

Indian Mathematical Heritage: Innovation, Science and Technology from Ancient Knowledge to Contemporary Learning

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Abstract—Indian Knowledge System (IKS) is an ongoing intellectual heritage in which knowledge is built on the basis of observation, reasoning, experimentation, computation and practical application. In the mathematical sciences, there was a central role and close link to astronomy, architecture, measurement, calendar-making, commerce and technological practice. The development of arithmetic, geometry, algebra, trigonometry, combinatorics and mathematical astronomy were all made significant contributions by ancient and medieval Indian mathematicians. In this paper, contributions of selected authors of the Indian origin namely Baudhayana, Pingala, Aryabhata, Brahmagupta, Mahavira, Bhaskara II and Madhava of Sangamagrama in terms of Innovation, Science and Technology have been studied. It emphasizes the knowledge of geometry which is found in the Śulba Sūtras, Pingala's combinatorial approach, Aryabhata's mathematical astronomy and trigonometry, Brahmagupta's use of zero and negative numbers, Bhaskara II's algebraic and computational techniques and Madhava's infinite series. The paper contends that these contributions are a body of knowledge that combines mathematical abstraction with the observation of nature and its practical needs. They are also touched upon as applicable to computational thinking and science, technology, engineering and mathematics (STEM) education, as well as to current learning of IKS.

Index Terms—Ancient Indian Mathematics, Aryabhata, Bhaskara II, Brahmagupta, Indian Knowledge System, Innovation, Madhava, Pingala, Science and Technology

I. INTRODUCTION

Mathematics has a long and diverse history on the Indian subcontinent, from the geometric techniques of the Śulba Sūtras through the mathematics of

classical astronomy to the infinite-series mathematics of the Kerala school [1], [2]. Mathematics did not exist in isolation as a purely theoretical pursuit; it was tightly tied to astronomy, architecture, ritual geometry, calendars, trade, measuring and calculating [3], [4]. Importance of Indian mathematics is not only with the discovery of specific results but also with the method of discovery which is through observation, abstraction, algorithmic reasoning, approximation, systematic calculation, and application of the results in practical situations. The present paper thus explores the Indian mathematics in the context of innovation, science and technology and its relevance in the present education system of India.

II. OBJECTIVES AND METHODOLOGY

The aim of the study is to analyse the major contribution of selected Indian mathematicians, innovative method and the relationship of mathematics with astronomy, science and technology and to understand the algorithmic and computational nature of selected methods. It also explores the potential of its heritage in the context of modern-day science, technology, engineering and mathematics (STEM) education and the Indian Knowledge System (IKS). It has a qualitative, historical and analytical approach to the paper with use of secondary sources and study of classical Indian mathematical traditions [5], [6]. Selected contributions are viewed in four ways: mathematical innovation, scientific application, technological/computational relevance, and contemporary educational relevance.

III. MATHEMATICS IN INDIAN KNOWLEDGE SYSTEM

Indian mathematical knowledge has evolved in the interactional process of several disciplines [7]. One of the special characteristics of mathematical literature was to give a few words of a rule, frequently in verse, and to accompany with explanations and examples. This helped to pass along computational procedures and algorithms. The Knowledge process could be shown as: *Observation* → *Mathematical Abstraction* → *Algorithm* → *Calculation* → *Scientific Application* → *Technological Practice*. This connection elucidates the innovative nature of Indian mathematics and its close ties with the practical needs.

IV. IMPORTANT ACHIEVEMENTS OF INDIAN MATHEMATICIANS

The Geometric Tradition was established by the 4th century BC philosopher-cum-mathematician Baudhayana. The Śulba Sūtras are one of the earliest surviving Indian mathematical texts [8], [9]. They are associated with the construction of ritual fire altars, and include rules for geometric construction, measurement and transformation of shapes. The Baudhayana Śulba Sūtra has a mathematical statement that is similar to what is now called the Pythagorean Theorem and mentions constructions of squares, rectangles and diagonals. A lot of measuring, proportional reasoning, area equivalence and geometric transformation were needed for such construction. So, geometry was used as a technology of measurement and construction, revealing how the practical need for technology could lead to systematic procedures in mathematics.

Chandaḥśāstra is an important example of mathematics arising from linguistics, and is associated with the Sanskrit poetic metres of Pingala [10], [11]. The use of different patterns of syllables resulted in systematic procedures based on combinations and recursive structures. In modern scholarship the algorithmic characteristics that are interpretable in the context of binary representation and combinatorial mathematics are identified. For a sequence with two different states, there are 2^n possible arrangements, which has been a way to conceptualize binary systems that are used in modern

computing. For Pingala, then, there is a historical precedent for: *Encoding* → *Enumeration* → *Algorithm*. But his work is not to be called the development of a modern electronic binary computer. It is important because it involves systematic, combinatorial and algorithmic reasoning.

Aryabhata (476–550 CE) is indeed central to the classical mathematics of India. His work, *Āryabhaṭīya*, written around 499 CE, systematically imparted mathematical and astronomical knowledge [12]. Aryabhata created and used a sine table for astronomical calculations thereby showing the use of trigonometry in astronomical observations. He also made a very close estimate of π : 3.1416 (modern decimal notation). His mathematical astronomy comprised calculations of planetary positions and eclipses, and time keeping. He gave a geometrical argument that was based on shadows, instead of myths, to explain eclipses. He is also connected with the development of the concept of the Earth's rotation as a cause of the apparent motion of the sky. Aryabhata's work is a reflection of the scientific process.

Brahmagupta (598–668 CE) explicitly gave rules for arithmetic with positive and negative numbers and zero in his *Brāhmasphuṭasiddhānta* of 628 CE [13], [14]. He gave rules for operations with zero and signed quantities and defined zero by subtraction. The number zero has two significant uses: as a place-holding value in position notation, and as a number in arithmetic. Positional notation and zero enabled representation and calculation of large numbers to be achieved. Brahmagupta also applied the ideas of 'fortune' and 'debt' to positive and negative numbers and he invented techniques to solve equations, quadratic problems, and indeterminate equations. His work illustrates the evolution from real numbers to abstract numbers.

The *Gaṇita Sāra Saṅgraha* of Mahavira (c. 900) gave a contribution to arithmetic and algebra [5]. Arithmetic operations, fractions, proportions, equations and practical problems were covered. One of its distinguishing characteristics was its focus on problems. Situations involving quantities, transactions, measurement and applications were used to present mathematical ideas. This is a way that is still educationally relevant as abstract mathematical concepts need to be tied to real-life problems.

Bhaskara II (1114–1185 CE) is one of the peaks of classical Indian mathematics. His great works are *Līlāvātī*, *Bījagaṇita* and the astronomical parts of *Siddhānta Śiromaṇi* [13], [15]. *Līlāvātī* has problems related to arithmetic, fractions, proportion, progressions, geometry, interest and combinations. *Bījagaṇita* is the study of equations, unknown quantities, surds, positive negative numbers and indeterminate equations. Bhaskara II is also credited with the cyclic solution of a few indeterminate quadratic equations, known as the Chakravala method. It may be considered from the modern computational point of view in the form of an iterative process: *Initial Approximation* → *Transformation* → *Improved Approximation* → *Repetition* → *Solution*. It is an efficient and systematic method, which is useful in the teaching of algorithmic thinking and computational mathematics. Another significant period of Indian mathematical creativity is associated with Madhava of Sangamagrama, who belonged to the Kerala school of mathematics and astronomy. He and his successors found the methods of infinite series that are linked to trigonometric functions and π [1], [2]. These methods signal a shift in the finite calculations to a systematic treatment of infinite processes. The series with these expansions in modern notation are connected with sine, cosine and arctangent functions, and are important in scientific computation and numerical approximation in the tradition of Madhava. Care must be taken in the understanding of the historical significance: modern calculus was the result of many traditions of mathematics and cannot be credited to a single scholar or culture [4].

V. MATHEMATICS AS A TECHNOLOGY OF SCIENCE

Mathematics had a close relationship with astronomy in India. Measuring angles, calculating planetary positions, forecasting eclipses, calculating time, and creating calendars and studying celestial cycles all involved astronomical calculations which used techniques for measurement. The efforts of Aryabhata, Brahmagupta, Bhaskara II and others show that mathematics was a scientific technology [1], [3]. There were mathematical problems that came from Astronomy, and Astronomy was possible to

calculate more accurately with mathematical techniques. This is very similar to the scientific modelling process of today.

VI. INNOVATIVENESS OF INDIAN MATHEMATICS

The examples selected illustrate some common patterns of innovation: (1) *Algorithmic Innovation*: The arithmetic, the extraction of roots, combinatorial enumeration, trigonometric calculation, kuṭṭaka and Chakravala constituted algorithmic innovations, with step-by-step procedures developed for them. (2) *Positional notation and systematic handling of zero*: This was the development of an efficient notation and calculation system for numbers. (3) *Geometric Innovation*: The tradition of Sulba (geometry) is an example of the application of geometry in practical construction and measurement. (4) *Astronomical Innovation*: Mathematical models used to observe and calculate the stars and other celestial bodies.

VII. RELEVANCE TO MODERN SCIENCE, TECHNOLOGY AND EDUCATION

Indian mathematics has much to offer to modern day STEM education [7]. Classical mathematical methods can be re-constructed in modern notation, pseudo-code and using computer programs. It could be used in education for: (1) reconstructing geometric constructions Śulba using GeoGebra; (2) programming the Chakravala or kuṭṭaka methods; (3) reproducing some of the astronomical calculations; (4) using combinatorics in exploring patterns of Pingala; (5) understanding how to approximate the solution of an equation by a series of Madhava-type terms; (6) visualizing trigonometric functions; and (7) comparing the numerical algorithms of the past with those of today. In such activities, the History of Mathematics + Mathematics + Computer Science is integrated, and IKS is not only a memorization topic, but also an interdisciplinary learning experience. The Indian mathematical tradition also has important examples of continuity (through the development of successive scientists/scholars, who explained, refined and extended earlier works), integration of knowledge, learning through problems, algorithmic reasoning, and observation and application.

VIII. DISCUSSION

The essays studied have shown that in India innovations in mathematics were often motivated by practical and scientific needs. Geometry grew with construction and with measurement; trigonometry grew with astronomy; Algebra grew with problems involving numbers from measurement; Combinatorial reasoning grew with the study of poetic meters; and the methods of infinite-series were applied to more and more precise astronomical and mathematical calculations. The methodological lesson is the most important. A continuous interplay between observation, abstraction, and application is what ancient Indian mathematicians showed as a means to innovation. Their work also defies a strict separation between pure and applied mathematics, as mathematical abstraction was sometimes the product of scientific and technological problems. The history of zero is a case in point of the power of such innovation. Positional notation and zero revolutionized the representation and calculation of numbers, and the explicit rules for adding numbers with zero by Brahmagupta marked an important step. Likewise, the Chakravala procedure illustrates the challenge of solving mathematical problems by repeatedly transforming them and systematically iterating. These examples are very relevant today as algorithms, numerical approximation, mathematical modelling and computational reasoning are increasingly important in science and technology.

IX. CONCLUSION

Ancient and medieval Indian mathematics is an important part of Indian Knowledge System and is a clear example of an uninterrupted tradition of mathematical innovations in this country. The mathematical methods of Baudhayana, Pingala, Aryabhata, Brahmagupta, Mahavira, Bhaskara II and Madhava illustrate the wide range of Indian mathematical ideas and scholarship. These should not be dismissed as “pure speculation” about the “invention” of modern mathematical ideas by one ancient mathematician. Mathematical progress is an accumulation and has involved interaction with scholars, schools and cultures. A historically responsible approach should pay tribute to the

originality of the Indian contributions as well as the wider development of mathematics in the world. The modern significance of this heritage is its capacity to stimulate investigations: techniques from the past may be reconstructed, programmed, visualized and compared with today’s algorithms. The mathematical astronomy field can be related to scientific modelling in the geometric constructions to engineering, the maths of the combinatorics to computer science and the numerical approximation to scientific computation. So Indian mathematical heritage is not just a legacy of the past, but a source of innovation to the future, connecting the heritage, the knowledge, the innovations, and the contemporary scientific education.

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