An Operations Research Approach to Transportation Cost Minimization

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***Abstract*—Transportation is an important component of supply chain and logistics management because the movement of goods from suppliers to destinations directly affects total operating cost. The Transportation Problem (TP) is a classical Operations Research model used to determine the optimal allocation of goods from several sources to several destinations while satisfying supply and demand constraints at minimum transportation cost. This paper presents an Operations Research approach to transportation cost minimization using mathematical modeling and standard optimization techniques. The proposed approach includes the North-West Corner Method, Least Cost Method, Vogel's Approximation Method (VAM), and the Modified Distribution Method (MODI). A numerical transportation problem is formulated and solved to demonstrate the methodology. The results show that an appropriate optimization procedure can reduce transportation cost and provide an efficient allocation plan. The study demonstrates the practical usefulness of transportation models for logistics, distribution, supply-chain management, and resource allocation.**

***Index Terms*—Operations Research, Transportation Problem, Linear Programming, Transportation Cost, VAM, MODI, Optimization, Resource Allocation**

I. INTRODUCTION

Operations Research (OR) serves as a foundational analytical methodology in administrative and industrial decision-making [1, 2]. Within contemporary sup-ply chain architectures and distribution networks, physical logistics represents one of the most substantial operational cost centers [1, 3].

The primary operational objective in logistics management is the efficient allocation of limited physical supply from production facilities, warehouses, or processing hubs to geographically dispersed demand nodes such as retail centers, regional distribution points, and end-user markets [4]. Because freight charges directly influence total operating overhead, optimizing transportation allocation schedules directly enhances competitive advantage and enterprise profitability [3].

Formulated mathematically, the Transportation Problem represents a specialized structural subset of Linear Programming (LP) [1, 5]. The primary mathematical objective is to allocate physical supply quantities across m source origins to fulfill specified demand quantities across n destinations while adhering to capacity limits and non-negativity boundaries [1]. Achieving an efficient allocation schedule requires systematic computational procedures capable of establishing an initial basic feasible solution (IBFS) [2, 3, 6]. Classical heuristic algorithms—specifically the North-West Corner Method (NWCM), Least Cost Method (LCM), and Vogel's Approximation Method (VAM)—serve as foundational approaches to construct valid initial distribution plans [3, 6, 7]. Subsequently, exact dual-variable optimization algorithms such as the Modified Distribution (MODI) Method evaluate non-basic distribution routes to iteratively refine allocations until global mathematical optimality is verified [3, 5, 6].

The core motivation of this study stems from the operational necessity for verified mathematical tools in freight management. Unoptimized logistics sched-ules frequently lead to misallocated vehicle capacity, underutilized supply hubs, and inflated distribution budgets. By synthesizing heuristic initialization methods with exact dual-variable optimization, this paper demonstrates how linear programming mitigates operational inefficiencies. The paper presents five independent numerical transportation models designed to evaluate baseline heuristic costs, penalty-based allocation mechanics, and exact dual optimality condition verification.

II. PROBLEM STATEMENT

1. Minimization of total transportation cost across multi-source and multi-destination logistical networks under deterministic operational parameters.
2. Efficient allocation of available supply capacities to meet destination demand requirements without violating network feasibility boundaries.
3. Computation of initial basic feasible solutions using structured heuristic rules to establish baseline distribution budgets.
4. Systematic evaluation and iterative refinement of transportation plan using exact dual optimization techniques to guarantee cost minimization.
5. Integration of mathematical transportation algorithms into practical managerial decision-making frameworks for logistics and resource allocation.

III. OBJECTIVES OF THE STUDY

1. Formulate multi-facility transportation problems into structured linear programming mathematical frameworks.
2. Determine initial basic feasible solutions for distribution networks using standardized mathematical allocation procedures.
3. Apply and compare the North-West Corner Method, Least Cost Method, and Vogel's Approximation Method in terms of solution quality and baseline cost.
4. Apply the Modified Distribution (MODI) Method to test allocation optimality and derive mathematically verified optimal routing plans.
5. Determine and interpret minimum transportation costs to provide action-able strategic guidance for industrial logistics management.

IV. LITERATURE REVIEW

The structural foundation of the Transportation Problem was established by Hitch-cock (1941), who introduced the classical formulation for distributing a single product from several sources to numerous localities at minimal total cost [1, 4, 6, 8]. Koopmans (1949) broadened this mathematical framework by applying transportation models to economic activity and industrial planning systems [8, 9]. The mathematical integration of transportation structures into linear programming theory was completed by Dantzig (1963), who demonstrated that the transportation matrix represents a constrained network simplex structure capable of specialized computational acceleration [5, 9].

A primary requirement in solving transportation linear programs is establishing an initial basic feasible solution [1, 4, 6]. Classical literature categorizes IBFS techniques based on algorithmic complexity and proximity to global optimality [4, 7]. The North-West Corner Method, analyzed by Mhlanga et al. (2014), represents a spatial heuristic that allocates units strictly according to top-left matrix positioning without considering unit shipping rates [4, 9]. Consequently, NWCM routinely yields higher initial operational expenditures [3, 4]. To incorporate unit cost considerations directly, Dantzig introduced the Least Cost Method (LCM), which prioritizes allocations based on minimum unit shipping rates [4, 5]. While LCM outperforms spatial heuristics, it remains susceptible to sub-optimal local assignments when low-cost cells consume capacity required for surrounding high-volume routes [2, 4].

Vogel's Approximation Method (VAM) was developed to overcome the localized limitations of standard heuristics by evaluating penalty costs [6, 11]. VAM computes penalties as the absolute difference between the two lowest unit costs in each row and column, directing allocations toward routes where deferred assignment would incur significant cost penalties [5, 11]. Empirical studies by Appati et al. (2015) and Ahmed et al. (2016) confirm that VAM consistently yields initial basic feasible solutions close to optimal values [2, 5].

To confirm whether an initial distribution plan achieves minimum overall cost, exact optimization

routines are required [4, 6, 8]. Charnes and Cooper developed the Stepping-Stone Method, which evaluates closed evaluation loops across non-basic decision variables [9].
This approach was refined into the Modified Distribution (MODI) Method (also known as the $u - v$ method), which computes dual multipliers to determine marginal cost changes for unallocated routes [5, 7, 9].

Modern literature emphasizes combining heuristic initialization with MODI optimization to systematically minimize distribution costs across commercial supply networks [3, 5].

V. MATHEMATICAL FORMULATION

The Transportation Problem is formulated as a constrained linear programming model [8]. Consider a distribution network consisting of m supply origins S_i ($i = 1, 2, \ldots, m$) and n demand destinations D_j ($j = 1, 2, \ldots, n$). Let a_i represent the total supply available at origin i, and let b_j represent the total demand required at destination j. The unit shipping cost between origin i and destination j is denoted by c_{ij}. The primary decision variable x_{ij} represents the physical quantity transported from origin i to destination j. The mathematical objective function minimizes total transportation expenditure across all active network routes:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad (1)$$

Subject to the following operational linear constraints:

1. Supply Capacity Constraints:
Total shipments originating from source i cannot exceed its available supply capacity:

$$\sum_{j=1}^{n} x_{ij} = a_i \quad \forall i = 1, 2, \ldots, m \quad (2)$$

2. Demand Requirements Constraints:
Total shipments delivered to destination j must fulfill its demand requirement:

$$\sum_{i=1}^{m} x_{ij} = b_j \quad \forall j = 1, 2, \ldots, n \quad (3)$$

3. Non-negativity Constraints:
Physical allocation quantities must be non-negative:
$x_{ij} \geq 0, \quad \forall i = 1, 2, \ldots, m$ and $\forall j = 1, 2, \ldots, n$ (4)

A transportation problem is classified as balanced when total system supply equals total system demand [8, 11]:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad (5)$$

If total supply exceeds total demand ($\sum a_i > \sum b_j$) or total demand exceeds total supply ($\sum b_j > \sum a_i$), a dummy destination or dummy source with zero-unit costs ($c_{ij} = 0$) are introduced to balance the matrix prior to optimization [11]. A basic feasible solution requires exactly $m + n - 1$ occupied basic cells, ensuring that the constraint equations remain linearly independent and non-degenerate [9, 10].

VI. METHODOLOGY

Stage 1: Data Collection
Data collection involves gathering quantitative parameters from supply chain networks, including source supply capacities, destination demand requirements, and per-unit transportation costs across all potential shipping routes. These empirical data points are verified for consistency to ensure that total available supply and destination demand constraints accurately reflect physical distribution conditions [8].

Stage 2: Construction of Transportation Table
The collected data is structured into an $m \times n$ transportation matrix displaying sources as rows, destinations as columns, unit costs within matrix cells, and rim conditions along the margins. The total system supply is evaluated against total system demand to determine matrix balance and establish necessary dummy rows or columns [1, 11].

Stage 3: Initial Feasible Solution

An initial basic feasible solution (IBFS) is derived using structured heuristic algorithms, such as the North-West Corner Method, Least Cost Method, or Vogel's Approximation Method. The resulting initial allocation matrix must satisfy all supply and demand rim conditions while containing exactly $m + n - 1$ occupied basic cells to prevent degeneracy [3, 6, 10].

Stage 4: Optimality Test

The dual variables u_i for supply sources and v_j for demand destinations are computed using occupied basic cells where $u_i + v_j = c_{ij}$ with $u_1 = 0$.

Opportunity costs $\Delta_{ij} = c_{ij} - (u_i + v_j)$ are then calculated for all unoccupied non-basic cells to evaluate potential cost reduction opportunities [5, 7].

Stage 5: Improvement of Solution

If any unoccupied cell exhibits a negative opportunity cost ($\Delta_{ij} < 0$), the cell with the most negative value is selected to enter the basis as a new basic variable. A closed evaluation loop is constructed through occupied cells to reallocate unit quantities along alternating positive and negative nodes while preserving rim feasibility [5, 10].

Stage 6: Final Transportation Plan

The allocation table is updated following loop adjustments, and dual variables and opportunity costs are iteratively recomputed until all non-basic cells satisfy $\Delta_{ij} \geq 0$.

Once non-negativity across all opportunity costs is confirmed, the allocation matrix represents the optimal minimum-cost distribution plan [3, 5, 10].

Stage 7: Comparison

The total transportation cost generated by the optimized distribution plan is compared against the baseline costs produced by initial heuristic methods. Quantitative cost savings and capacity efficiency gains are evaluated to illustrate the operational value of mathematical optimization in logistics planning [4, 8].

VII. NUMERICAL EXAMPLES AND SOLUTIONS

7.1. Question 1: Balanced Transportation Problem

STEP 1 — MAIN QUESTION TABLE

Source	W_1	W_2	W_3	W_4	Supply
F_1	4	6	8	13	20
F_2	5	11	9	7	30
F_3	10	8	7	6	25
Demand	10	25	20	20	75

STEP 2 — ALLOCATION / UNITS TABLE

Allocation Route	Units	Unit Cost	Total Cost
$F_1 \rightarrow W_1$	10	4	40
$F_1 \rightarrow W_2$	10	6	60
$F_2 \rightarrow W_2$	15	11	165
$F_2 \rightarrow W_3$	15	9	135
$F_3 \rightarrow W_3$	5	7	35
$F_3 \rightarrow W_4$	20	6	120

STEP 3 — MINIMUM TRANSPORTATION COST

Total Transportation Cost $= \Sigma$ (Units Allocated $\times$Unit Transportation Cost)

$Z = (10 \times 4) + (10 \times 6) + (15 \times 11) + (15 \times 9) + (5 \times 7) + (20 \times 6)$

$Z = 40 + 60 + 165 + 135 + 35 + 120 = 555$

Note: Total Supply = 20 + 30 + 25 = 75; Total Demand = 10 + 25 + 20 + 20 = 75. The problem is balanced.

Initial Transportation Cost = 555

7.2. Question 2: Initial Basic Feasible Solutionusing North-West Corner Method

STEP 1 — MAIN QUESTION TABLE

Source	D_1	D_2	D_3	Supply
S_1	2	3	1	20
S_2	5	4	8	30
S_3	5	6	8	25
Demand	10	35	30	75

STEP 2 — ALLOCATION / UNITS TABLE

Allocation Route	Units	Unit Cost	Total Cost
$S_1 \rightarrow D_1$	10	2	20
$S_1 \rightarrow D_2$	10	3	30
$S_2 \rightarrow D_2$	25	4	100
$S_2 \rightarrow D_3$	5	8	40
$S_3 \rightarrow D_3$	25	8	200

STEP 3 — MINIMUM TRANSPORTATION COST

Total Transportation Cost $= \Sigma$ (Units Allocated ×Unit Transportation Cost)

$Z = (10 \times 2) + (10 \times 3) + (25 \times 4) + (5 \times 8) + (25 \times 8)$

$Z = 20 + 30 + 100 + 40 + 200 = 390$

Initial Transportation Cost = 390

7.3. Question 3: Initial Basic Feasible Solution using Least Cost Method

STEP 1 — MAIN QUESTION TABLE

Source	$D1$	$D2$	$D3$	Supply
$S1$	6	4	1	25
$S2$	3	8	7	30
$S3$	4	6	5	20
Demand	20	30	25	75

STEP 2 — ALLOCATION / UNITS TABLE

Allocation Route	Units	Unit Cost	Total Cost
$S1 \rightarrow D3$	25	1	25
$S2 \rightarrow D1$	20	3	60
$S2 \rightarrow D2$	10	8	80
$S3 \rightarrow D2$	20	6	120

STEP 3 — MINIMUM TRANSPORTATION COST

Total Transportation Cost $= \Sigma$ (Units Allocated ×Unit Transportation Cost)

$Z = (25 \times 1) + (20 \times 3) + (10 \times 8) + (20 \times 6)$

$Z = 25 + 60 + 80 + 120 = 285$

Initial Transportation Cost = 285

7.4. Question 4: Vogel's Approximation Method (VAM)

STEP 1 — MAIN QUESTION TABLE

Source	$D1$	$D2$	$D3$	Supply
$S1$	10	2	20	15
$S2$	5	12	15	25
$S3$	8	7	6	20
Demand	10	25	25	60

STEP 2 — ALLOCATION / UNITS TABLE

Allocation Route	Units	Unit Cost	Total Cost
$S1 \rightarrow D2$	15	2	30
$S2 \rightarrow D1$	10	5	50
$S2 \rightarrow D2$	10	12	120
$S2 \rightarrow D3$	5	15	75
$S3 \rightarrow D3$	20	6	120

STEP 3 — MINIMUM TRANSPORTATION COST

Total Transportation Cost $= \Sigma$ (Units Allocated ×Unit Transportation Cost)

$Z = (15 \times 2) + (10 \times 5) + (10 \times 12) + (5 \times 15) + (20 \times 6)$

$Z = 30 + 50 + 120 + 75 + 120 = 395$

Initial Transportation Cost = 395

7.5. Question 5: Optimum Solution by MODI Method

STEP 1 — MAIN QUESTION TABLE

Source	$D1$	$D2$	$D3$	Supply
$S1$	3	1	7	15
$S2$	4	6	5	25
Demand	10	15	15	40

STEP 2 — ALLOCATION / UNITS TABLE

Allocation Route	Units	Unit Cost	Total Cost
$S1 \rightarrow D2$	15	1	15
$S2 \rightarrow D1$	10	4	40
$S2 \rightarrow D3$	15	5	75

STEP 3 — MINIMUM TRANSPORTATION COST

Total Transportation Cost $= \Sigma$ (Units Allocated ×Unit Transportation Cost)

$Z = (15 \times 1) + (10 \times 4) + (15 \times 5)$

$Z = 15 + 40 + 75 = 130$

Minimum Transportation Cost = 130

VIII. RESULTS AND DISCUSSION

The empirical numerical evaluations demonstrate the computational behavior and economic impact of different transportation algorithms across varying net-work dimensions. A detailed comparison of the verified numerical outcomes provides clear insights into solution efficiency:

Table 1: Summary of Numerical Evaluation Results

Problem	Method Applied	Solution Type	Total Units Shipped
Q1	North-West Corner Method	Initial Basic Feasible	75
Q3	North-West Corner Method	Initial Basic Feasible	75
Q4	Least Cost Method	Initial Basic Feasible	75

Q5	Vogel's Approximation Method	Initial Basic Feasible	60
Q6	Modified Distribution (MODI)	Mathematically Optimal	40

Comparing initial basic feasible solution methods confirms established theoretical performance hierarchies. The North-West Corner Method (applied in Q1 and Q3) generates allocations strictly based on matrix positioning without incorporating unit transportation costs. Consequently, NWCM allocations frequently assign large unit volumes to high-cost routes, resulting in elevated baseline expenditures ($Z = 555$ for Q1 and $Z = 390$ for Q3). In contrast, the Least Cost Method (applied in Q4) actively scans the cost matrix to prioritize routes with the lowest unit shipping rates. In Q4, LCM immediately captures the minimal unit cost cell ($S1$, $D3$) at unit cost 1 and ($S2$, $D1$) at unit cost 3, achieving an initial basic feasible cost of 285. Vogel's Approximation Method (applied in Q5) incorporates penalty calculations to evaluate the economic consequence of failing to allocate to the lowest-cost route. In Q5, VAM prioritizes penalty differentials, allocating 20 units to route ($S3$, $D3$) at unit cost 6 and 15 units to ($S1$, $D2$) at unit cost 2, yielding an initial basic feasible cost of 395. VAM prevents high-penalty misallocations by accounting for cost trade-offs across entire rows and columns.

Finally, the Modified Distribution (MODI) Method (applied in Q6) demonstrates the necessity of dual-variable optimization. Starting from an initial feasible state, MODI calculates dual multipliers (u_i, v_j) to evaluate non-basic opportunity costs ($\Delta_{ij} = c_{ij} - u_i - v_j$). By reallocating units along closed adjustment loops, MODI systematically eliminates negative opportunity costs, reducing the initial allocation cost to a verified optimal minimum of 130.

IX. PRACTICAL APPLICATIONS

9.1. Supply Chain Management

Transportation models enable manufacturing enterprises to optimize multi-echelon supply distribution networks across manufacturing plants, regional warehouses, and distribution centers. By minimizing freight expenditures while meeting operational fulfillment schedules, enterprises achieve significant cost savings and improve supply chain responsiveness.

9.2. Agricultural Distribution

Agricultural distribution networks apply transportation optimization to allocate perishable produce from primary harvest regions to processing facilities and urban consumer markets. Efficient routing minimizes transit times, reduces product spoilage, and optimizes fleet utilization during peak seasonal harvest cycles [4].

9.3. Food and Essential Resource Distribution

Humanitarian organizations and relief agencies deploy transportation linear pro-grams to coordinate the emergency distribution of food, clean water, and essential resources to disaster-affected populations. Optimization frameworks ensure equitable resource allocation while minimizing logistic expenditures under severe budgetary constraints [1].

9.4. Retail Distribution

Omnichannel retail corporations utilize transportation algorithms to streamline inventory replenishment from automated fulfillment hubs to physical retail store-fronts. Optimizing truckload allocations reduces freight delivery costs, prevents stockout events, and maintains balanced regional inventory levels [4].

9.5. Healthcare Logistics

Healthcare networks deploy transportation cost minimization models to distribute pharmaceuticals, medical equipment, and personal protective assets from central depositories to regional hospitals. Precise resource allocation ensures timely delivery of critical medical goods while operating within strict institutional bud-gets [1, 8].

X. ADVANTAGES OF THE PROPOSED APPROACH

1. Direct Reduction in Freight Expenditure: Systematic application of transportation algorithms identifies minimal-cost distribution paths, yielding

direct savings in operational distribution budgets.
2. Efficient Resource Allocation: Mathematical modeling prevents the underutilization or overloading of supply centers by ensuring exact alignment between source capacities and destination requirements.
3. Structured Decision-Making Framework: The multi-stage optimization process eliminates subjective decision bias by providing a reproducible, mathematically grounded methodology for network managers.
4. Optimal Capacity Utilization: Supply constraints are rigorously enforced, guaranteeing that production centers operate within valid output limits without exceeding facility thresholds.
5. Exact Optimality Verification: Incorporating the MODI method provides explicit mathematical proof of optimality through dual variable evaluation, ensuring no further cost reduction is possible.
6. Scalability Across Logistics Networks: The underlying linear programming structure scales efficiently from small regional networks to complex global supply chain architectures.

XI. LIMITATIONS

1. Deterministic Cost Assumptions: Standard transportation models assume unit shipping costs remain constant, ignoring real-world fuel price fluctuations, volume discounts, and spot-market rate changes.
2. Static Supply and Demand Parameters: Supply availability and destination demand are treated as known fixed constants, whereas practical sup-ply networks experience demand volatility and supply disruptions.
3. External Transportation Disruptions: Fixed network models do not dynamically accommodate unexpected traffic congestion, severe weather events, or vehicle breakdown delays.
4. Linear Function Constraints: The model assumes strictly linear cost relationships, which may fail to capture non-linear scale economies or step-cost pricing structures in freight transport.
5. Simplified Network Topology: Basic transportation structures model direct point-to-point shipments, omitting complex multi-stop transshipment nodes and cross-docking dynamics.

XII. PROPOSED RESEARCH FRAMEWORK

The proposed framework provides a comprehensive roadmap for conducting transportation optimization studies. The framework initiates with rigorous problem definition and empirical data collection, establishing the core parameter inputs required for matrix formulation. Following structural balance verification, the model executes heuristic initialization (NWCM, LCM, or VAM) to form a feasible baseline.

Table 2: Sequential Phases of the Proposed Transportation Research Framework

Phase	Stage Name	Description / Function
Phase 1	Problem Identification	Define supply origins, destination nodes, and operational constraints.
Phase 2	Data Collection	Gather supply capacities, demand requirements, and unit transportation costs.
Phase 3	Transportation Table Construction	Structure the cost matrix, verify rim conditions, and balance supply and demand.
Phase 4	Initial Feasible Solution	Execute heuristic methods (NWCM, LCM, VAM) to derive initial allocations.
Phase 5	Optimality Test	Compute dual potentials (u_i, v_j) and evaluate opportunity costs (Δ_{ij}).
Phase 6	MODI Improvement	Construct closed evaluation loops to adjust allocations until all $\Delta_{ij} \geq 0$.
Phase 7	Final Transportation Plan	Finalize optimal allocation schedule satisfying all supply and demand limits.
Phase 8	Cost Comparison	Analyze cost differentials across methods to quantify optimization savings.
Phase 9	Practical Interpretation	Translate optimized outputs into actionable operational logistics plans.

The framework then transitions into dual-variable optimization using the MODI method, iteratively adjusting allocation loops until all non-basic evaluation metrics confirm mathematical optimality. Finally, optimized outcomes are translated into actionable operational logistics plans.

XIII. CONCLUSION

This study demonstrates the application of Operations Research methodology to transportation cost minimization within logistics and supply chain systems. Transportation problems formulated as linear programs provide a structured approach to managing physical distribution networks. The numerical evaluations confirm that initial basic feasible solution methods vary in computational efficiency and proximity to minimal costs. Heuristics like NWCM establish base-line feasibility based on spatial placement, whereas LCM and VAM incorporate cost and penalty metrics to achieve tighter initial bounds.

The application of the Modified Distribution (MODI) Method highlights the necessity of formal optimization. By systematically evaluating dual variables (u_i, v_j) and opportunity costs (Δ_{ij}), MODI converts initial feasible allocations into mathematically verified global optima. As demonstrated in Question 6, optimal reallocation reduced total shipping expenditures to a verified minimum of 130. In practical environments, adopting structured transportation modeling enables logistics managers to lower freight costs, improve capacity utilization, and maintain service level commitments across diverse distribution sectors. Integrating mathematical optimization into supply chain operations provides an essential framework for maintaining cost control and distribution efficiency.

XIV. FUTURE SCOPE

Future research in transportation optimization can build upon static models by incorporating advanced dynamic frameworks. Key areas for prospective re-search include:

1. Stochastic and Fuzzy Transportation Modeling: Integrating uncertain supply levels, fluctuating destination demands, and probabilistic transit times using fuzzy logic and robust stochastic programming.
2. Dynamic Multi-Period Routing: Expanding single-period static matrices into multi-period dynamic optimization frameworks that capture inventory carrying costs and temporal demand shifts.
3. Sustainable and Green Logistics: Incorporating carbon emission penal-ties, environmental impact metrics, and fuel consumption functions along-side traditional economic cost objectives.
4. Multi-Objective Optimization: Developing Pareto-optimal solution procedures that balance competing operational criteria, such as minimizing total cost, reducing delivery lead times, and maximizing service reliability.
5. AI-Assisted Algorithmic Solvers: Combining deep reinforcement learn-ing and metaheuristic algorithms with exact mathematical programming to accelerate optimal route generation across large-scale logistics networks.

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