Graph - Theoretic Optimization of Transportation Networks Using Shortest-Path Algorithms

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***Abstract*—Graph theory is an important branch of discrete mathematics that provides mathematical models for representing and analyzing interconnected systems. Transportation networks can naturally be represented as weighted graphs, where locations are modeled as vertices, roads as edges, and distance, travel time, or transportation cost as edge weights. This research investigates the application of graph-theoretic methods for optimizing routes in transportation networks. A mathematical graph model is developed to represent connections between different locations, and shortest-path algorithms, particularly Dijkstra's algorithm, are applied to identify efficient routes between selected source and destination vertices. Different route conditions can be analyzed by modifying edge weights or considering changes in network connectivity. The study evaluates the effectiveness of graph-based shortest-path optimization in minimizing travel distance or cost and demonstrates how mathematical graph models can support practical transportation decision-making. The proposed approach provides a foundation for extending graph-based optimization to dynamic transportation networks, real-time traffic conditions, and intelligent route-planning systems.**

***Index Terms*—Graph Theory, Transportation Networks, Graph-Theoretic Optimization, Weighted Graphs, Shortest Path, Dijkstra's Algorithm, Route Optimization, Network Optimization, Discrete Mathematics, Mathematical Modeling**

I. INTRODUCTION

Graph theory is an important branch of discrete mathematics that provides a mathematical framework for representing relationships and connections among different objects [1], [2]. A graph consists of vertices and edges, where vertices represent entities or locations and edges represent relationships or connections between them. The foundation of graph theory can be traced to Leonhard Euler's solution to the Seven Bridges of Königsberg problem in 1736, which established an important mathematical approach to studying connected structures. Since then, graph theory has developed into a widely applicable area of mathematics for modeling and analyzing complex interconnected systems.

One of the important applications of graph theory is the representation of transportation networks. In a transportation system, locations such as junctions, stations, or cities can be represented by vertices, while roads or routes connecting them can be represented by edges [1], [2], [5]. When numerical values such as distance, travel time, or transportation cost are associated with these edges, the resulting structure can be represented as a weighted graph. Such a mathematical representation allows different routes between locations to be compared systematically rather than relying only on direct or visually obvious routes. The use of weighted graphs for transportation modeling therefore provides a foundation for mathematical route optimization.

A fundamental problem in transportation-network analysis is the shortest-path problem, in which the objective is to determine a path between two vertices having the minimum total edge weight. If the weights represent travel time, the shortest path corresponds to the route requiring the minimum modeled travel time.

For a path containing edge weights (), its total weight is given by

$$W(P) = w_1 + w_2 + \cdots + w_k$$

The shortest path is therefore the feasible path having the smallest value of (W) [2], [3].

Dijkstra's algorithm is a classical graph-theoretic method for solving the shortest-path problem in weighted graphs with non-negative edge weights [3]. The algorithm progressively determines the minimum known distance from a selected source vertex to other vertices and can therefore be applied to identify efficient routes in transportation and network systems. Its applications include road transportation, navigation, delivery services, internet routing, and emergency vehicle routing.

Although a shortest path can be determined for a given weighted network, transportation conditions may change because of factors such as increased travel time on a road or temporary unavailability of a connection. Such changes can modify the edge weights or connectivity of the graph and may consequently change the optimal route. Therefore, studying the response of shortest-path solutions to controlled changes in a transportation network provides a useful mathematical perspective on route optimization and network flexibility [6]-[8].

The present study develops a weighted graph model of a transportation network and applies Dijkstra's shortest-path algorithm to determine minimum-travel-time routes between a selected source and destination. The study further examines the effect of changes in edge weights and network connectivity through different transportation scenarios. By comparing the resulting shortest paths and minimum travel times, the research aims to demonstrate how graph-theoretic modeling can be used to analyze route selection and the sensitivity of transportation networks to changing conditions.

II. RESEARCH PROBLEM

Transportation networks consist of multiple interconnected locations and routes, creating several possible paths between a source and a destination. Selecting an efficient route becomes challenging when different roads have different travel times and when network conditions change due to congestion or temporary road unavailability. A mathematical representation is therefore required to systematically evaluate alternative routes and identify an optimal path.

Graph theory provides a suitable mathematical framework for representing transportation networks as weighted graphs, in which locations are modeled as vertices, roads as edges, and travel times as edge weights [1], [4], [5]. The shortest-path problem can then be formulated as the determination of a path between a selected source and destination that minimizes the sum of the weights of its constituent edges. Dijkstra's algorithm provides an established method for solving this problem when the edge weights are non-negative [3].

However, a transportation network may undergo changes in edge weights or connectivity. An increase in travel time on a particular road may make an alternative route more efficient, while the closure of a road may eliminate previously available paths. Therefore, the problem addressed in this study is to mathematically model a transportation network as a weighted graph, determine its shortest routes using Dijkstra's algorithm, and examine how changes in travel-time weights and network connectivity influence the resulting optimal routes and minimum travel time.

III. RESEARCH GAP

Graph theory and shortest-path algorithms have been extensively studied and applied to transportation and network optimization. Weighted graph models provide an effective mathematical representation of transportation systems, while shortest-path algorithms such as Dijkstra's algorithm can be used to identify routes with minimum total weight [3]-[5]. Existing studies have also investigated transportation networks under dynamic, uncertain, and changing conditions [6]-[8].

However, a simple and mathematically transparent analysis of how shortest -path solutions respond to controlled changes in individual edge weights and network connectivity can provide useful insight into the behavior of transportation networks. In particular, changes such as increased travel time on a road or temporary road closure can alter the set of feasible and optimal routes, while alternative connections may preserve the minimum travel time.

Therefore, this study focuses on systematically examining the sensitivity of shortest -path solutions to changes in transportation-network conditions. A weighted transportation network is modeled mathematically, Dijkstra's algorithm is applied under normal conditions, and the resulting shortest paths are compared with those obtained after controlled changes in edge weights and network connectivity. The study aims to provide a clear mathematical and computational analysis of route selection and network flexibility under the defined scenarios.

IV. OBJECTIVES

The main objectives of this study are:

1. To model a transportation network using a weighted graph, where locations are represented by vertices, roads by edges, and travel time by edge weights.
2. To formulate the shortest-path problem mathematically for determining the minimum-travel-time route between a selected source and destination.
3. To apply Dijkstra's algorithm to identify shortest routes in the proposed weighted transportation network.
4. To examine the effect of changes in edge weights on the selection of optimal transportation routes.
5. To analyze the effect of road unavailability on shortest-path solutions by modifying the connectivity of the transportation network.
6. To compare the shortest-path results under different network scenarios in terms of optimal route, minimum travel time, and the availability of alternative optimal routes.
7. To verify the mathematical results computationally and evaluate the usefulness of graph-theoretic modeling for transportation-route optimization.

V. LITERATURE REVIEW

Graph theory has been widely used as a mathematical framework for representing and analyzing transportation networks. In graph-based transportation models, locations or transportation facilities can be represented as vertices, while roads or connections are represented as edges. Additional characteristics such as distance, travel time, and cost can be assigned as edge weights. Guze reviewed graph-theoretic methods for transportation-network modeling and emphasized the usefulness of graph structures, connectivity, and graph algorithms in transportation-network analysis and optimization [4].

Derrible and Kennedy examined the application of graph theory and network science to transit - network design and reviewed how network characteristics can be used to understand and improve transportation systems. Their work demonstrates that graph-theoretic approaches can provide useful information for transportation planning and network design [5].

Shortest-path optimization is one of the fundamental applications of graph theory in transportation networks. In a weighted network, the shortest-path problem seeks a route between two locations that minimizes the sum of the weights associated with the edges of the route. Classical algorithms such as Dijkstra's algorithm provide efficient solutions when edge weights are non-negative.

However, transportation networks are not always static. Travel times may vary because of congestion, traffic conditions, and other changes in network conditions. Hall studied the fastest-path problem in networks with random and time-dependent travel times and showed that conventional shortest-path methods such as Dijkstra's algorithm do not necessarily determine the minimum expected travel-time path when travel times are stochastic and time-dependent [6]. Similarly, Sung et al. investigated shortest paths in networks with time-dependent flow speeds and developed a modified approach based on Dijkstra's label-setting algorithm for such transportation conditions [7].

Further research has considered transportation networks affected by disruptions and changing conditions. Sever et al. investigated the dynamic shortest-path problem with time-dependent stochastic disruptions, where the objective is to determine routes with minimum expected travel time using historical and real-time information [8]. Their work demonstrates that transportation routing becomes more complex when network conditions change over time or are uncertain.

Recent research has increasingly focused on dynamic and uncertain transportation networks in which travel times and network conditions may change over time. These studies demonstrate that conventional shortest-path approaches may require modification when travel conditions become time-dependent or stochastic [6]-[8].

The literature therefore indicates that graph theory provides a well-established mathematical basis for transportation-network modeling and that shortest-path algorithms are important tools for route optimization. At the same time, advanced research increasingly addresses dynamic, stochastic, and data-driven transportation conditions. The present study focuses on a simpler and mathematically transparent weighted-graph model and investigates how controlled changes in edge weights and network connectivity influence shortest-path solutions obtained using Dijkstra's algorithm. This provides a focused mathematical analysis of route selection, alternative paths, and network flexibility under different controlled transportation scenarios.

VI. MATHEMATICAL BACKGROUND

A. Graph

A graph is an ordered pair G=(V, E) where (V) is a non-empty set of vertices and (E) is a set of edges connecting pairs of vertices. In a transportation network, vertices can represent locations or junctions, while edges represent roads or routes connecting them [1], [2].

B. Vertex

A vertex is a fundamental element of a graph that represents an entity or location. In the proposed transportation model, each vertex represents a transportation location or junction [1], [2].

C. Edge

An edge represents a connection between two vertices. In the transportation model, an edge represents a road connecting two locations [1], [2].

D. Weighted Graph

A weighted graph is a graph in which a numerical value is assigned to each edge. The weight may represent quantities such as distance, travel time, cost, capacity, or risk.

In this study, edge weights represent travel time in minutes.

Thus, the transportation network is represented as a weighted graph G=(V,E,w), where (w(e)) denotes the travel time associated with edge (e)[1], [2].

E. Path

A path is a sequence of vertices in which consecutive vertices are connected by edges. For example, A-C-D-F-H-I represents a path from (A) to (I) in the proposed transportation network [1], [2].

F. Total Weight of a Path

If a path consists of edges having weights $w_1, w_2, \dots,$ w_k then its total weight is calculated as

$W(P) = \sum_{\{i=1\}}{\{k\}}w$

or equivalently, $W(P) = w_1 + w_2 + \cdots + w_k.i$

In this study, (W(P)) represents the total travel time of a route [2]. For example, for the route A-C-D-F-H-I, the travel time is $W(P) = 3 + 4 + 5 + 4 + 2 = 18$ min $u\ tes.$

G. Shortest Path

The shortest-path problem aims to determine a path between two vertices for which the total edge weight is minimum. In this study, the source vertex is (A) and the destination vertex is (I). When edge weights represent travel time, the shortest path corresponds to the route with the minimum total travel time. The total weight of a path is given by $W(P) = w_1 + w_2 + \cdots +$ w_k

where (W(P)) represents the total weight of the path and represents *(w1, w2, w3, ..., wk)* the weights of the individual edges in that path. The optimal shortest path (P*) is the path having the minimum total weight: $W(P^*) = \min W(P)$ [2], [3]

Here, (P*) represents an optimal shortest path, and (W(P)) represents the total travel time of a path.

H. Dijkstra's Algorithm

Dijkstra's algorithm is a shortest-path algorithm used to determine the minimum distance from a selected source vertex to other vertices in a weighted graph with non-negative edge weights. It progressively selects the unvisited vertex with the smallest tentative distance and updates the distances of its neighboring vertices [3].

For the proposed transportation network, the source vertex is (A), and Dijkstra's algorithm is used to determine the shortest routes to the destination vertex (I).

The basic steps are:

1. Assign distance (0) to the source vertex and infinity to all other vertices.
2. Select the unvisited vertex having the smallest tentative distance.
3. Examine its adjacent vertices.
4. Calculate the distance through the selected vertex.
5. Update a neighboring vertex if the newly calculated distance is smaller.
6. Mark the selected vertex as visited.
7. Repeat the process until the required shortest path is determined.
8. Trace the predecessor vertices to reconstruct the shortest route.

I. Multiple Shortest Paths

A weighted graph may contain more than one path having the same minimum total weight. In such a case, the graph has multiple shortest paths.

For example, in the baseline transportation network, A-C-D-F-H-I has a total travel time 18 minutes, while A-C-E-F-H-I has a total travel time of 18 minutes

Therefore, both routes are optimal under the defined network conditions. The existence of multiple shortest paths is considered when analyzing the transportation scenarios in this study [1], [2].

VII. PROPOSED TRANSPORTATION NETWORK MODEL

A. Network Description

In this study, a transportation network is represented using a weighted graph consisting of nine locations, denoted by A, B, C, D, E, F, G, H, and I. Each location is represented by a vertex, while a road connecting two locations is represented by an edge. The weight assigned to each edge represents the estimated travel time between the corresponding locations, measured in minutes.

The proposed network is designed to provide multiple alternative routes between the source and destination, allowing the effect of changes in travel time and road availability to be analyzed.

B. Set of Vertices and Edges

The set of vertices in the proposed transportation network is defined as: V= {A, B, C, D, E, F, G, H, I}

Each vertex represents a transportation location or junction.

The edges represent the available road connections between these locations. The corresponding travel-time weights are presented in Table 1.

C. Edge-Weight Assignment

In the proposed model, travel time is selected as the edge weight because it provides a direct measure for comparing the efficiency of alternative transportation routes.

Each edge is assigned a non-negative travel-time value in minutes.

Table I. WEIGHTED EDGES OF THE PROPOSED TRANSPORTATION NETWORK

Edge	Travel Time (minutes)
A–B	4
A–C	3
B–C	2
B–D	5
B–E	7
C–D	4
C–E	6
D–E	2
D–F	5
E–F	3
E–G	6
F–G	2
F–H	4
G–H	3
G–I	5
H–I	2

D. Source and Destination

For the shortest-path analysis, vertex A is selected as the source location and vertex I is selected as the destination location. Therefore, the objective is to determine a path from A to I having the minimum total travel time.

E. Weighted Network Representation

The complete weighted transportation network used in this study is shown in Figure 1. The vertices represent transportation locations, the connecting lines represent roads, and the numerical values assigned to the edges represent travel time in minutes. The network provides multiple possible routes between A and I, which enables the analysis of shortest-path changes under different transportation conditions.

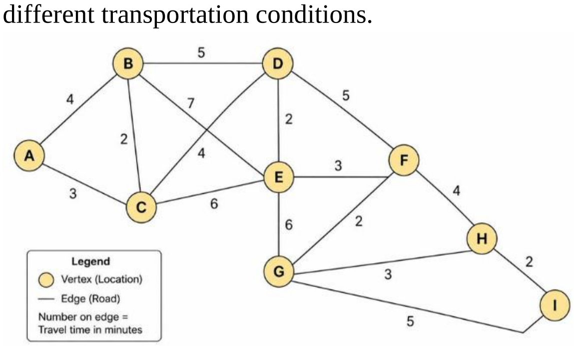

Figure 1. Weighted Transportation Network Used for Shortest-Path Analysis

VIII. METHODOLOGY

A. Research Design

This study follows a mathematical and computational approach to analyze shortest -path optimization in a transportation network. The transportation network is represented as a weighted graph, where locations are modeled as vertices, roads as edges, and travel time as edge weights. Dijkstra's algorithm is then applied to determine the shortest routes between the selected source and destination.

The study further examines how changes in transportation-network conditions affect the shortest-path solution. Controlled changes are introduced by increasing selected edge weights and removing a road connection. The resulting shortest paths and minimum travel times are then compared with the baseline network.

B. Construction of the Transportation Network

The proposed transportation network consists of nine vertices:

V= {A, B, C, D, E, F, G, H, I}

Each vertex represents a transportation location, and each edge represents a road connecting two locations. The edge weights represent travel time in minutes.

Vertex A is selected as the source and vertex I as the destination.

The weighted network and its edge values are presented in Figure 1 and Table 1.

C. Application of Dijkstra's Algorithm

Dijkstra's algorithm is applied to the proposed weighted graph because all edge weights are non-negative [3]. The algorithm begins by assigning a distance of zero to the source vertex A and infinity to all other vertices.

The following procedure is used:

1. Set the distance of A to 0 and all other vertices to infinity.
2. Select the unvisited vertex with the smallest tentative distance.
3. Examine all its connected neighboring vertices.
4. Calculate the new tentative distance through the selected vertex.
5. Update the distance if the newly calculated value is smaller.
6. Mark the selected vertex as visited.
7. Repeat the process until the destination is reached.
8. Trace the predecessor vertices to obtain the shortest route.

D. Baseline Analysis

First, Dijkstra's algorithm is applied to the original transportation network without any modifications. The shortest route from A to I and its minimum travel time are determined.

This baseline result is used for comparison with the modified transportation-network scenarios.

E. Scenario-Based Analysis

To investigate the effect of changing transportation conditions, four scenarios are considered.

Scenario 1 - Normal Conditions:

The original network is analyzed without any modification.

Scenario 2 - Congestion on C-D:

The travel time of edge C–D is increased from 4 minutes to 10 minutes to represent increased congestion.

Scenario 3 - Road Closure:

The edge D–F is removed from the network to represent temporary road unavailability.

Scenario 4 - Severe Congestion on F-H:

The travel time of edge F–H is increased from 4 minutes to 12 minutes to represent severe congestion.

For each scenario, Dijkstra's algorithm is reapplied and the resulting shortest route and minimum travel time are recorded.

F. Comparative Analysis

The results obtained from the different scenarios are compared using the following parameters:

- Shortest route from A to I
- Minimum travel time
- Number of shortest paths with the same minimum travel time
- Change in route selection
- Change in minimum travel time

The percentage change in minimum travel time is calculated using:

$$\text{Percentage Change} = \frac{\text{New Travel Time} - \text{Original Travel Time}}{\text{Original Travel Time}} \times 100$$

G. Computational Verification

The calculated shortest paths are verified computationally using Python. The same vertices, edges, weights, source, destination, and scenario modifications are used during computational verification.

The purpose of this verification is to confirm the mathematical calculations and ensure consistency between the theoretical and computational results.

IX. RESULTS AND ANALYSIS

A. Baseline Analysis — Normal Conditions

Under normal transportation conditions, all edges retain their original travel-time weights. Dijkstra's algorithm is applied to determine the shortest route from source vertex A to destination vertex I.

The minimum travel time obtained is: 18 minutes

The analysis shows that multiple routes have the same minimum travel time. Three optimal routes are identified:

Route 1:
A−C−D−F−H−I
3+4+5+4+2=18

Route 2:
A−C−E−F−H−I
3+6+3+4+2=18

Route 3:
A−C−D−E−F−H−I
3+4+2+3+4+2=18

Thus, the baseline network has three shortest paths, each requiring 18 minutes.

B. Scenario 1 — Normal Conditions

The baseline condition serves as the reference scenario.

Parameter	Result
Source	A
Destination	I
Minimum travel time	18 minutes
Number of shortest paths	3

The presence of multiple shortest paths indicates that the proposed network provides alternative routes with equal minimum travel time.

C. Scenario 2 — Increased Travel Time on C–D

To represent congestion, the travel time of edge C–D is increased from 4 minutes to 10 minutes.

The route:
A−C−D−F−H−I
now has a travel time of:
3+10+5+4+2=24

Therefore, it is no longer an optimal route. The shortest route becomes:
A−C−E−F−H−I
with:
3+6+3+4+2=18

Thus, the minimum travel time remains 18 minutes, but the number of shortest paths decreases.

D. Scenario 3 — Closure of Road D–F

In this scenario, edge D–F is removed to represent temporary road unavailability. The route: A−C−D−F−H−I is no longer feasible.

However, alternative routes remain available. The minimum travel time remains: 18 minutes

Two shortest routes remain: A−C−E−F−H−I and A−C−D−E−F−H−I

Both require 18 minutes. This demonstrates that the presence of alternative connections can maintain the minimum travel time even when one road becomes unavailable.

E. Scenario 4 — Severe Congestion on F–H

In the final scenario, the travel time of edge F–H is increased from 4 minutes to 12 minutes.

The route: A−C−D−F−H−I
now requires: 3+4+5+12+2=26

Therefore, alternative routes through G become more efficient. Three shortest routes are identified:

1. A−C−D−F−G−I
   3+4+5+2+5=19
2. A−C−D−E−F−G−I
   3+4+2+3+2+5=19
3. A−C−E−F−G−I
   3+6+3+2+5=19

Thus, the minimum travel time becomes: 19 minutes

Compared with the baseline value of 18 minutes, the increase is:

$$Percentage\ Change = \frac{19-18}{18} \times 100 = 5.56\ \%$$

Therefore, severe congestion on F–H increases the minimum modeled travel time by 5.56 %.

F. Comparative Results

Table 2. Comparison of Shortest-Path Results Under Different Network Conditions

Scenario	Network condition	Minimum travel time	Main result
1	Normal conditions	18 min	Three shortest paths
2	C–D congestion	18 min	One shortest path remains
3	D–F road closure	18 min	Alternative routes maintain travel time
4	F–H severe congestion	19 min	Three shortest paths through G; minimum travel time increases to 19 min

Overall observation

The results demonstrate that changes in edge weights and network connectivity can alter the shortest-path solution. Moderate changes may change the selected route without increasing the minimum travel time, whereas a sufficiently large increase in travel time can increase the minimum achievable travel time.

X. DISCUSSION

The results demonstrate that the shortest-path solution in a weighted transportation network depends on both the structure of the network and the weights assigned to its edges. Under normal conditions, the proposed network provides multiple routes having the same minimum travel time of 18 minutes. This indicates that a transportation network can contain more than one mathematically optimal route between a source and destination.

The second scenario demonstrates the effect of increased travel time on a single road. When the travel time of edge C–D is increased from 4 minutes to 10 minutes, routes using this edge become less favorable. The shortest route therefore shifts toward an alternative connection through E. However, the minimum travel time remains 18 minutes. This indicates that a change in an individual edge weight does not necessarily increase the overall minimum travel time when an alternative route with equivalent cost is available.

The road-closure scenario provides further evidence of the importance of network connectivity and redundancy. When edge D–F is removed, the network continues to provide alternative routes between the source and destination. As a result, the minimum travel time remains 18 minutes. From a mathematical perspective, this shows that the existence of alternative paths can reduce the effect of the loss of an individual connection.

The fourth scenario produces a more significant change. Increasing the travel time of edge F–H from 4 minutes to 12 minutes makes routes using this connection less efficient. The shortest route is consequently redirected through G, and the minimum travel time increases from 18 minutes to 19 minutes. This represents an increase of approximately 5.56%. The result demonstrates that sufficiently large changes in edge weights can affect both the selected optimal route and the minimum achievable travel time.

Overall, the scenario analysis demonstrates the sensitivity of shortest-path solutions to changes in transportation-network conditions. The results also show that network redundancy can provide alternative optimal routes when a particular road becomes congested or unavailable. Therefore, weighted graph modeling provides a clear mathematical framework for examining route selection and the effect of network changes.

The findings are consistent with the fundamental principle of shortest -path optimization: the optimal route is determined by the combined weights of its edges rather than by the number of roads in the route. Thus, a route containing more edges may still be preferable if its total travel time is lower [1], [2].

XI. LIMITATIONS OF THE STUDY

Although the proposed weighted-graph approach provides a clear mathematical framework for shortest-path analysis, the study has certain limitations.

1. Abstract transportation network:

The network used in this study consists of hypothetical locations and road connections. It does not represent a specific real-world city or transportation system.

2. Assigned travel-time values:
The edge weights are predefined travel times used for mathematical analysis. They are not obtained from real-time traffic or GPS data.

3. Static conditions within each scenario:
Each scenario assumes fixed edge weights during the analysis. In real transportation systems, travel times may continuously change because of traffic, weather, accidents, and other factors.

4. Limited network size:
The proposed network contains nine vertices and a limited number of edges. Larger transportation networks may involve substantially greater computational complexity.

5. Dijkstra's algorithm constraint:
Dijkstra's algorithm is applicable to networks with non-negative edge weights. It does not directly handle negative edge weights [3].

6. Simplified transportation factors:
The model considers travel time as the primary optimization criterion and does not simultaneously incorporate factors such as fuel consumption, monetary cost, traffic density, road capacity, or environmental impact.

7. Limited number of scenarios:
Only selected changes in edge weights and network connectivity are examined. More extensive scenario analysis could provide a broader understanding of transportation-network behavior.

These limitations indicate that the proposed model should be considered a controlled mathematical representation rather than a complete real-world traffic-routing system. Nevertheless, it provides a transparent framework for examining how changes in transportation-network conditions influence shortest-path solutions.

XII. CONCLUSION

This study demonstrates the application of graph theory to shortest-path optimization in a modeled transportation network. The transportation system was represented as a weighted graph in which locations were modeled as vertices, roads as edges, and travel time as edge weights. Dijkstra’s algorithm was applied to determine minimum-travel-time routes between the selected source and destination.

Under normal network conditions, the analysis identified multiple shortest paths with a minimum travel time of 18 minutes. The scenario-based analysis further demonstrated that changes in transportation-network conditions can influence route selection. Increasing the travel time of an individual road caused the shortest route to shift toward an alternative connection, while the closure of a road could be accommodated when sufficient alternative paths were available. Under severe congestion, the minimum travel time increased from 18 to 19 minutes, representing a 5.56% increase.

The results demonstrate that the shortest-path solution depends on both the edge weights and connectivity of the network. The presence of alternative routes can provide network flexibility and maintain efficient travel even when individual connections become unavailable or less efficient. Thus, weighted graph modeling combined with Dijkstra’s algorithm provides a transparent mathematical approach for analyzing transportation-route optimization under controlled network conditions.

Overall, the study shows that graph-theoretic methods can effectively represent transportation networks and provide a systematic basis for evaluating route changes under different conditions. The proposed model also provides a foundation for extending the analysis to larger networks, real-world transportation data, and dynamically changing traffic conditions.

XIII. FUTURE SCOPE

The present study provides a basic mathematical framework for analyzing shortest paths in a weighted transportation network. The proposed approach can be extended in several directions to make the model more representative of real-world transportation systems.

1. Real-world transportation data: The hypothetical network can be replaced with actual road-network data obtained from transportation databases, GPS systems, or digital maps.
2. Real-time traffic conditions: Edge weights can be updated dynamically using real-time traffic

information so that changing travel times can be incorporated into route selection.

3. Larger transportation networks: The proposed approach can be applied to larger networks containing hundreds or thousands of locations and road connections.
4. Multiple optimization factors: Future models can consider additional factors such as distance, fuel consumption, transportation cost, congestion level, and environmental impact along with travel time.
5. Dynamic shortest-path algorithms: More advanced algorithms can be investigated for networks in which edge weights continuously change with time.
6. Artificial intelligence and machine learning: Traffic prediction techniques and intelligent routing methods can be integrated with graph-based models to predict changing travel conditions and improve route selection.
7. Uncertainty modeling: Fuzzy or stochastic graph models can be explored to represent uncertainty in travel time caused by unpredictable traffic conditions, accidents, weather, or road disruptions.

These extensions can help transform the proposed mathematical model into a more comprehensive transportation-route optimization framework capable of handling larger, dynamic, and real-world transportation networks.

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