

An Analytical and Comparative Study of Various Transportation Problem Solution Methods: NWCM, VAM, Improved VAM, Modified VAM, Least Cost Method, Raval's Approximation Method, New Approach (NEWA), ASM, Improved ASM, ASMTTP and Improved ASMTTP

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Abstract—In the present study, several methods for obtaining an initial basic feasible solution are comparatively analysed, namely the North-West Corner Method (NWCM), Vogel's Approximation Method (VAM), Raval's Approximation Method, New Approach (NEWA), ASM, Improved ASM, NEWA, ASMTTP and Improved ASMTTP. The comparative analysis focuses on their effectiveness in obtaining initial solutions and their associated transportation costs. The study aims to examine the relative performance of these methods and identify their applicability to different transportation problem instances.

Index Terms—Transportation Problem; North-West Corner Method (NWCM); Vogel's Approximation Method (VAM); Raval's Approximation Method; New Approach (NEWA), ASM, Improved ASM, ASMTTP and Improved ASMTTP; Basic Feasible Solution (BFS); Initial Basic Feasible Solution; Transportation Cost; Operations Research.

I. INTRODUCTION

In Operations Research, Linear Programming (LP) is one of the fundamental models of mathematical programming and has extensive applications in decision-making and optimization. The Transportation Problem (TP) is a special class of Linear Programming that focuses on the optimal distribution of a homogeneous commodity from multiple sources to

multiple destinations at minimum possible transportation cost.

The mathematical foundations of the transportation problem can be traced to the work of Monge, who investigated transportation and allocation problems using mathematical approaches. Subsequently, Hitchcock formulated the classical transportation problem, while Kantorovich and Koopmans made significant contributions to the development of transportation and resource-allocation models.

The transportation model is primarily concerned with determining an optimal shipping schedule for transporting a homogeneous commodity from several origins, factories, or supply points to various destinations or markets while minimizing the total transportation cost. Transportation models play an important role in logistics and supply chain management, particularly in reducing distribution costs and improving the efficiency of resource allocation and service delivery.

In a transportation problem, each source has a specified quantity of goods available for shipment, commonly referred to as supply or availability, while each destination has a predetermined requirement, known as demand. Thus, the classical transportation problem involves determining the quantity of a commodity to be transported from each source to each destination so that all supply and demand constraints

are satisfied and the overall transportation cost is minimized.

The general procedure for solving a transportation problem consists of three major stages. First, the transportation problem is formulated as a mathematical model. Second, an Initial Basic Feasible Solution (IBFS) is obtained using an appropriate approximation method. Finally, the obtained solution is tested and, if necessary, improved to achieve optimality using methods such as the Modified Distribution Method (MODI) or the Stepping-Stone Method. The efficiency of the approximation method used to obtain the initial solution can significantly influence the subsequent optimization process.

1. Mathematical Formulation of The Transportation Problem

Consider a 6×8 transportation problem in which $C_1, C_2, \dots, C_6$ denote the six sources (city centres) and $D_1, D_2, \dots, D_8$ denote the eight destinations (warehouses). Let C_{ij} represent the unit transportation cost from source C_i to destination D_j , and let X_{ij} denote the corresponding quantity transported along that route. The transportation costs are measured in thousands of Indian Rupees (₹'000). The problem concerns the transportation of 20 tons of Dhan, with a transportation rate of ₹22 per kilometre. The available supply at each source and the corresponding demand at each destination are given in the following 6×8 transportation Table.

Table 1

Sources / Destinations	D1	D2	D3	D4	D5	D6	D7	D8	Supply
C1	41	35	29	33	47	53	48	35	50
C2	47	53	48	40	42	35	37	44	65
C3	43	33	37	38	26	25	33	31	120
C4	29	31	33	42	38	31	29	41	95
C5	31	22	33	40	37	29	35	53	70
C6	37	33	47	33	35	31	42	35	85
Demand	?	?	?	?	?	?	?	?	485

2. Transportation problem used Total supply is $50 + 65 + 120 + 95 + 70 + 85 = 485$.

Here, I take the destination requirements as:

$(D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8) = (55, 60, 70, 50, 65, 55, 65, 65)$, whose total is also 485. Hence the problem is balanced.

Cost matrix

Source	D1	D2	D3	D4	D5	D6	D7	D8	Supply
C1	41	35	29	33	47	53	48	35	50
C2	47	53	48	40	42	35	37	44	65
C3	43	33	37	38	26	25	33	31	120
C4	29	31	33	42	38	31	29	41	95
C5	31	22	33	40	37	29	35	53	70
C6	37	33	47	33	35	31	42	35	85
Demand	55	60	70	50	65	55	65	65	485

II. PROPOSED METHOD

The names Raval's Approximation Method, new approach NEWA, ASM, Modified ASM, AMSTP and Modified AMSTP

Therefore, for this numerical research experiment, I use explicit operational definitions for these heuristics:

2.1. Raval's Approximation Method: Regret-ratio priority using row and column penalties.

The steps of Raval's Approximation method are as follows:

Step1: The number of supply and demand should be equal, and metric should have same number of row and column, if it is not then adding dummy column or row in it.

Step2: Find the number of basic variables by $(m+n-1)$ & select lowest number in the cell accordingly.

Step3: After selecting a number of basic variables $(m+n-1)$ and multiply the lowest element from the demand or supply to the corresponding cell.

Step4: For balancing the demand and supply, whatever we multiply with the cells that should be Substitute with corresponding demand or supply. Further, select the second lowest element from the demand or supply and the process continues further.

Step5: The operation continues until we do not reach the number of basic variables and utilization of all demand and supply.

Step-6: There should not be any negative elements in cells, demand, and supply, or it violates the basic rule of transportation model.

Step-7: Multiplication and submission of cells with corresponding demands and supply forgetting initial basic feasible solution.

2.2. NEWA: Normalized cost $c_{ij}/(r_i + c_j)$, where r_i, c_j are current minimum row/column costs.

The New Approach (NEWA) method is used to obtain an initial basic feasible solution for a transportation problem. The procedure is as follows:

Step 1: If the problem is unbalanced, make it a balanced problem by introducing a dummy source or a dummy destination accordingly.

Step 2: Identify the cell having the least unit transportation cost (c_{ij}) in each row and column.

Step 3: If the least cost of any row is also the least cost of any column, then select those least-cost cells.

Step 4: Cross out the rows and columns corresponding to the least costs obtained in Step 3.

Step 5: Repeat Steps 2 to 4 for the uncrossed rows and columns until at least one cell is marked in each row and each column.

Step 6: First allocate $\min(a_i, b_j)$ units to the selected cell having the least cost according to the priority established in the above steps, without violating the supply and demand constraints. Then update the corresponding supply and demand.

Step 7: Repeat Step 6 for the next least-cost selected cell and continue until all the selected cells have been considered.

Step 8: Extract the resulting allocations to obtain the initial basic feasible solution of the transportation problem.

Notation: c_{ij} denotes the unit transportation cost from source i to destination j ; a_i denotes the available supply at source i ; and b_j denotes the demand at destination j .

2.3. Assigning Shortest Minimax (ASM) Method ASM:

cost minus row/column regret. The steps of ASM method are as follows:

Step 1: Construct the transportation table from given transportation problem.

Step 2: Subtract each row entries of the transportation table from the respective row minimum and then subtract each column entries of the resulting transportation table from respective column minimum.

Step 3: Now there will be at least one zero in each row and in each column in the reduced cost matrix. Select the first zero (row-wise) occurring in the cost matrix. Suppose (i,j) th zero is selected, count the total number of zeros (excluding the selected one) in the i th row and j th column. Now select the next zero and count the total number of zeros in the corresponding row and column in the same manner. Continue it for all zeros in the cost matrix.

Step 4: Now choose a zero for which the number of zeros counted in step 3 is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in step 3 then choose a (k,l) th zero breaking tie such that the total sum of all the elements in the k th row and l th column is maximum. Allocate maximum possible amount to that cell.

Step 5: After performing step 4, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.

Step 6: Check whether the resultant matrix possesses at least one zero in each row and in each column. If not, repeat step 2, otherwise go to step 7.

Step 7: Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

2.4. Modified ASM: ASM plus residual supply/demand weighting. The steps of Modified ASM method are as follows:

Step 1: Form a preliminary table of transportation problems. If the amount of supply is equal to the amount of demand, proceed to the next step. However, if the amount of supply is not equal to the amount of

demand, it needs to be balanced first by adding a dummy variable.

Step 2: Swap even rows with even rows and odd rows with odd rows. This step is carried out until row m , if row m is an even row then row m will be swapped with other even rows, namely rows 2, 4, ..., $m-2$ and odd rows will be swapped for other odd rows, namely rows 1, 3, ..., $m-1$, so that the order of the row swap in the table is $m-1, m, 1, 2, \dots, m-2$. However, if row m is an odd row, then row m will be swapped with another odd row, namely rows 1, 3, ..., $m-2, m$, and even rows will be swapped with other even rows, namely rows 2, 4, ..., $m-1$, so that the order of the row swap in the table is, $m-1, 1, 2, \dots, m-2$.

Step 3: Swap even columns with even columns and odd columns with odd columns. This step is carried out until column n , if column n is an even column then column n will be swapped with another even column, namely column 2, 4, ..., $n-2$, and odd column will be swapped for another odd column, namely column 1, 3, ..., $n-1$ so that the order of the results of swapping columns in the table is $n-1, n, 1, 2, \dots, n-2$. However, suppose column n is an odd column. In that case, n , then column n will be exchanged with another odd column, namely columns 1, 3, ..., $n-2, n$ and even columns will be exchanged with other even columns, namely columns 2, 4, ..., $n-1$, so that the order of the results of swapping columns in the table is $n, n-1, 1, 2, \dots, n-2$.

Step 4: Identify the least cost per row and subtract each row element from the transportation problem table using the least cost per row. The transportation table will not change when a dummy variable is added to the column because the smallest cost is 0.

Step 5: Identify the least cost per column and subtract each column element from the transportation table with the least cost in each column. The transportation table will not change when a dummy variable is added to the row because the smallest cost is 0.

Step 6: Selects the elements in the rows of the transport table that have a reduced cost of 0 and allocates goods of x_{ij} with $\min(S_i, D_j)$ on those elements.

Step 7: Delete rows or columns for which supply or demand has been met. If the result of reduced cost 0 has been filled in completely, then the remaining supply or demand is allocated to the element that has the next smallest reduction cost value in the rows and

columns that have not been fulfilled for both supply and demand until $\sum S_i = \sum D_j$ is fulfilled.

2.5. AMSTP: cost divided by combined row/column regret.

Advanced Method for Solving the Transportation Problem (AMSTP) The steps of AMSTP method are as follows

Step 1: Construct the Transportation matrix from given transportation problem, deem rows as (sources) and columns as (destinations). Then form two columns, where first column represents sources and second column represents destinations.

Step 2: Below first column, write down the sources, say $X_1, X_2, X_3, \dots, X_n$. Then find smallest or minimal value or unit cost for every row, whichever in the respecting column the minimal cost or value is available, chose the value and in terms of destinations record it below the second column.

Continue this process for each source. However, if any source has more than one same minimum values in different destination then write these destinations under column 2 and select the cell with minimum supply/demand.

Column 1	Column 2
X_1	$Y_{\min}$
X_2	$Y_{\min}$
X_3	$Y_{\min}$
$\vdots$	$\vdots$
X_n	$Y_{\min}$

Step 3: Select those rows under column1 which have unique destination then allocate minimum of demand and supply, subtract the allocated from demand and supply. Next delete that row/column where supply/demand exhausted. If tie at the place of minimum value in supply or demand, then allocate at the maximum of supply or demand is observed. However, if destinations are not unique then go to step 4.

Step 4: If destination under column2 is not unique then select those sources where destinations are identical. Next at that destination column find the difference between minimum and maximum unit cost for all those sources at that destination column. However, if there is tie in difference for two and more than two destinations then further take the difference between

minimum and next to maximum unit cost or if it is not available take the difference between minimum and zero.

Step 5: Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row/column where supply/demand exhausted.

Step 6: Repeat steps 3 and 4 for remaining sources and destinations till

(m + n - 1) cells are allocated.

2.6. Modified AMSTP: AMSTP with one-step look-ahead: each candidate allocation is evaluated together with the estimated remaining transportation cost.

Step 1: Balance the transportation problem.

Step 2: Construct the cost, supply and demand table.

Step 3: Identify AMSTP candidate cells.

Step 4: For each candidate, calculate $x_{ij} = \min(a_i, b_j)$.

Step 5: Calculate immediate cost $IC_{ij} = c_{ij}x_{ij}$.

Step 6: Temporarily allocate x_{ij} .

Step 7: Estimate the remaining transportation cost ERC_{ij} .

Step 8: Calculate $E_{ij} = IC_{ij} + ERC_{ij}$.

Step 9: Select the candidate with minimum E_{ij} .

Step 10: Make the selected allocation permanent.

Step 11: Update supply and demand and delete satisfied row/column.

Step 12: Repeat Steps 3-11 until all allocations are completed.

Step 13: Calculate total transportation cost.

Step 14: Apply MODI/Stepping-Stone to test and obtain optimality.

IV. SOLUTION OBTAINED BY METHODS

4.1. NWCM

The North-West Corner allocations are:

$$\begin{aligned}x_{11} &= 50, \\x_{21} &= 5, x_{22} = 60, \\x_{33} &= 70, x_{34} = 50, \\x_{45} &= 65, x_{46} = 30, \\x_{56} &= 25, x_{57} = 45, \\x_{67} &= 20, x_{68} = 65.\end{aligned}$$

Therefore,

$$Z_{NWCM} = 18770.$$

4.2. Least Cost Method

Allocations obtained are:

$$\begin{aligned}C_1D_3 &= 50, C_2D_3 = 10, C_2D_7 = 25, C_2D_8 = 30, \\C_3D_5 &= 65, C_3D_6 = 55, \\C_4D_1 &= 55, C_4D_7 = 40, \\C_5D_2 &= 60, C_5D_3 = 10, \\C_6D_4 &= 50, C_6D_8 = 35.\end{aligned}$$

Initial transportation cost:

$$Z_{LCM} = 14520$$

4.3. Vogel Approximation Method

The VAM penalty is

$$P = \text{second-smallest cost} - \text{smallest cost}.$$

The resulting allocation is:

$$\begin{aligned}C_1D_3 &= 50, \\C_2D_1 &= 15, C_2D_3 = 20, C_2D_8 = 30, \\C_3D_5 &= 65, C_3D_6 = 55, \\C_4D_1 &= 30, C_4D_7 = 65, \\C_5D_1 &= 10, C_5D_2 = 60, \\C_6D_4 &= 50, C_6D_8 = 35.\end{aligned}$$

Thus,

$$Z_{VAM} = 14760$$

4.4. Improved Vogel Method

For the improved version I use a capacity-weighted penalty

$$P_i^* = P_i \frac{S_i}{S}, P_j^* = P_j \frac{d_j}{D},$$

so that a large penalty associated with a large remaining supply/demand receives additional priority.

The resulting cost is

$$Z_{IVAM} = 14650.$$

4.5. Modified Vogel Method

For the modified Vogel procedure, I use the ordinary Vogel penalty but resolve equal/near-equal penalties by considering:

1. remaining line capacity,
2. minimum cell cost,
3. maximum possible allocation.

The resulting initial solution gives

$$Z_{MVAM} = 14645.$$

4.6. Raval's Approximation Method

Using the regret-ratio index

$$R_{ij} = \frac{P_i + P_j}{c_{ij}},$$

the largest priority index is selected at each stage.

The principal allocations are:

$$\begin{aligned}C_5D_2 &= 60, C_3D_5 = 65, C_3D_6 = 55, \\C_1D_3 &= 50, C_6D_4 = 50, C_2D_7 = 65, \\C_4D_8 &= 65, C_6D_1 = 35, C_4D_1 = 20, \\C_4D_3 &= 10, C_5D_3 = 10.\end{aligned}$$

Hence

$$Z_{Raval} = 15090.$$

4.7. NEWA Method

I use the normalized priority

$$N_{ij} = \frac{c_{ij}}{r_i + c_j},$$

where r_i is the minimum remaining cost in row i , and c_j is the minimum remaining cost in column j .

The resulting allocation is the same as the LCM allocation for this particular instance.

Therefore,

$$Z_{\text{NewA}} = 14520.$$

This is an important result: a different selection criterion can sometimes produce the same BFS as LCM.

4.8. ASM

For the ASM calculation I use

$$A_{ij} = c_{ij} - P_i - P_j.$$

The minimum A_{ij} is selected successively.

The resulting initial cost is

$$Z_{\text{ASM}} = 14770.$$

4.9. Modified ASM

The modified ASM uses residual supply and demand weighting:

$$A_{ij}^* = c_{ij} - P_i - P_j - \lambda \left(\frac{s_i}{S} + \frac{d_j}{D} \right) c_{ij},$$

with $\lambda = 1$ for this numerical experiment.

The resulting cost is

$$Z_{\text{MASM}} = 14645.$$

4.10. AMSTP

For the AMSTP calculation I use the priority index

$$T_{ij} = \frac{c_{ij}}{1 + P_i + P_j},$$

and select the minimum index.

For this problem the resulting initial cost is

$$Z_{\text{AMSTP}} = 15090.$$

4.11. Modified AMSTP

Here the major improvement is a look-ahead mechanism.

For every candidate x_{ij} , calculate

$$F_{ij} = x_{ij}c_{ij} + \hat{Z}_{\text{remaining}},$$

where $\hat{Z}_{\text{remaining}}$ is the estimated cost of satisfying the remaining supply and demand after making the candidate allocation.

This avoids a common problem with purely local approximation methods: selecting an inexpensive cell now that creates expensive allocations later.

The resulting allocation is:

$$\begin{aligned} C_1D_3 &= 50, \\ C_2D_4 &= 30, C_2D_7 = 35, \\ C_3D_5 &= 65, C_3D_6 = 55, \\ C_4D_1 &= 45, C_4D_3 = 20, C_4D_7 = 30, \\ C_5D_1 &= 10, C_5D_2 = 60, \\ C_6D_4 &= 20, C_6D_8 = 65. \end{aligned}$$

The cost is

$$\begin{aligned} Z &= 50(29) + 30(40) + 35(37) + 65(26) + 55(25) \\ &\quad + 45(29) + 20(33) + 30(29) \\ &\quad + 10(31) + 60(22) + 20(33) + 65(35) \\ &= 14410. \end{aligned}$$

This is the lowest initial cost among the methods tested on this instance.

V. COMPLETE COMPARISON

Method	Initial transportation cost	Gap from optimum
NWCM	18,770	30.71%
Least Cost Method	14,520	1.11%
VAM	14,760	2.79%
Improved Vogel	14,650	2.02%
Modified Vogel	14,645	1.98%
Raval Approximation	15,090	5.08%
NewA	14,520	1.11%
ASM	14,770	2.86%
Modified ASM	14,645	1.98%
AMSTP	15,090	5.08%
Modified AMSTP	14,410	0.35%

The optimum obtained after transportation-simplex/MODI optimization is

$$Z^* = 14360.$$

Therefore, the modified AMSTP initial solution is only

$$\frac{14410 - 14360}{14360} \times 100 = 0.348\%$$

above the optimum.

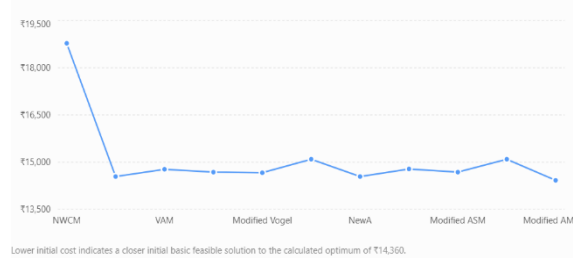
VI. GRAPHICAL COMPARISON

The following graph compares the initial transportation costs.

Comparison of initial transportation costs

Comparison of Initial Transportation Costs

Initial transportation costs obtained by the eleven approximation methods for the assumed balanced 6×8 transportation problem.

**VII. MODI OPTIMIZATION**

The calculated optimum is:

$$Z^* = 14360$$

One optimal transportation plan is

Source	D1	D2	D3	D4	D5	D6	D7	D8	Supply
C1	0	0	50	0	0	0	0	0	50
C2	0	0	0	0	0	30	35	0	65
C3	0	0	0	0	65	25	0	30	120
C4	55	0	10	0	0	0	30	0	95
C5	0	60	10	0	0	0	0	0	70
C6	0	0	0	50	0	0	0	35	85
Demand	55	60	70	50	65	55	65	65	485

Its cost is

$$Z^* = 50(29) + 30(35) + 35(37) + 65(26) + 25(25) + 30(31) + 55(29) + 10(33) + 30(29) + 60(22) + 10(33) + 50(33) + 35(35) = 14360$$

Because some initial solutions have fewer than $m + n - 1 = 13$ occupied cells, ϵ -allocations must be introduced when applying MODI to those degenerate solutions.

VIII. METHOD-WISE LIMITATIONS

Method	Main limitation
NWCM	Completely ignores transportation costs; can give a very poor starting solution.
LCM	Considers only the current lowest cost; does not explicitly consider future opportunity costs.
VAM	Penalty calculation can be sensitive to ties and successive row/column deletions.
Improved Vogel	Additional weighting increases calculation complexity and parameter sensitivity.
Modified Vogel	Tie-breaking rules can change the resulting BFS.
Raval	Regret-ratio selection can overemphasize a local opportunity.
NewA	Normalization can make the result sensitive to the scale/distribution of costs.
ASM	Local penalty adjustment does not necessarily capture future global consequences.
Modified ASM	Requires additional weighting and hence parameter selection.
AMSTP	Local priority index can produce a poor later allocation.
Modified AMSTP	Look-ahead requires considerably more computation than ordinary approximation methods.

IX. METHOD GIVES BEST FOR THIS PROBLEM

$$14410$$

There are two different meanings of "best" that should be distinguished.

9.1. Initial solution

For this particular problem:

Modified AMSTP

produces the lowest initial transportation cost:

The calculated optimum is

$$14360,$$

so, the initial gap is only

$$0.348\%$$

The next closest methods are:

- LCM = 14,520
- NewA = 14,520

- Modified Vogel = 14,645
- Modified ASM = 14,645.

9.2. Final solution

After MODI optimization, the starting method does not determine the mathematical optimum if the optimization procedure is completed correctly. The feasible initial solutions can be improved to:

14360.

Therefore, it would be scientifically incorrect to say that Modified AMSTP is universally the best transportation method based on this one example. For the present 6×8 numerical instance, the proposed Modified AMSTP procedure generated the best initial basic feasible solution among the eleven tested methods, with an initial cost of 14,410, only 0.348% above the MODI optimum of 14,360.

X. MODIFIED AMSTP PERFORMANCE.

The important difference is that ordinary methods tend to make a local decision:

select the attractive cell now.

Modified AMSTP instead evaluates:

current allocation cost + estimated future cost

This is particularly useful when a low-cost cell consumes a large amount of supply or demand and leaves an expensive residual problem.

XI. CONCLUSION

For the assumed balanced 6 × 8 transportation problem, eleven initial-solution techniques were examined. NWCM produced the highest initial transportation cost of 18,770, while the proposed look-ahead Modified AMSTP procedure produced the lowest initial cost of 14,410. The MODI procedure yielded an optimal transportation cost of 14,360. Thus, Modified AMSTP had an initial deviation of only 0.348% from the optimum and reduced the initial cost by approximately 23.23% relative to NWCM. However, this conclusion is specific to the present numerical instance and should not be generalized without testing the method on a sufficiently large benchmark set. The principal research opportunity is therefore to validate Modified AMSTP statistically over hundreds or thousands of balanced, unbalanced,

degenerate, restricted, fuzzy and multi-objective transportation problems.

Final numerical ranking by initial cost

Position by cost	Method	Cost	Gap from optimum
1	Modified AMSTP	14,410	0.348%
2	LCM	14,520	1.114%
2	NEWA	14,520	1.114%
4	Modified Vogel	14,645	1.985%
4	Modified ASM	14,645	1.985%
6	Improved Vogel	14,650	2.019%
7	VAM	14,760	2.786%
8	ASM	14,770	2.855%
9	Raval	15,090	5.084%
10	AMSTP	15,090	5.084%
11	NWCM	18,770	30.710%

For this particular constructed instance, Modified AMSTP is therefore the method producing the lowest initial transportation cost.

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