and societal costs which actually affects educational and vocational performance, family and social disruption and has a negative contact with the societal criminal justice system negative due to the effects from co-drinkers. Drinking alcohol by youths in the society has drastically affected costs, such as, the cost of low productivity, law enforcement costs and the cost of direct health care. It is seen and confirmed nowadays people especially youths involved in consuming alcohol while mixing it with Crystal Meth popularly called ICE. This mixture (Alcohol + Ice) caused a lot of damage to the brain which always result to mental disorder (Madness) and even Death if not checked. Excessive drinking of alcohol which is the abuse of alcohol consumption contributed to a variety of health related problems such as kidney failure, liver cirrhosis, heart cancer, cardiovascular disease, accident, body parts deformation, mental abnormality and for the most part sudden death.

According to World Medical Association (2016), on statement of reducing the Global impact of alcohol on health and society, that the excessive drinking of alcohol is a major causal and contributing factor in injuries and sudden or premature deaths due to motor accidents, sudden falls, frequent suicides among youths, fire outbreaks, drowning and violence. This is because alcohol drinking has been one of the most

commonly used chemical substance and stimulant for intoxication by boys and girls in the society. This is experienced during social activities like sports, games, sexual activities, several reception ceremonies and functions in the society. Alcohol drinking which is the result of alcohol addiction is a major cause of frequent death for millions youth yearly according to the World Status Report on Alcohol and Health from the World Health Organisation (WHO), (2017). In the report, Alcohol drinking kills more than HIV and AIDS, Tuberculosis (TB), violence and other diseases.

Previous studies reveal the used of Mathematics model to investigates alcohol drinking with the influence of private and public addiction treatment centers. Khajji et al., (2020) investigated and discovered that the basic reproduction number is very important to determine the dynamic of behavior of the system. Parameters were identified to have high importance on the reproduction number.

Imtiaz et al. (2017) discovered in their study that alcohol drinking is a risk factor for frequent accident and some ill-health problems. In their publication on alcohol consumption as a risk factor for TB disease, alcohol drinking is responsible for 17% of TB and 35% increase risk for developing active TB cases. According to Samuel, et al. (2021), alcohol consumption is seen in the area of social interactions such as Drinking Bars, has caused or triggered transmission or outbreak of TB sickness in various institutionalized settings which include Prison and among homeless population. TB treatment failure and poor relieve from TB sickness are caused by excessive alcohol drinking. Also, loss to follow, high mortality for drug-susceptible and drug resistant TB result from the excessive intake of alcohol amongst youths.

Diseases like sexually transmitted disease (STD) and HIV/AIDS infections were studied to be associated with alcohol drinking. Cork R.L. and Clerk D.B. (2005) and Djomegni, et al. (2020) confirmed in their studies that there is high transmission dynamics of STDs and HIV/AIDS in the society due to high intake of alcohol among youths.

Carcy R.B. (1995), studied how alcohol consumption has negative impacts on work productivity. According to the reports from the study, the outcome of the alcohol intake affects person's performance at work thus caused sickness or injury that keeps them off work temporally daily. This makes it difficult for the alcohol drinkers to concentrate and focus on their jobs

thereby affects the work outputs. It is seen in the society that poverty is a process where by an individual fail to reach or attain a minimum level of wellbeing and it is a socio-economic problem facing mankind.

Some of the infectious disease like HIV/AIDS according to Fisher, et al. (2007) can result to death due to the frequent drinking of alcohol. The excessive intake of alcohol can affect the immune system thereby causing chronic ill-health that can lead to death. Also, Jordan G. (2007), did a study and discovered that poverty as a results of absence from job can lead to poor nutrition or improper diet. This can result to malnutrition which can cause heavy sickness and leads to death thereafter.

The researcher is motivated from two directions;

- (i) To study the effects of alcohol drinking using a mathematical model which may help to predict trends in alcohol related problems in the society.
- (ii) To investigate that the application of mathematics in the effects of alcohol drinking can improve knowledge in both fields.

Previous studies such as Chinnaduraia and Athithanb, (2020) proposed and evaluated a non-linear system of Differential Equation. The study modelled poverty dynamics with effects of alcohol consumption. Also, Tesema et al. dynamics with the effects of alcohol consumption. They classified the entire population into four (4) distinct compartments such as, the Susceptible population (N), the Poverty population (P), the Alcohol consumption population (A) and the Recovery population (R). So in this study, there is need to improve on the previous development. There is an addition of one compartment to make it five distinct compartments. These are the Susceptible Population (S), the alcohol Drinkers Population, the Poverty Population (P), the Madness Population (M) and the Recovery Population (R).

Background of the Study: It is discovered nowadays in our culture and society that the psychoactive liquid substance called ethanol popularly known as alcohol is widely consumed or drank. According to Rehm et al. (2013), its consumption is highly associated with socio-economic health implication. As verified by Koob and Le Moal, (2006), the effect of drinking alcohol happened in different ways. Some depend on amount, frequency, drinking pattern, age, gender and

even genetic influence. In their study, they investigated and concluded that alcohol consumption has adverse effect on the Liver function, mental health and central nervous system.

Alcohol actually is seen as a small water-soluble molecule or liquid which is rapidly absorbed directly from the gastrointestinal tract of an individual. Its absorption and distribution was studied by Lieber, (1997). The study showed that 20% of alcohol is absorbed in the stomach and 80% of it is absorbed in the small intestine. Factors such as presence of food, delay gastric emptying and reduced peak blood alcohol concentration (BAC) influencers of the rate at which the liquid alcohol is usually absorbed in the body.

Based on the metabolism of alcohol, Zakhari, (2006) discovered that the movement of alcohol (metabolism) occurs in the liver and acetaldehyde which is highly reactive toxic metabolism which is responsible for tissue damage, hangover symptoms and carcinogenicity.

Koob and Le Moal, (2006), also studied and discovered that alcohol drinking affects endogenous Opioid system. It stimulates endorphin and releases enhancing euphoria and pleasure and thereafter promotes dependence. Excessive consumption of alcohol leads to neuro-adaptive changes, such as receptor downregulation and altered neurotransmitter synthesis, which underline tolerance, withdrawal and cravings.

Alcohol drinking also affects immune system. Alcohol impairs innate and adaptive immunity, affecting macrophage and T-cell function, increasing susceptibility to infections. Szabo and Mandrekar, (2009), study shows that alcohol when taken enhances GABAergic inhibition and inhibits glutamatergic excitation, leading to sedation, impaired judgement and motor dysfunction. Alcohol intake also increases dopaminergic activity in the brain's reward pathway, contributing to addictive and leading to mental disorder, Koob and Le Moal, (2006).

Taylor et al. (2010) studied and found out that alcohol drinking has something to do with motor and cognitive impairment. According to the research, alcohol intake impairs coordination, reaction time and executive function, which results in an increased risk of accident, falls and unsafe behaviors.

Alcohol drinking also has behavioural and emotional effects. According to Steele and Joseph, (1990),

alcohol intake affects emotional regulation and social behavior. In this, an individual experiences disinhibition, aggressions and emotional lability.

Objectives of the Study: The main Objective of the study is to understand the effects of dynamics of drinking alcohol addiction on poverty and madness among youths in the society.

The Mathematical model is formulated in the subsequent section and performs the model analysis, with the numerical simulation stated, results and discussion are presented and finally conclusion of the study.

II. FORMULATION OF MATHEMATICAL MODEL

For effective study the researcher considered a total population denoted by $N(t)$. This total population is then divided into subgroups or classes. This includes; Susceptible population (S), Alcohol population (A), Poverty population (P), Madness population (M) and Recovered population (R). Putting all the subgroups of the total population, we have,

$$N(t) = S(t) + A(t) + P(t) + M(t) + R(t)$$

Some assumptions were made in the case of developing the mathematical model. These assumptions were;

- . The susceptible population increases with constant recruitment rate ω
- . Susceptible people join group A and P with contact rate α and β respectively
- . Alcohol drinkers progress to group P, M and R with poverty rate θ due to alcohol, madness rate γ due to addicted alcohol intake and recovery rate ϕ due to education, support, therapy and counselling.
- . Poor people join group M and R with rates μ and $\emptyset$ respectively.
- . The (abnormal) mad people join group S with rate σ .
- . All groups of people suffer with natural death rate δ .
- . The death of people in group P happens due to high thinking with rate ρ . There is no death due to madness and all parameters in the model are nonnegative.

The description of state variable and parameters of the system (Equation 1) are given in Table 1 below. The flow chart of the mathematical model is also given in Figure 1.

Based on the assumptions and the flow chart, the governing system of differential equations describing the dynamics is;

$$\frac{dS}{dt} = \omega - \alpha SA - \beta SP + \sigma M - \partial S \quad 1$$

With initial conditions $S(0)=S_0 > 0$, $A(0) = 0$, $P(0) = P_0 > 0$, $M(0) = M_0 \geq 0$ and $R(0) = R_0 \geq 0$.

III. MATHEMATICAL MODEL FORMULATION

In this section, we will consider the existence and uniqueness of the model, its boundedness, the equilibrium points and the reproductive number exhibited by the model, the endemic point and the stability of the analysis.

The consideration of the existence uniqueness of the model is of priority. This is because the reliability of the model sometimes is reduced, in that some solutions

make ambiguous predictions. Note also that a nonnegative quantity of the population indicates positivity of that population. Also, boundedness of the solution of the model used help to clear the fact that the natural population and biological process are limited. That means there is no infinite alcohol addicted population.

Looking at the posed of the Equation 1, the right hand side of the equation is continuous and indicates continuous partial derivatives of all orders in a domain D of the positive octant $R_+^5 = \{(S, A, P, M, R): S > 0, A, P, M, R \geq 0\}$, thus making the solution in the nonnegative octant bounded. Hence, there is existence and uniqueness of the solution of the model system with nonnegative initial condition.

Table 1: Description of state variables and parameters

Variable/parameter	Description of the variables and parameters
S	Size of susceptible population at time t
A	Size of alcohol drinkers at time t
P	Size of poor population at time t
M	Size of mentally ill population at time t
R	Size of recovered population at time t
α	Contact rate of susceptible and alcohol drinkers people
β	Contact rate of susceptible and poor people
θ	Poverty rate due to alcohol intake
φ	Recovery rate of mentally ill people because of treatment
γ	Madness rate due to excess alcohol consumption
$\emptyset$	Madness rate because of chronic poverty
μ	Poverty recovery rate due to empowerment
σ	Rate at which madness wanes off due to treatment, therapy and counselling
ω	Recruitment rate
∂	Natural death rate from all compartment
ρ	Death rate due to high poverty

Re-interpretation of the model; assume S recruited at rate ω (birth/migration), die at rate ∂ , can become alcohol users (A) by contact with A at rate α , can become poor (P) by contact with P at rate β . Alcohol users can become poor at rate θ due to poverty from alcohol, or mentally ill at rate γ due to alcohol consumption, poor can become mentally ill at rate $\emptyset$ due to chronic poverty. Mentally ill can recover to R, at rate σ . Recovered people can maybe go back to S at some rate? Not specified. Poor can recover at rate μ from poverty (likely go back to S). Also, death due to

poverty at rate ρ from P, and natural death at rate ∂ from any compartment.

Considering these assumed variables and parameters;

$S \rightarrow A$ at rate αSA

$S \rightarrow P$ at rate βSP due to economic circumstances

$A \rightarrow P$ at rate θA

$A \rightarrow M$ at rate γA (alcohol causing mental illness)

$P \rightarrow M$ at rate $\emptyset P$ (chronic poverty leading to madness)

$A \rightarrow R$ at rate φA (treatment for alcohol abuse)

$M \rightarrow R$ at rate σM (treatment for mental illness)

$P \rightarrow S$ at rate μP (poverty recovery)

Death: natural ∂ from all compartment; death due to poverty ρP from P
Recruitment ω into S.

Positivity

Looking at the positivity of the model, an equation from equation 1 and the initial conditions, then the solution of the system of equation 1 is positive for all $t \geq 0$.

Building the differential equations with the variables and parameters, it yields

$$\frac{dS}{dt} = \omega - \alpha SA - \beta SP - \partial S + \sigma M \quad 1$$

$$\frac{dA}{dt} = \alpha SA - \theta A - \varphi A - \gamma A - \partial A \quad 2$$

$$\frac{dP}{dt} = \beta SP + \theta A - \emptyset P - \mu P - \rho P \quad 3$$

$$\frac{dM}{dt} = \gamma A + \emptyset P - \sigma M - \partial M \quad 4$$

$$\frac{dR}{dt} = \varphi A + \sigma M - \partial R \quad 5$$

Solving the ODE using method of separation of variables:

From the equation $\frac{dA}{dt} = \alpha SA - \theta A - \varphi A - \gamma A - \partial A$

Let $k = \beta S - (\alpha + \theta + \varphi + \gamma)$ then,

$\frac{dA}{dt} = kA$, separating the variables we have,

$\frac{dA}{A} = Kdt$, integrating both sides:

$\ln A = kt + C$, where C is the integration constant.

$A = e^{kt+C} = e^C e^{kt}$,

Let $e^C = A_0$ (a positive constant, but the sign will be handled by the initial conditions). Thus,

$A(t) = A_0 e^{kt} = A_0 e^{\{\beta S - (\alpha + \theta + \varphi + \gamma)\}t}$

The solution for Equation 1 above is

$S(t) = \frac{\omega}{\partial} (1 - e^{-\partial t})$, $A(t) = 0$

The same process is applicable to other parameters.

Boundedness of the Model

The solution of the system of equation 1 which contain $S(t)$, $A(t)$, $P(t)$, $M(t)$, and $R(t)$ where a positive invariant region

$\forall = \{(S(t), A(t), P(t), M(t), R(t)) \in R^5 : 0 < N(t) \leq \{N(t), \frac{\omega}{\partial}\}\}$

This is shown in the flow chart of the model below.

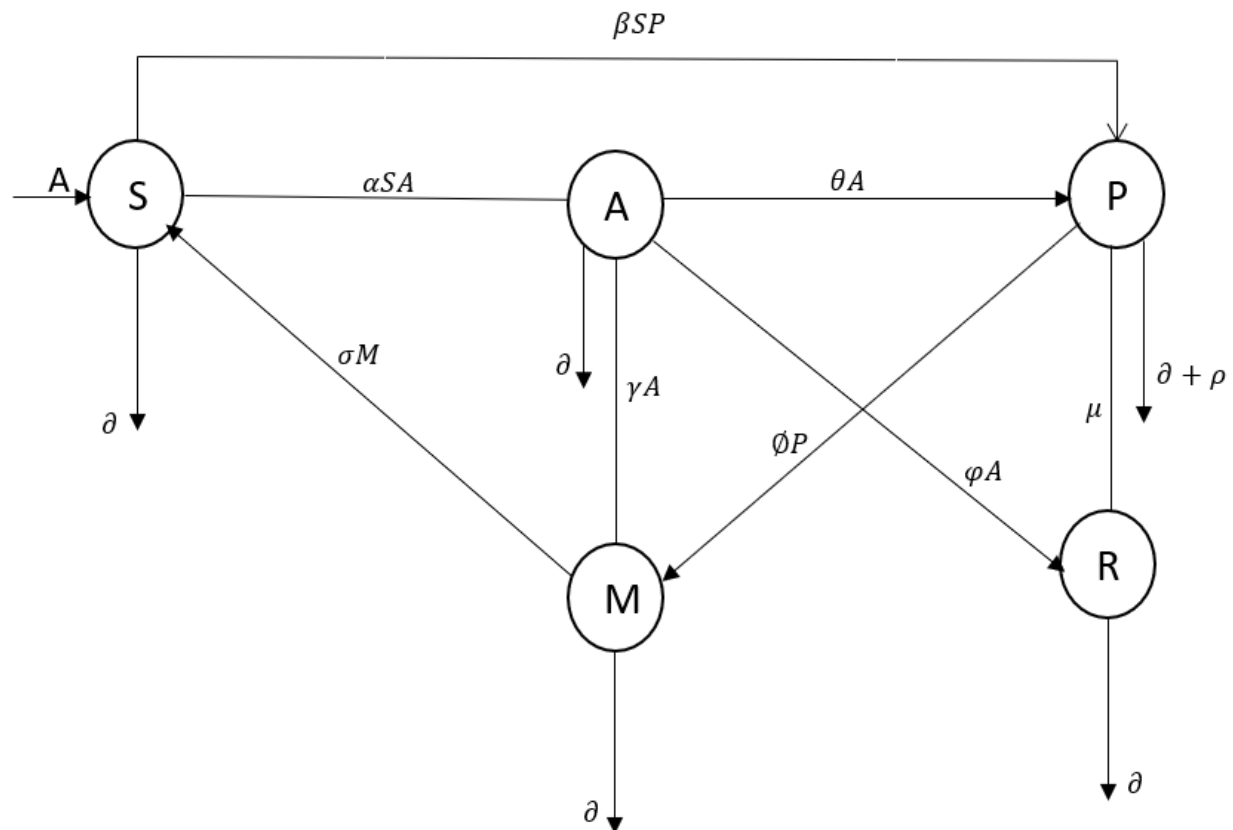


FIGURE 1: Flow chart of the Mathematical Model

Assume the total population is

$$N(t) = S(t) + A(t) + P(t) + M(t) + R(t)$$

Then,

$$\frac{dN}{dt} = \omega - \partial N - \rho P \leq \omega - \partial N \quad 6$$

Hence, solving the inequality in equation 6, we have

$$N(t) \leq \frac{\omega}{\partial} + \left(\frac{\partial N_0}{\partial} \right) e^{-\partial t} \quad 7a$$

$$\psi = \{S(t), A(t), P(t), M(t), R(t) \in R_+^5: 0 < N(t) \leq \max \left(N_0, \frac{\omega}{\partial} \right)\} \quad 7b$$

Thus, is the proof the boundedness of the region in equation 1.

Equilibrium Points and Reproduction Number:

The three (3) equilibrium points stated below are exhibited in the mathematical model of equation 1. These are ; the Alcohol-free equilibrium (E_A), the poverty-free equilibrium point (E_P) and the mentally-ill-free equilibrium point (E_m)

The Alcohol and the poverty-free equilibrium points: These is a stability-state solution where there is no poverty; $P = 0$ and Alcohol addiction ($A = 0$)

$$F = \begin{bmatrix} \alpha \frac{\omega}{\partial} & 0 & 0 \\ 0 & \beta \frac{\omega}{\partial} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} \theta + \phi + \partial + \gamma & 0 & 0 \\ -\theta & \partial + \phi + \rho + \mu & 0 \\ -\gamma & -\phi & \sigma + \partial \end{bmatrix}$$

And hence,

$$FV^{-1} = \begin{bmatrix} \frac{\alpha\omega}{\partial C_1} & 0 & 0 \\ \frac{\beta\theta\omega}{C_1 C_2} & \frac{\omega\beta}{\partial C_2} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ the eigenvalues of } FV^{-1} \text{ are}$$

$$\lambda_1 = \frac{\alpha\omega}{(\theta + \phi + \partial + \gamma)\partial}, \lambda_2 = \frac{\beta\omega}{(\partial + \phi + \rho + \mu)\partial}, \lambda_3 = 0 \quad 9$$

Let,

$$R_{01} = \frac{\alpha\omega}{(\theta + \phi + \partial + \gamma)\partial} \quad \text{and} \quad R_{02} = \frac{\beta\omega}{(\partial + \phi + \rho + \mu)\partial}$$

Thus, the basic reproduction number R_0 is given by $R_0 = \max(R_{01}, R_{02})$ where R_{01} represent the reproduction number related to alcohol addiction, while R_{02} correspondent to the reproduction number associated with the poverty rate.

It is noted that the basic reproduction number R_0 depend on the contact rate and is independent of the madness waning-off rate σ .

$$F = \begin{pmatrix} \alpha S^* & 0 & 0 \\ 0 & \beta S^* & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \theta + \phi + \partial + \gamma & 0 & 0 \\ -\theta & \partial + \phi + \rho + \mu & 0 \\ -\gamma & -\phi & \sigma + \partial \end{pmatrix}$$

Where, $C_1 = (\beta + \phi + \mu + \gamma)$, $C_2 = (\partial + \phi + \rho + \mu)$ and $C_3 = (\partial + \sigma)$. Since this is a lower triangular

Where $N(0)=N_0$ is the initial population. In equation 7a, the right-right side expression is decreasing if $N_0 > \frac{\omega}{\partial}$ and increasing if $N_0 < \frac{\omega}{\partial}$. Thus,

$0 < N(0) \leq \max \left\{ N_0, \frac{\omega}{\partial} \right\}$. Therefore, the solution of the system of equation 1 is bounded in the region:

Thus, the poverty-free equilibrium point E_P is

$$E_P = \left\{ (S^0, A^0, P^0, M^0, R^0) = \left(\frac{\omega}{\partial}, 0, 0, 0, 0 \right) \right\} \quad 8$$

The Reproduction Number (R_0). R_0 is the basic reproduction number that measure the average of secondary new excess alcohol drinkers and secondary poverty caused by single excess alcohol drinkers and a single poor people in an entirely susceptible population during the mean addicted and poverty period in a population. It measures the spread of alcohol addiction and the poverty spread.

Using the generation matrix FV^{-1} , it can be shown thus;

Alcohol-free Equilibrium point (E_A). This is the steady-state solution where there is no alcohol drinkers that is no alcohol addicted persons in the population.

It is given as

$$S^* = \frac{\partial + \phi + \rho + \mu}{\beta} = \frac{S^*}{R_{02}} \quad 10$$

Thus, E_A exist if $R_{02} > 1$

The Basic Reproduction Number (R_{02}). This number measures the average number of secondary of new excess alcohol drinkers which are caused by a single excess alcohol drinker in an entire susceptible population the mean addicted period in a population. This R_{0A} is evaluated using the next generation matrix method.

Thus, from the model of equation 1, we have,

$E_A = (S^*, A^*, P^*, M^*, R^*)$, for the generation matrix, we have,

matrix, its eigenvalues are the diagonal elements and hence,

$$\lambda_1 = \frac{\alpha S^*}{\theta + \varphi + \partial + \gamma}, \lambda_2 = \frac{\beta S^*}{\partial + \varphi + \rho + \mu} = 1, \lambda_3 = 0 \quad 11$$

Thus, R_{0A} is the spectral radius of the dominant eigenvalues of next generation matrix FV^{-1}

$$R_{0A} = \max \left\{ \frac{R_{01}}{R_{02}}, 1 \right\} \quad 12$$

Therefore, the basic reproduction number of alcohol addiction is

$$R_{0A} = \max \left\{ \frac{R_{01}}{R_{02}}, 1 \right\}$$

Mentally ill-Free Equilibrium Point (E_m). This occurs when poverty and alcohol addiction persist in the society. Thus, the Mentally ill -Free Equilibrium Point is given as;

$$E_m = (S^*, A^*, P^*, M^*, R^*) \quad 13$$

We need to consider the Alcohol-Free Equilibrium Point E_A and the Poverty-Free Equilibrium Point E_P . For local stability Alcohol-free equilibrium point E_A which is locally asymptotically stable if $R_0 < 1$. We use the Jacobian matrix of the model in equation 1.

$$J(E_P) = \begin{bmatrix} -\alpha A^* - \beta P^* & -\alpha S^* & -\beta S^* & \sigma & 0 \\ \alpha A^* & \alpha S^* - C_1 & 0 & 0 & 0 \\ \beta P^* & \theta & \beta S^* - C_2 & 0 & 0 \\ 0 & \gamma & \emptyset & -C_3 & 0 \\ 0 & \varphi & \mu & 0 & -\partial \end{bmatrix}$$

The first two eigenvalues are

$$\lambda_1 = -\partial \text{ and } \lambda_2 = \alpha S^* - C_1 = C_1 \left(\frac{R_{01}}{R_{02}} - 1 \right) = C_1 \left[\left(\frac{R_{01} - R_{02}}{R_{02}} \right) \right]$$

Hence, $\lambda_2 < 0$ if $R_{01} < R_{02}$. The values of λ_3, λ_4 , and λ_5 are as follows;

$$J_1 = \begin{bmatrix} -(\beta P^* + \partial) & -C_2 & \sigma \\ \beta P^* & 0 & 0 \\ 0 & \emptyset & -C_3 \end{bmatrix} = \begin{bmatrix} V_1 & V_2 \\ V_3 & V_4 \end{bmatrix}$$

$$\text{Where } V_1 = \begin{bmatrix} -(\beta P^*) & -C_2 \\ \beta P^* & 0 \end{bmatrix}, V_2 = \begin{bmatrix} \partial \\ 0 \end{bmatrix}, V_3 = [0 \quad \emptyset] \text{ and } V_4 = (-C_3)$$

The characteristics equation of the 2 x 2 matrix J_1 is

$$\det(J_1 - \lambda I_2) = \begin{bmatrix} V_1 - \lambda I_2 & V_2 \\ V_3 & V_4 - \lambda \end{bmatrix} = 0, \text{ this implies}$$

$$\det(V_1 - \lambda I_3) \det\{(V_4 - \lambda_5 I_1) - V_3(V_4 - \lambda_5 I_1)^{-1} V_2\} = 0$$

Here, note that,

$$V_3((V_4 - \lambda_5 I_1)^{-1}) V_2 = (0 \quad \emptyset) \begin{bmatrix} -\sigma \\ C_3 + \lambda_5 \\ 0 \end{bmatrix} = 0 \quad 14$$

which leads

$$(\det(V_1 - \lambda I_2)) (\det(C_4 - \lambda I_1)) = 0 \quad 15$$

Thus, $\det(V_1 - \lambda I_2) = 0$ or $\det(V_4 - \lambda_1) = 0$

Hence, $\det(C_4 - \lambda I_1) = 0$ implies $\lambda = -(\sigma + \partial)$. Then,

$\lambda_3 = -C_3 = -(\partial + \sigma)$, which is real and negative. Hence, V_1 have negative real part if,

$\det(V_1) > 0$ and trace (V_1) < 0 thus,

$$\det(V_1) = \frac{C_2 \beta S^0 (R_{02} - 1)(\partial + \sigma)}{R_{02}(\partial C_2 + \sigma(\partial + \rho + \mu))} > 0 \text{ if } R_{02} > 1$$

$$J(E_A) = \begin{bmatrix} -\partial & -\alpha S^0 & -\beta S^0 & \sigma & 0 \\ 0 & \alpha S^0 C_1 & 0 & 0 & 0 \\ 0 & \theta & \beta S^0 - C_2 & 0 & 0 \\ 0 & \gamma & \emptyset & -C_3 & 0 \\ 0 & \varphi & \mu & 0 & -\partial \end{bmatrix}$$

After some manipulations, the eigenvalues of the matrix are;

$$\lambda_1 = \lambda_2 = \partial, \lambda_3 = -k_3 = -(\sigma + \partial)$$

$$\lambda_4 = \alpha \frac{\omega}{\partial} - (\partial + \theta + \gamma + \varphi) = (\partial + \theta + \gamma + \varphi)(R_0 - 1)$$

Thus, λ_4 and λ_5 are negative if $R_{01} < 1$ and $R_{02} < 1$ respectively.

Hence, there is need for the poverty-free equilibrium point E_P which is locally asymptotically stable if $R_0 < 1$.

The Poverty-Free Equilibrium Point E_P

Therefore, the alcohol-free equilibrium point (E_A) is locally asymptotically stable if $R_{02} > 1$ and $R_{02} > R_{01}$
 The Mentally ill-Free Equilibrium Point (E_M) is locally asymptotically stable if $R_0 > 0$ and $R_{OA} > 1$,
 Also, consider the Jacobian matrix of the system in equation 1,

$$J(E_M) = \begin{bmatrix} -\alpha A^{**} - \partial & -C_1 & -\beta S^{**} & \sigma & 0 \\ \alpha A^{**} & 0 & 0 & 0 & 0 \\ \beta P^{**} & \theta & \beta S^{**} - C_2 & 0 & \partial \\ 0 & \gamma & \emptyset & -(\sigma + \partial) & 0 \\ 0 & \varphi & \mu & 0 & -\partial \end{bmatrix} \quad 16$$

One can easily observe that one of the eigenvalues of the Jacobian matrix is $\lambda_1 = -\partial$ which is real and negative. The remaining eigenvalues are obtained from the matrix also.

$$Q(E_M) = \begin{bmatrix} -(\alpha A^{**} + \beta P^{**} + \partial) & -S^{**} & -\beta S^{**} & \sigma \\ \alpha A^{**} & 0 & 0 & 0 \\ \beta P^{**} & \theta & (-\beta S^{**} - C_2) & 0 \\ 0 & \gamma & \emptyset & -C_2 \end{bmatrix}$$

Then, the matrix $Q(E_M)$ in equation 16 is $J(E_M) = \begin{bmatrix} V_1 & V_2 \\ V_3 & V_4 \end{bmatrix}$, where,

$$V_1 = \begin{bmatrix} -(\alpha A^{**} + \beta P^{**} + \partial) & -\alpha S^{**} & -\beta S^{**} \\ \alpha A^{**} & 0 & 0 \\ \beta P^{**} & \theta & \beta S^{**} - C_2 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} \sigma \\ 0 \\ 0 \end{bmatrix}, V_3 = [0 \quad \gamma \quad \emptyset] \text{ and } V_4 = (-C_3)$$

The polynomial characteristics equation of the two matrices is

$$\det(V_1 - \lambda I_3) \det(V_4 - \lambda I_1) - V_3(V_4 - \lambda I_1)^{-1}V_2 = 0 \quad 17$$

But,

$$V_3(V_4 - \lambda I_1)^{-1}V_2 = (0 \quad \gamma \quad \emptyset) \begin{bmatrix} \frac{-\sigma}{C_3 + \lambda} \\ 0 \\ 0 \end{bmatrix} = 0 \quad 18$$

Hence, $\det(V_1 - \lambda I_3) \det(V_4 - \lambda I_1)$

From the second equation of 16, we have $\lambda_2 = -C_3 = -(\partial + \sigma)$ which is real and negative.

Using Routh-Hurwitz criteria to determine the sign of the remaining eigenvalues as $a_1 > 0, a_2 > 0, a_3 > 0, a_1 a_2 > a_3$

When $a_1 = -\text{trace}(V_1)$, a_2 represent the sum of the 2 x 2 principal minors of V_1 and $a_3 = -\det(V_1)$, from matrix V_1

$$a_1 = ((\alpha A^{**} + \beta P^{**} + \partial) + (\rho + \emptyset + \partial + \mu)) \left(1 - \frac{R_{02}}{R_{01}}\right) > 0$$

Provided that $R_{01} > 1$ and $R_{02} < R_{01}$, In addition

$$a_2 = \left[(\alpha A^{**} + \beta P^{**} + \partial)(\rho + \emptyset + \partial + \mu) \left(1 - \frac{R_{02}}{R_{01}}\right) + \beta^2 S^{**} P^{**} \right] > 0$$

Whenever $R_{01} > 1$ and $R_{01} < R_{02}$. Furthermore,

$$a_3 = -\det(V_1) = \left(\alpha A^{**} (C_1 C_2 \left(1 - \frac{R_{02}}{R_{01}}\right)) + \theta \beta S^{**} \right) > 0$$

If $R_{01} > 1$ and $R_{01} > R_{02}$.

Finally, if we multiply the expression for a_1 and a_2 , it is straightforward to show that the condition $a_1 a_2 - a_3 > 0$ is satisfied when

$R_{01} > 1$ and $R_{01} > R_{02}$. Thus, by the Routh-Hurwitz criteria, the mentally ill-free equilibrium point (E_m) is locally asymptotically stable if $R_0 > 1$ and $R_{OA} > 1$

IV. NUMERICAL SIMULATION AND DISCUSSION

In order to ascertain the qualitative analysis obtained in the previous sections of the study, the numerical solutions to equation 1 is shown in the subsequent tables and figures. Considering the values generated for each of the parameters stated in table 2 with initial conditions $S(0)=10,000$, $A(0)=25,000$, $P(0)=17,000$, $M(0)=6,000$ and $R(0)=4,400$ which are shown in figure 2, 3 and 4 below.

Figure 2 shows that the Alcohol-free equilibrium point is locally asymptotically stable with $R_{01} = 0.1782 < 1$ and $R_{02} = 0.02459 < 1$. This confirmed that the qualitative analysis and the solution of the model system converges to the Alcohol-free equilibrium point.

Replacing the $\beta = 3 \times 10^{-7}$ with $\beta = 3 \times 10^{-5}$, while other parameter values in the table are kept constant. We have $R_{01} = 0.2459$ and $R_{02} = 2.4570$ and $R_{0A} > 1$. This also confirmed and show that the solution converges to Alcohol-free equilibrium point. Also, in figure 5 where $\beta = 10^{-6}$ in place of 10^{-8} and other parameter values are fixed, we have $R_{01} = 12.4301$ and $R_{02} = 4.2459$. ($R_{01} > 1$ and $R_{02} > 1$). This also supported that the solution converges to the Mentally ill-free equilibrium point.

In figure 6, the poverty-free equilibrium point is locally asymptotically stable on the first strain, Alcohol addiction $A(t)$ while the second strain, the poverty $P(t)$ persists in the society.

In figure 7 with use of the initial conditions, the mentally ill-free equilibrium is locally asymptotically stable.

In this system, both strains, Alcohol addiction $A(t)$ and the poverty state $P(t)$ persist in the society. The qualitative analysis and numerical results are summarized as;

- * Region I when ($R_{01} < 1, R_{02} < 1$): The Alcohol-free equilibrium point is stable.
- * Region II when ($R_{01} < 1, R_{02} > 1$): The poverty-free equilibrium point exist and is stable.
- * Region III when ($R_{01} > 1, R_{02} < 1$): The mentally ill-free equilibrium point exist and is stable.
- * Region IV when ($R_{01} > 1, R_{02} > 1$ and $R_{02} < R_{01}$): The mentally ill-free equilibrium point exist and is stable, Alcohol addiction is dominant.

* Region V when ($R_{01} > 1, R_{02} > 1$ and $R_{02} < R_{01}$): The mentally ill-free equilibrium point exist and is stable, poverty is dominant as shown in figure 8.

In figure 9 as α decrease both alcohol addiction (alcohol addicted people) and poverty level (poor people) decline, the decrease is slow for the poor population whereas is rapid for alcohol addicted people.

In figure 10 as β decreases, the Alcohol addicted population increase while the poor population decrease.

Looking at the occurrence of poverty $P(t)$ and the drinking of alcohol $A(t)$, the contact rate β plays a more significant role in spreading poverty than α .

We can reduce the mentally ill people $M(t)$ which are caused by chronic poverty and excessive drinking of alcohol, if $\alpha = \beta$ and $\rho = 0$ by comparing ($\gamma = 10^{-2}$, $\phi = 10^{-15}$) with ($\gamma = 10^{-15}$, $\phi = 10^{-2}$) and ($\gamma = 10^{-2}$, $\phi = 10^{-1}$) with ($\gamma = 10^{-1}$, $\phi = 10^{-2}$). The result indicates that the reduction of mentally ill people due to poverty (ϕ) is greater than the one due to alcohol addicted drinking (γ).

Summarily, when comparing the reduction of mentally ill population through therapy and counselling due to chronic poverty (ϕ) and due to addicted drinking of alcohol (γ). Madness is more strongly influenced by poverty than by alcohol drinking.

V. CONCLUSION AND RECOMMENDATIONS

In this study, a mathematical model was stated to really examine the habits of drinking alcohol and how it causes poverty and madness in the society. The total population N is sub-divided into five (5) compartments such as Susceptible Population (S), Alcohol addiction population (A), Poverty population (P) due to excess alcohol drinking, Madness population (M) sequel to the abuse of alcohol and the Recovered population (R).

The mathematical model was able to satisfies both boundedness and positivity condition, thus is biologically meaningful. It was also dynamic in that the parameter values were analyzed qualitatively and numerically. The model also contains three equilibrium points such as the alcohol-free equilibrium point (E_A), the poverty-free equilibrium point (E_P) and the mentally ill-free equilibrium point

(E_M) which all exist unconditionally at $R_{02} > 1$, $R_0 > 1$ and $R_{0A} > 1$ and two basic reproduction numbers R_{01} and R_{02} are identified where,

$$R_{01} = \frac{\alpha\omega}{(\theta+\phi+\delta+\gamma)\delta} \text{ and } R_{02} = \frac{\beta\omega}{(\delta+\phi+\rho+\mu)\delta}$$

In the study, the alcohol-free equilibrium point is locally asymptotically stable if $R_{0A} = 1$, the poverty-free equilibrium point is locally asymptotically stable if $R_0 < 1$ and the mentally ill-free equilibrium point is locally asymptotically stable at $R_0 > 1$ and $R_{0A} > 1$. The Ode45 of MATLAB was used for the numerical simulation which revealed the validity of the qualitative analysis.

VI. RECOMMENDATIONS

Based on the findings of the study, I therefore recommend that;

- 1 Indiscriminate selling of alcohol drinks should be checked and stopped so as to reduce excessive alcohol consumption in the society.
- 2 A guidance and counselling unit should be established in each of the local government areas of the state so as to guide and counsel the relieved and

freed ones. This will surely help them not to go back to the act again.

3 The Zonal and General Hospitals in the state should be revamped and equipped so as to treat and care for reported cases.

4 The future researchers on the same or related problem should direct the study on the bifurcation analysis of the model and extend the system of equation 1 to control theory.

5 This research should be replicated out by adding more parameters where appropriate values assign to them.

VII. CONTRIBUTIONS TO THE BODY OF KNOWLEDGE

The outcome of the study is necessary because it will be a guide and foundation for other researchers that will embark on similar or related problem in future. The government, law enforcement agencies and well-meaning individuals in the society will use the result of the research to checkmate the drinking habit of youths in the society. This will also reduce the level of poverty and the number of ill persons in the society.

Figures

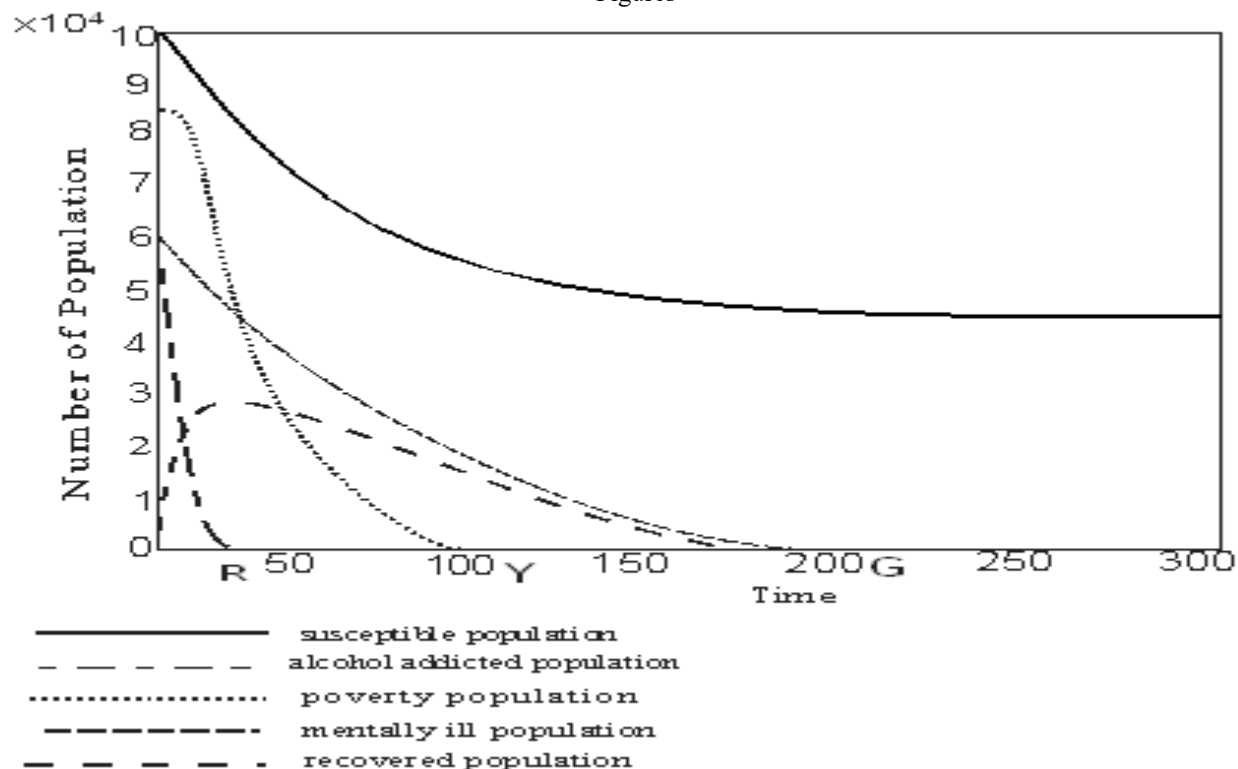


FIGURE 2: Time series plot of state variable for $R_{01}= 0.1797$ and $R_{02}= 0.02468$

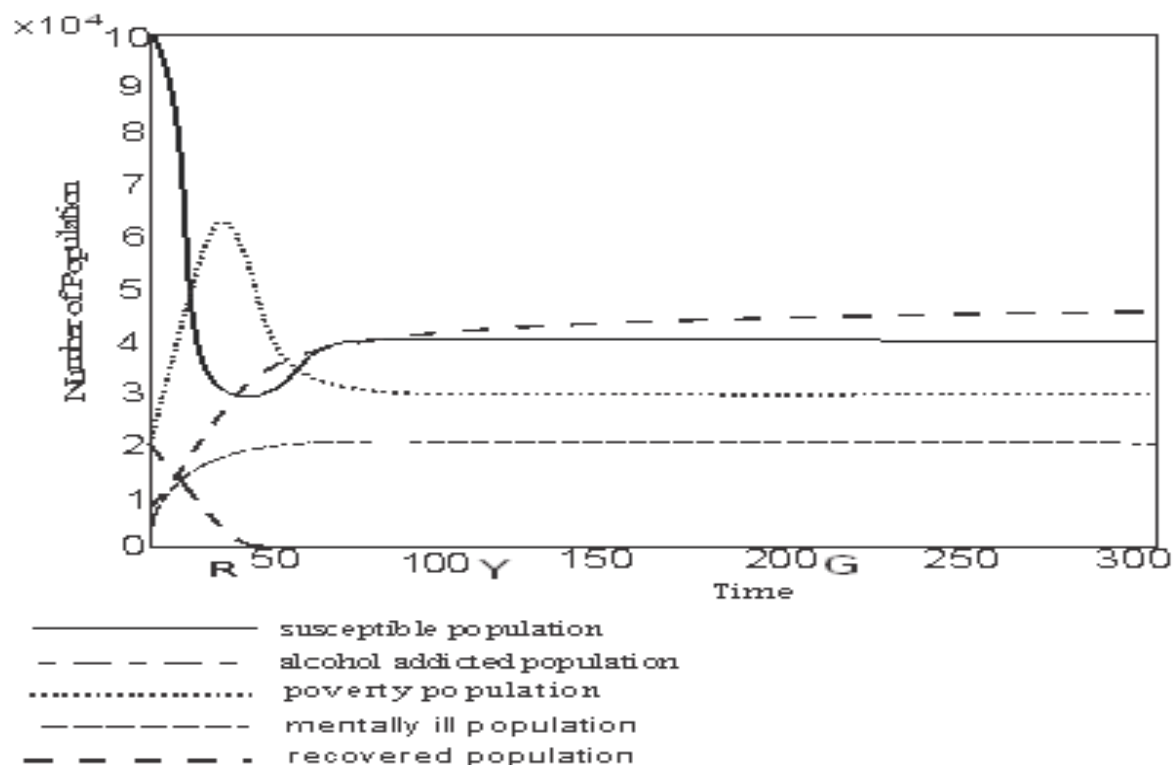


FIGURE 3: Time series plot of state variable for $R_{01} = 0.2468$ and $R_{02} = 2.458$

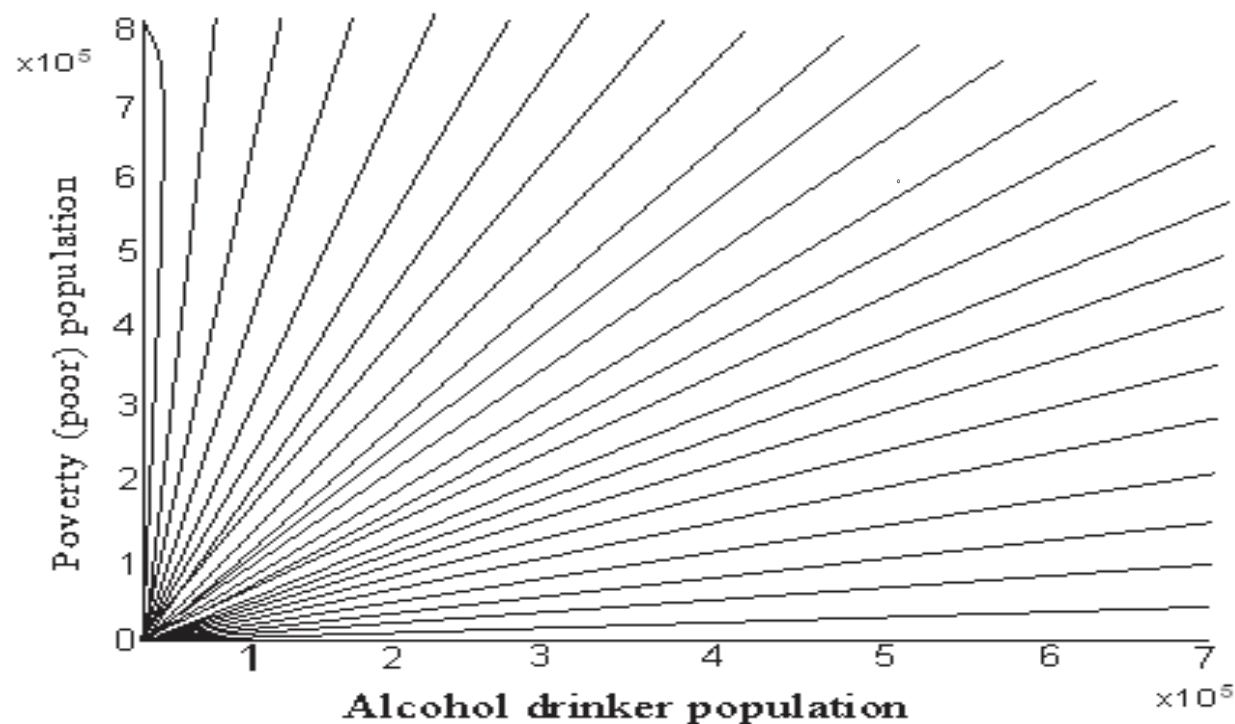


FIGURE 5: The parameter values are the same as the parameter values of figure 2.

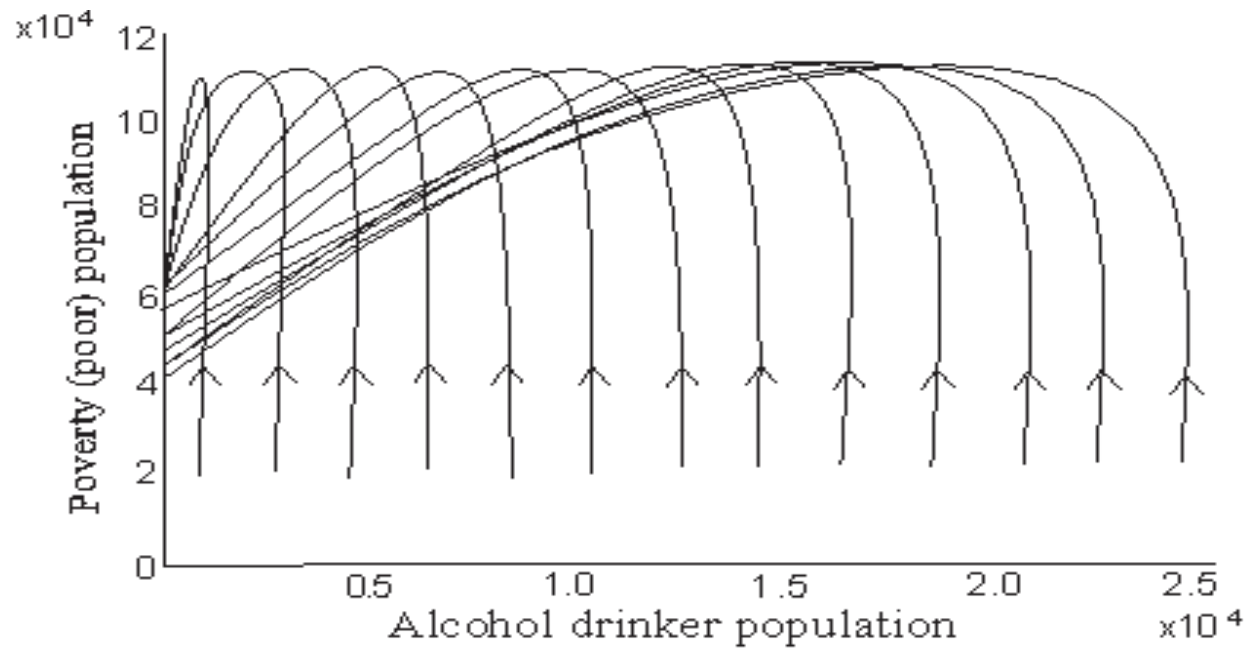


FIGURE 6: The parameter values anre the same as the parameter values of figure 3.

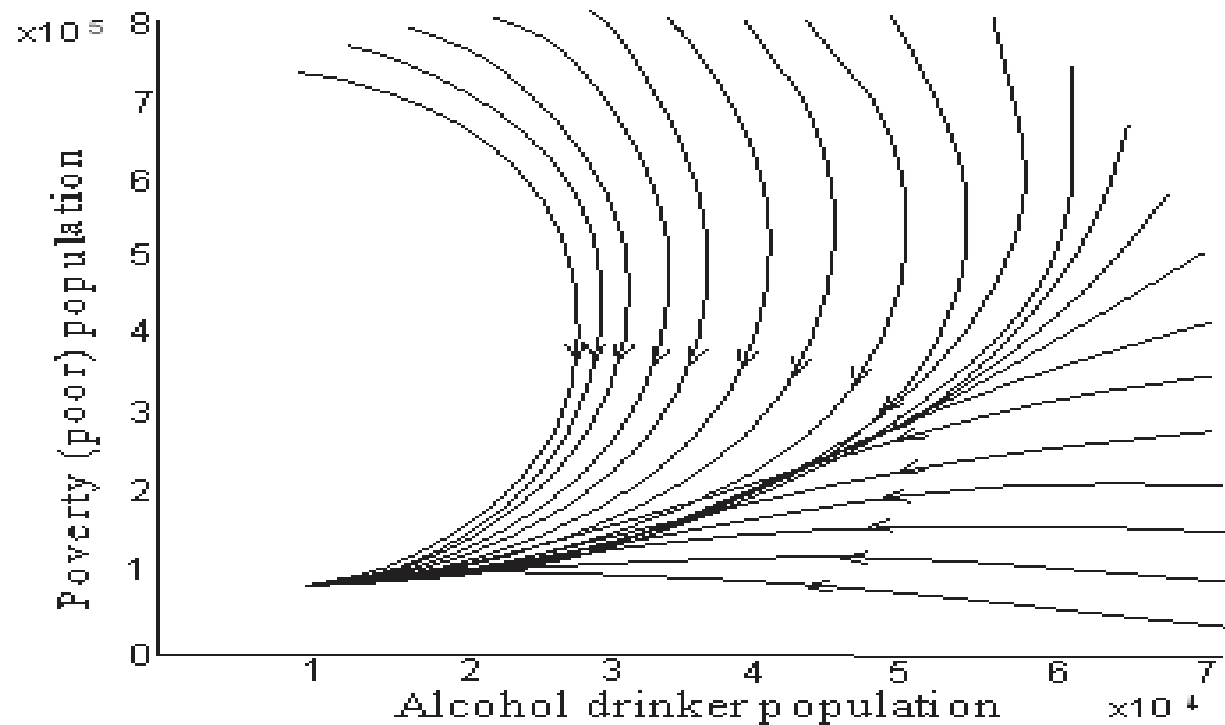


FIGURE 7: The parameter values are the same as the parameter values of figure 4.

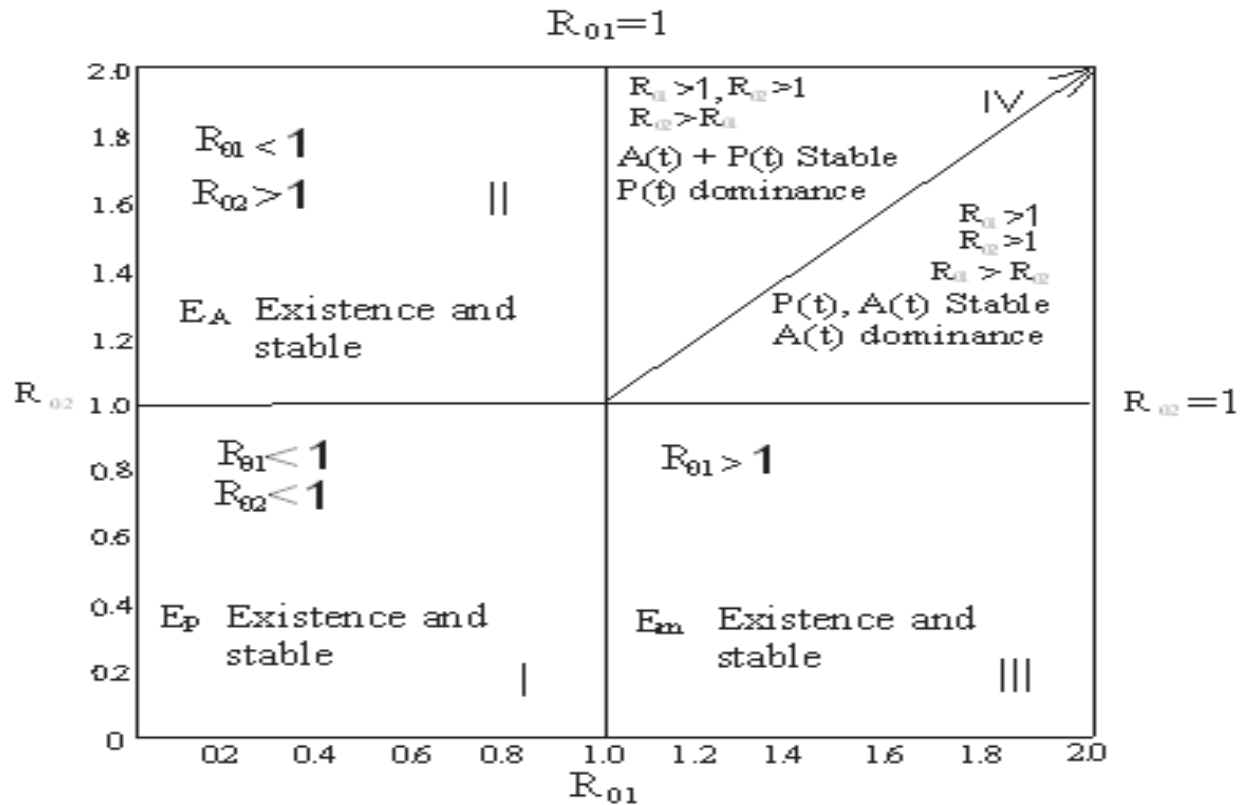


FIGURE 8: Existence and stability condition of the equilibrium points

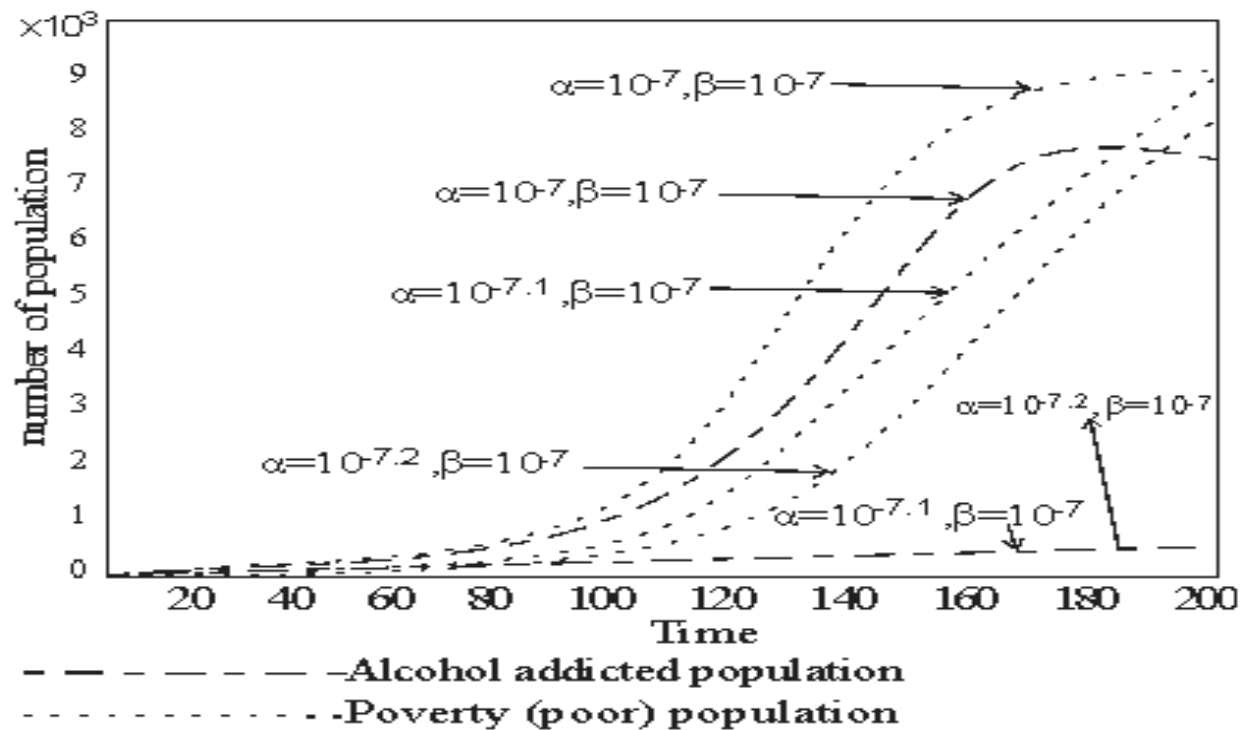


Figure 9: Effect of α on the poverty prevalence

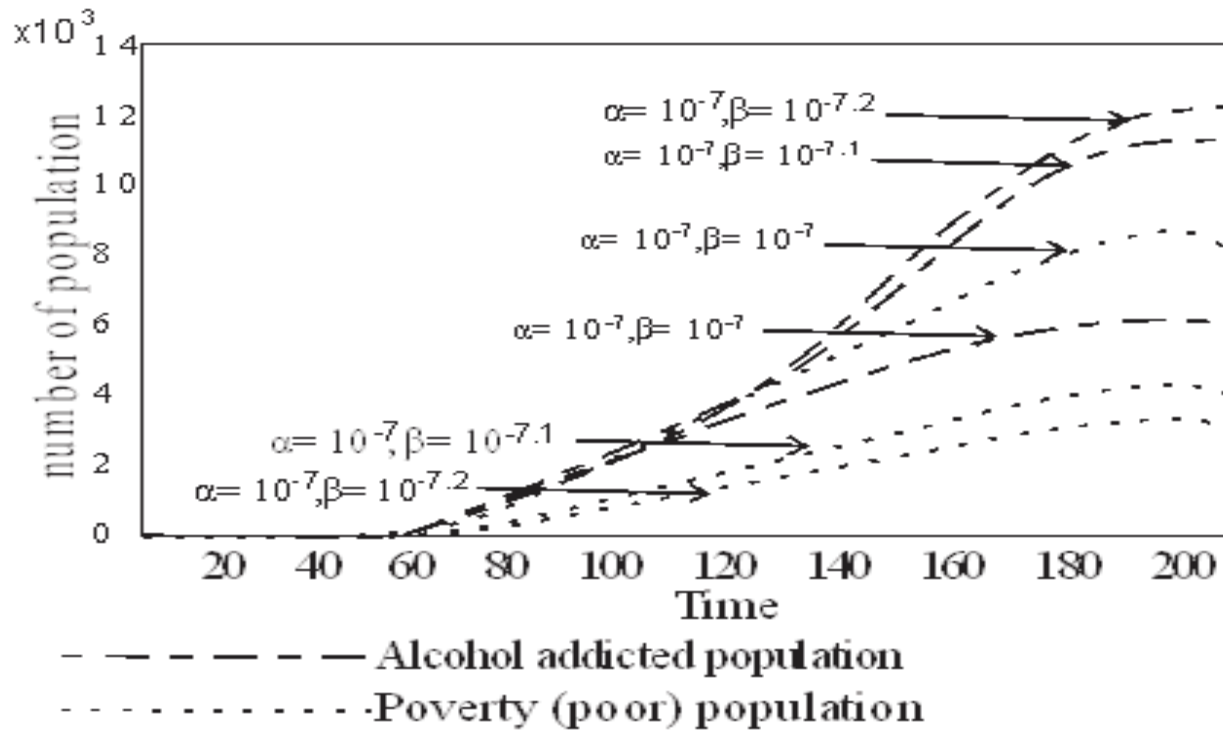


Figure 10: Effect of β on the poverty (poor) population

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