Linear Programming Problem for Agricultural Crop Planning: A Literature-Based Analytical Study with Re-Computation and Critical Audit of Two Published Models

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***Abstract*—Crop planning, the decision of how much land to commit to each crop under limited land, capital, inputs and labour, is a textbook application of linear programming (LP), yet published applications differ widely in rigour and reporting quality. This literature-based analytical study traces the development of LP in farm planning and examines two recent applications in depth: a seven-crop cash-crop model for a small-scale farm in Limpopo, South Africa (Muzhinji & Ndou, 2022) and an eight-crop mixed-cropping farm model for Rajasthan, India (Bhatia & Rana, 2020). For each paper we extract the real-life context, objective function, constraints, assumptions, solution method and reported results, and then re-solve the published models independently in Python (SciPy/HiGHS). The re-computation reproduces the first paper's optimum (gross income R740,780; net income R612,112) but exposes reporting inconsistencies: a headline improvement of 37.8% (38.7% in the text) against 63.2% computed, two swapped rows in the sensitivity table, and a rounded plan that exceeds the seed budget. It also shows that about 81% of the improvement over the farmer's plan comes from cultivating previously idle land rather than from re-mixing crops. For the second paper, the reported optimum (100,375) is reproduced only when one land cap is omitted; the fully constrained model gives 3,102 and none of its resource limits binds. Extending the first model, we find that a net-income objective gives the same plan, that a 30% diversification cap costs only 0.5% of net income, and that in a 5,000-draw price-uncertainty simulation the fixed plan stays within 1.1% of the re-optimised value on average. We conclude with reporting, validation and extension recommendations toward stochastic, multi-objective and rotation-based models.**

***Index Terms*—linear programming; crop planning; operations research; resource allocation; sensitivity analysis; shadow prices; farm management**

I. INTRODUCTION

1.1. Operations research and the crop-planning problem

Operations Research (OR) is the discipline that applies mathematical models, optimisation and analytics to decisions about the allocation of scarce resources. Its core workflow is to describe a real decision in terms of decision variables, an objective to be maximised or minimised, and constraints that capture physical and economic limits; to solve the resulting model with an algorithm; and to interpret the solution for decision makers. Linear programming (LP), formalised by Dantzig and developed in his classic text (Dantzig, 1963), is the most widely taught OR technique because many allocation problems can be written with a linear objective and linear constraints, and because efficient algorithms such as the simplex method solve them reliably.

Agriculture is a natural home for LP. A farmer or a regional planner must decide how much land to give to each crop while land, capital, seed, fertiliser, agro-chemicals, water and labour are all limited, while markets absorb only so much produce, and while some crops are agronomically or commercially compulsory. Early agricultural treatments of LP (Heady & Candler, 1958) and later farm-planning texts (Dent et al., 1986) established the "crop mix" or "product mix" model as the standard formulation. Glen (1987) provided an OR-oriented survey of farm-planning models and observed that although the approach is characteristic of operations research, the profession's direct involvement in agricultural planning had been limited, with much of the published work coming from agricultural economics. Weintraub and Romero

(2006) later reviewed OR models for the management of agricultural and forestry resources, covering planning problems at farm and regional-sector level and the difficulties LP models face in practice.

1.2. Problem statement and rationale

A large and growing body of applied papers formulates LP crop-planning models for individual farms and regions. Many are short case studies that report an "optimal crop mix" and a percentage gain over current practice. For students, practitioners and policy users, two questions follow. First, are the models formulated and documented well enough to be reproduced? Second, do the reported gains mean what the abstracts say they mean? Because crop-planning LPs are small, published tables of coefficients allow the model to be re-solved in seconds, which makes a reproducibility audit cheap and informative. Reproduction is also a good way to learn how the model works: which resources bind, what an extra hectare or rupee is worth, and how fragile the plan is to price changes.

1.3. Objectives and research questions

This study is a literature-based analytical project in operations research. Its objective is to understand how LPP has been applied to agricultural crop planning by analysing published research, to explain the modelling choices and solution methods used, and to critically reflect on formulation, assumptions and results.

Three research questions guide the work:

- RQ1. How have published LPP studies formulated the crop-planning problem (objective, constraints, assumptions, problem type) and solved it?
- RQ2. Can the headline results of two recent, openly available applications be reproduced from their published data, and what does an independent re-computation reveal?
- RQ3. What additional managerial insight (shadow prices, diversification, price risk) can be obtained by extending the model, and what research gaps does this expose?

1.4. Contributions and structure

The paper makes four contributions. (i) It consolidates the literature on LP-based crop planning into a compact synthesis and gap analysis (Section 2). (ii) It documents two published models in a common notation, including full parameter tables (Sections 3 to 5). (iii) It independently re-solves both models and reports a structured audit of the discrepancies, an exercise rarely reported for applied LP papers. (iv) It extends the first model with net-income, diversification, ranging and price-uncertainty analyses that the original paper did not undertake (Section 6). Section 7 discusses implications, strengths, limitations and extensions, and Section 8 concludes.

II. LITERATURE REVIEW

2.1. Linear programming in farm planning

LP entered agricultural economics in the 1950s as a tool for farm-management advice, and farm-planning texts and surveys present it as the standard starting point for more elaborate models (Dent et al., 1986; Glen, 1987). The standard farm LP has activities (crops or livestock enterprises) as variables, resource constraints on land, labour, capital and inputs, and an objective of gross margin or net income. Its strengths are transparency, computational ease and, through duality, shadow prices that value each scarce resource. Its main weaknesses are the assumptions built into LP itself: proportionality (constant returns per hectare), additivity (no interaction between crops), divisibility (any fractional hectare is allowed) and certainty (prices, yields and resource limits are known). Weintraub and Romero (2006) note that LP models at farm level present difficulties and discuss zero-one and multi-period variants, which motivates the extensions reviewed below.

2.2. Deterministic crop-mix applications

Applied LP crop-mix studies are numerous. Sarker et al. (1997) determined an optimum crop mix for crop cultivation in Bangladesh, and Sarker and Jaim (2014) built representative-farm LP models from survey data for a coastal area of Bangladesh, reporting that existing resource allocations were sub-optimal and that reallocation could raise productivity and income. Muzhinji and Ndou (2022; analysed in Section 4) review Zimbabwean studies in which LP raised income by 35% (Majeke, 2013, as cited in Muzhinji & Ndou, 2022) and the profit margin by 76% (Buzuzi & Buzuzi, 2018, as cited in Muzhinji & Ndou, 2022) relative to traditional plans, and note that Patel et al. (2017, as cited in Muzhinji & Ndou, 2022) applied LP to agricultural land allocation in Patan district, India.

A common pattern across these studies is a single-season, single-objective model with a handful of resources, solved by the simplex method with a spreadsheet, MATLAB or LINGO, and validated only by comparison with farmers' current practice.

2.3. Extensions: uncertainty, multiple objectives, rotations and water

Several streams extend the basic model. For risk, Rădulescu and Rădulescu (2012) proposed a portfolio-selection decision support tool for crop planning under risk, and Kumari et al. (2014, as cited in Muzhinji & Ndou, 2022) used fuzzy multi-objective LP for allocating land to vegetable crops under uncertain profits. For multiple criteria, Sarker and Quaddus (2002) modelled a nationwide crop-planning problem with a multiple-criteria decision-making tool, while Adekanmbi and Olugbara (2015) and Adekanmbi et al. (2014) applied generalised differential evolution, a metaheuristic, to multi-objective crop-mix planning. Where integrality matters, Filippi et al. (2017) formulated mixed-integer linear programming models for optimal crop selection, and Jolayemi and Olaomi (1995) proposed a mathematical programming procedure for selecting crops in mixed-cropping schemes. Rotation effects were addressed by Castellazzi et al. (2008), who proposed a systematic representation of crop rotations, and Dury et al. (2012) reviewed models supporting cropping-plan and rotation decisions. Water allocation has been modelled by Rao et al. (1990) for multicrop seasonal and intraseasonal irrigation, by Shangguan et al. (2002) for regional allocation under deficit irrigation, and by Li and Guo (2014) with a multi-objective model under multiple uncertainties. Decision-support and simulation approaches include an integrated expert system for optimal crop planning (Nevo et al., 1994) and agent-based spatial models (Berger, 2001).

Table 1 *Representative Studies on Crop Planning and Their Relevance to This Study*

Study	Technique / focus	Setting	Relevance to this study
Glen (1987)	Survey of farm-planning models (crop and livestock)	General	OR-oriented framing of farm planning
Sarker et al. (1997)	Optimum crop mix model	Bangladesh	Deterministic crop-mix LP benchmark
Sarker & Jaim (2014)	Representative-farm LP from survey data	Coastal Bangladesh	Survey-based LP with reallocation gains
Sarker & Quaddus (2002)	Multiple-criteria decision making for crop planning	Nationwide	Multi-criteria alternative to single-objective LP
Filippi et al. (2017)	Mixed-integer LP for crop selection	General	Integer extension of crop selection
Adekanmbi & Olugbara (2015)	Multi-objective generalised differential evolution	General	Metaheuristic alternative to LP
Rădulescu & Rădulescu (2012)	Portfolio selection for crop planning under risk	General	Risk-based alternative to LP
Castellazzi et al. (2008)	Systematic representation of crop rotations	General	Rotation effects absent from audited models
Muzhinji & Ndou (2022)	Single-objective LP, seven cash crops	Limpopo, South Africa	Audited in Section 4
Bhatia & Rana (2020)	Multi-objective formulation, solved as LP (LINGO)	Rajasthan, India	Audited in Section 5

Note. Entries summarise the titles and abstracts of the cited works; "Relevance" is the present authors' characterisation.

2.4. Research gap

Three gaps emerge. First, although many papers report a percentage improvement over farmers' plans, few

state whether the comparison plan uses the same land, which can overstate the value of optimisation itself. Second, published sensitivity analyses are usually limited to one-at-a-time ±10% changes in coefficients and right-hand sides, without shadow-price interpretation, and are rarely checked for labelling errors.

Third, deterministic LP papers rarely quantify how much is lost by ignoring price risk or by omitting diversification requirements. This paper addresses these gaps through re-computation and extended analysis of two openly available models.

III. METHODOLOGY

3.1. Selection of research articles

Two papers were selected from open-access, peer-reviewed journals indexed in repositories such as RePEc/IDEAS and AgEcon Search and a university digital common. Selection criteria were: (a) LP is the central technique; (b) the problem is a real farm or regional crop-planning decision based on collected data; (c) the full objective function, constraints and coefficient tables are published, so that the model can be re-solved; (d) the studies differ in geography, problem scale and software; and (e) recency (2020 to 2022). Table 2 summarises the selected papers.

Table 2 *Overview of the Two Selected Papers*

Feature	Paper 1: Muzhinji & Ndou (2022)	Paper 2: Bhatia & Rana (2020)
Journal	African Journal of Food, Agriculture, Nutrition and Development, 22(7)	Applications and Applied Mathematics: An International Journal, 15(2)
Region and domain	Vhembe District, Limpopo, South Africa; vegetable cash-crop farming	Districts of Rajasthan, India; arid, mixed-crop and livestock farming
Decision	Hectares of seven crops on a 7-ha farm	Hectares of eight crops in a mixed-cropping system
Objective	Maximise gross income (stated aim: net income)	Three objectives formulated (revenue, cost, water); single revenue objective solved
Problem type	Deterministic, single-objective, continuous LP; 7 variables, 8 resource constraints	Deterministic LP; 8 variables, 8 resource and 8 bound constraints
Solution method	Two-phase simplex method	LP solved with LINGO 18.0; sensitivity report
Software	MATLAB	LINGO 18.0
Data	Farmer's business plan, 2019–2021	Farmer interviews and survey, districts of Rajasthan
Validation	Comparison with farmer's plan; ±10% sensitivity	Sensitivity (ranging) report

Note. Information compiled from the two papers.

3.2. Analytical framework

Each paper was analysed against five questions that mirror the project brief: the real-life context (problem, domain, why OR is suitable); the modelling (objective, constraints, problem type, mathematical model, assumptions); the solution methodology (technique, software, validation); the key findings and their implications; and a critical reflection on strengths, limitations and possible extensions. The analysis of Paper 1 is in Section 4, of Paper 2 in Section 5, and the extension study in Section 6.

3.3. A unified LP model for crop planning

To compare the papers, both are written in one notation. Let $j = 1,\ldots,n$ index crops and $i = 1,\ldots,m$ index resources. Let x_j be the area (ha) planted to crop j, r_j the gross income per hectare, a_{ij} the amount of resource i used per hectare of crop j, b_i the availability of resource i, and l_j and u_j the minimum and maximum area for crop j. The generic crop-planning LP is:

$$\text{Maximise } Z = \Sigma_j r_j x_j \quad (1)$$

$$\text{subject to } \Sigma_j a_{ij} x_j \le b_i, \quad i = 1,\ldots,m \quad (2)$$

$$\Sigma_j x_j \le A \text{ (land)}, \quad l_j \le x_j \le u_j, \quad x_j \ge 0 \quad (3)$$

Replacing r_j by the gross margin $g_j = r_j - \Sigma_i c_{ij}$ (where c_{ij} is the cost of input i per hectare) turns the objective

into net income. By LP duality, each binding constraint *i* has a shadow price $y_i \geq 0$, the increase in Z per extra unit of b_i, valid within a stated range; a variable fixed at a bound has a reduced cost showing how much the objective would change per extra hectare forced above its minimum. These quantities are central to the managerial interpretation in later sections.

3.4 Re-computation and audit protocol

All published coefficient tables were transcribed and the models were re-solved in Python 3 with SciPy 1.17 (Virtanen et al., 2020) using the HiGHS solver (Huangfu & Hall, 2018). The audit proceeded in five steps: (1) solve the model exactly as printed; (2) test whether the published solution is feasible and whether its objective value matches; (3) compare the printed constraints with the printed data tables for internal consistency; (4) extract shadow prices, reduced costs and slack; and (5) repeat the authors' sensitivity experiments and compare results. Differences are reported as findings about the publication, not as criticism of the authors' intent, and are subject to the caveat that we only observe what is printed. The extended analyses in Section 6 use the same solver and the Paper 1 data.

IV. PAPER 1: CASH-CROP COMBINATION FOR A SMALL-SCALE FARM IN LIMPOPO

4.1 Real-life context

Muzhinji and Ndou (2022) study a small-scale vegetable and cash-crop farmer in Vhembe District, Limpopo Province, South Africa, who farms 7 ha and grows green maize ("green mealies"), cabbages, tomatoes, mustard, spinach, butternut and sweet potatoes for supermarkets, restaurants and local markets. The farmer had allocated land using experience, trial and error and comparison with neighbours, a practice the authors note can leave resources poorly allocated. For the coming season the farmer wished to commit R140,000 of capital and needed to decide how many hectares each crop should receive. The domain is agriculture (small-scale commercial horticulture), and the decision is a classic product-mix problem.

OR is suitable because the decision involves several scarce resources (land, seed, fertiliser, chemicals, fungicides, labour, other costs and capital), several competing activities with different returns per hectare and different resource intensities, and a clear economic objective. An LP formulation can evaluate millions of possible allocations implicitly, guarantee optimality, and report the economic value of each limiting resource, none of which is possible by experience or trial and error.

4.2 Mathematical model

Let $x_1,\ldots,x_7$ denote hectares of green maize, cabbages, tomatoes, mustard, spinach, butternut and sweet potatoes. Table 3 gives the per-hectare data used in the model (from the authors' planning table for 2020–2021) together with the resource limits. The model published by the authors is:

$$\text{Max } Z = 100{,}000x_1 + 140{,}000x_2 + 160{,}000x_3 + 60{,}000x_4 + 44{,}000x_5 + 44{,}000x_6 + 50{,}000x_7 \quad (4)$$

subject to:

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 7$ (land)

$900x_1 + 9{,}000x_2 + 8{,}000x_3 + 4{,}000x_4 + 4{,}500x_5 + 1{,}000x_6 + 4{,}500x_7 \leq 40{,}000$ (seed)

$200x_1 + 200x_2 + 300x_3 \leq 1{,}000$ (chemicals); similarly for fertiliser, fungicides,

labour, other costs and capital (Table 3);

$x_1 \geq 1$, $x_2 \geq 1$, $x_3 \geq 0.8$, $x_4 \geq 0.8$, $x_5, x_6, x_7 \geq 0.4$; $x_5, x_6, x_7 \leq 0.8$; all $x_j \geq 0$.

Table 3 *Per-Hectare Data and Resource Limits in Paper 1 (South African Rand)*

Crop	Gross income	Seed	Fert.	Chem.	Fung.	Labour	Other	Direct cost	Margin
Green maize	100,000	900	4,000	200	500	4,000	5,000	14,600	85,400
Cabbage	140,000	9,000	4,000	200	200	4,000	4,000	21,400	118,600
Tomato	160,000	8,000	4,000	300	3,000	4,000	4,000	23,300	136,700
Mustard	60,000	4,000	3,000	0	300	4,000	3,000	14,300	45,700
Spinach	44,000	4,500	5,000	0	400	4,000	3,000	16,900	27,100

Crop	Gross income	Seed	Fert.	Chem.	Fung.	Labour	Other	Direct cost	Margin
Butternut	44,000	1,000	4,500	0	1,000	4,000	2,000	12,500	31,500
Sweet potato	50,000	4,500	5,000	0	700	4,000	5,000	19,200	30,800
Limit	–	40,000	30,000	1,000	7,000	30,000	30,000	140,000	–

Note. Source: Muzhinji and Ndou (2022), planning table for 2020–2021. Direct cost and margin are computed by the present authors as the sum of the six input rows and gross income minus direct cost. The land limit is 7 ha. Fert. = fertiliser; Chem. = chemicals; Fung. = fungicides.

Problem type, objective and constraints

This is a deterministic, single-objective, continuous LP with 7 decision variables, 8 resource inequalities (land, seed, fertiliser, chemicals, fungicides, labour, other costs, capital), 7 lower-bound and 3 upper-bound constraints. The objective maximises gross income (Equation 4), while the title and discussion refer to net income; because input costs are limited by the resource constraints, maximising gross income under those limits is a reasonable proxy, an issue we test in Section 6.1. The lower bounds reflect customer demand and the farmer's requirement to produce all crops, and the upper bounds on the three minor crops reflect the farmer's decision to allocate between 0.4 and 0.8 ha to them.

Assumptions

The authors state four assumptions: (i) crop prices and yields per hectare are fixed during the planning period; (ii) resource availability is fixed; (iii) the land is equally suitable for all crops; and (iv) crop combination depends on crop types only, not cultivation methods. Implicitly the model also assumes proportionality (resource use scales linearly with area), additivity (no crop interactions or rotation effects), divisibility, a single season, and that all output is sold at the assumed income.

4.3 Solution methodology

The problem is converted to standard form by adding slack variables to ≤ constraints and subtracting surplus variables from ≥ constraints. The authors solve it with the two-phase simplex method in MATLAB. Validation is by (a) comparing the LP plan with the farmer's previous allocation and (b) a sensitivity analysis in which each crop's income coefficient and each resource limit is changed by ±10% and the effect on optimal income is classified as low, medium or none.

4.4 Reported results

The reported optimum allocates 1.16 ha to green maize, 2.64 ha to cabbages, 0.8 ha to tomatoes, 1.2 ha to mustard and 0.4 ha each to spinach, butternut and sweet potatoes, giving gross income of R740,800, expenditure of R128,672 and net income of R612,128. The farmer's plan used 5 ha (2 ha of maize and 0.5 ha of each other crop) and realised gross income of R449,000, expenditure of R74,000 and net income of R375,000. The authors conclude that the LP increased the "profit margin" by 37.8% (abstract and conclusion) or 38.7% (results text) and that cabbages and maize are more profitable than spinach and sweet potatoes. Sensitivity analysis shows low sensitivity to most coefficients, with cabbage income and the chemicals limit showing medium sensitivity.

4.5 Independent re-computation and audit

Solving the model exactly as printed returns Z = R740,780.25 with x_1 = 1.1605 ha and x_2 = 2.6395 ha, the other variables being as reported. The published plan (1.16 and 2.64 ha) is therefore the exact optimum rounded to two decimals, which raises the computed gross income to R740,800 and, because seed is binding, uses R40,004 of seed against a R40,000 limit. Table 4 compares the plans, and Figure 1 shows the land allocation.

Table 4 *Farmer's Plan, Published LP Plan and Exact LP Optimum in Paper 1*

Item	Farmer's plan	Published LP	Exact LP	Share of land (exact)
Green maize (ha)	2.00	1.16	1.1605	16.6%
Cabbage (ha)	0.50	2.64	2.6395	37.7%

Item	Farmer's plan	Published LP	Exact LP	Share of land (exact)
Tomato (ha)	0.50	0.80	0.8000	11.4%
Mustard (ha)	0.50	1.20	1.2000	17.1%
Spinach (ha)	0.50	0.40	0.4000	5.7%
Butternut (ha)	0.50	0.40	0.4000	5.7%
Sweet potato (ha)	0.50	0.40	0.4000	5.7%
Land used (ha)	5.00	7.00	7.00	100%
Gross income (R)	449,000	740,800	740,780	
Expenditure (R)	74,000	128,672	128,669	
Net income (R)	375,000	612,128	612,112	

Note. Farmer's plan and published LP figures from Muzhinji and Ndou (2022); exact LP values re-computed by the present authors with HiGHS.

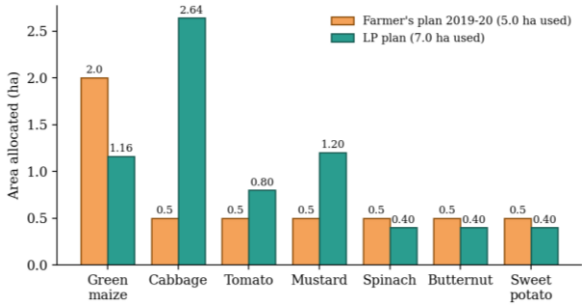

Figure 1 Land Allocation: Farmer's Plan versus LP Plan in Paper 1

Note. Areas in hectares. The farmer's plan leaves 2 ha idle; the LP plan uses all 7 ha and shifts land toward cabbages and mustard. Data: Muzhinji and Ndou (2022). The audit produced seven findings (Table 5). Three are most consequential for interpretatio

n. First, the headline improvement is misreported. The net income rises from R375,000 to R612,128, an increase of 63.2% (gross income rises 65.0%). The figure of 38.7% in the paper equals the gain (R237,128) divided by the *optimal* net income, a different statistic from "increase over the base", and the abstract's 37.8% appears to be a transcription variant of it. Second, the sensitivity table has two swapped rows: re-solving the model with the seed limit reduced by 10% gives a 2.67% fall in gross income and a 0.87% rise when it is increased by 10%, exactly the numbers printed in the paper's "fertiliser" row, while the fertiliser results (a 0.99% fall and no gain) match the paper's "seed" row. Seed is therefore a binding, medium-sensitivity resource, not a "low" or "none" one as the labels imply. Third, the comparison with the farmer's plan confounds two effects: the farmer left 2 ha unplanted, whereas the LP uses all 7 ha, so much of the gain stems from land use, not crop choice (Section 6.3).

Table 5 *Audit Findings for Paper 1*

#	Aspect	As published	Independent re-computation	Implication
1	Headline gain	37.8% (abstract), 38.7% (text)	Net +63.2%; gross +65.0%; 38.7% equals gain ÷ optimal net	Gain understated and statistic mislabelled
2	Rounded plan	1.16 ha maize, 2.64 ha cabbage	Exact: 1.1605, 2.6395 ha; published plan uses R40,004 seed (limit R40,000)	Published plan infeasible by R4; trivial but avoidable
3	Coefficient consistency	Constraints for fertiliser, other costs and capital use values differing from planning table	Spinach fertiliser 4,500 vs 5,000; mustard/spinach "other" 1,500 vs 3,000; capital tomato 29,300 vs 23,300, sweet potato 19,500 vs 19,200	Constraints are slack, so optimum unchanged (verified); reproducibility impaired
4	Sensitivity table	Seed: −0.99%, 0%; Fertiliser: −2.67%, +0.86%	Seed: −2.67%, +0.87%; Fertiliser: −0.99%, 0%	Rows swapped; seed is binding

#	Aspect	As published	Independent re-computation	Implication
5	Objective	Title/text: net income; model: gross income	Same plan under net objective (Section 6.1)	Benign here; not guaranteed in general
6	Comparison base	Farmer's plan uses 5 of 7 ha	About 81% of gain from idle land	Optimisation benefit overstated
7	Text versus table	Text: cabbages, tomatoes, butternut, sweet potato each "1 ha" for incomes	Table 1 shows 0.5 ha each	Baseline income per hectare ambiguous

Note. Findings 1 to 4 and 6 are based on numerical re-computation; findings 5 and 7 are based on reading the paper. Percentages are of gross income unless stated.

Resource values and binding constraints

Table 6 reports resource use at the exact optimum and the shadow prices from the dual solution. Land, seed and chemicals bind; all other resources have slack. One additional hectare of land is worth R40,247 of gross income (R29,305 of net income), each extra rand of seed budget R4.94 of gross income and each extra rand of chemicals budget R276.54. The chemicals result is striking: tomatoes, the highest-income crop (R160,000 per ha), are held at their 0.8 ha minimum because they consume 300 rand of chemicals per hectare against a 1,000-rand limit; their reduced cost is −R2,716 per extra hectare. Similarly, the 0.4 ha floors on spinach and sweet potatoes cost the farm R18,469 and R12,469 of gross income per hectare respectively relative to the best alternative use of land and inputs. These interpretations are absent from the original paper but follow directly from its own model.

Table 6 *Resource Use and Shadow Prices at the Exact Optimum (Paper 1)*

Resource	Used	Limit	Slack	Shadow price (gross)	Shadow price (net)
Land (ha)	7.00	7.00	0	R40,247 per ha	R29,305 per ha
Seed (R)	40,000	40,000	0	R4.94 per R	R4.10 per R
Chemicals (R)	1,000	1,000	0	R276.54 per R	R262.03 per R
Fertiliser (R)	27,800	30,000	2,200	0	0
Fungicides (R)	4,708	7,000	2,292	0	0
Labour (R)	28,000	30,000	2,000	0	0
Other costs (R)	27,160	30,000	2,840	0	0
Capital (R)	128,669	140,000	11,331	0	0

Note. Computed by the present authors using the planning-table coefficients (Table 3). Shadow prices are the change in the objective per unit increase in the limit and hold only within the range over which the optimal basis is unchanged.

V. PAPER 2: INTEGRATED FARM MODEL FOR RAJASTHAN

5.1 Real-life context

Bhatia and Rana (2020) analyse cropping practice in districts of Rajasthan, an arid and semi-arid Indian state where most cultivation is rain-fed and water is the dominant constraint. Farmer interviews revealed that crops with higher economic benefit (for example tomatoes and cucumber) were not cultivated because of high input cost and short shelf life, that small farmers owning 1 to 7 bighas commonly practise mixed cropping, and that integrating livestock (goats, cows, buffaloes, poultry) lowers fertilisation cost. The farm considered grows eight crops: grams, wheat, lemon, chillies, gourd, okra, tomatoes and coriander (the paper lists tomatoes both among the farm's crops and among crops that farmers avoid). The domain is arid-zone smallholder agriculture, and the decision is how to allocate land, with attention to water, in a mixed-cropping setting.

OR is suitable because mixed cropping, limited water and multiple cost heads create a multi-resource

allocation problem for which LP offers transparent, quickly solved models and a sensitivity report that tells decision makers which parameters matter.

5.2 Mathematical model

The authors first formulate a seasonal multi-objective model with three objectives: maximise total farm revenue (including a mixed-cropping interaction term in which the effect e_{kj} of crop k on crop j takes values −1, 0 or +1), minimise total farm cost (harvesting, capital and operating costs), and minimise water requirement. Constraints (their Equations 2 to 8) cover seed procurement, fertiliser and pesticide requirements, planting cost, harvesting cost, land, total capital and water availability. Because the interaction term multiplies two area variables, the revenue objective as formulated is bilinear and therefore not linear.

The model that is actually solved is the simpler single-objective LP in the paper's Table 4. It maximises $Z = 52x_1 + 80x_2 + 55x_3 + 60x_4 + 75x_5 + 75x_6 + 50x_7 + 70x_8$ subject to a 6-unit land cap on each crop and eight resource constraints (machine cost, labour, fertilisers, field preparation, water cost, seeds, harvesting cost, cultivation cost). Table 7 reproduces the coefficients.

Table 7 *Coefficients of the Model Solved in Paper 2*

Resource	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Limit
Machine cost	8	9	7	6	4	8	9	8	20,000
Labour	4	3	6	5	2	1	4	4	7,000
Fertilisers	4	3	3	2	5	6	1	5	15,000
Field preparation	8	4	8	9	6	5	6	4	8,000
Water cost	3	2	4	1	7	8	4	4	20,000
Seeds	9	2	4	5	8	7	8	3	32,000
Harvesting cost	8	6	5	7	2	4	5	7	20,000
Cultivation cost	7	6	5	7	2	4	5	2	22,000
Objective coefficient	52	80	55	60	75	75	50	70	maximise

Note. Source: Bhatia and Rana (2020), Table 4. In addition, each $x_j \leq 6$ ("land availability"). Units as printed in the paper.

Problem type, objective, constraints and assumptions

As solved, the model is a deterministic single-objective LP with 8 variables and 16 constraints (8 resource, 8 bound). The objective is revenue (profit-coefficient) maximisation. Assumptions stated by the authors are that climatic and market fluctuations and other stochastic parameters (soil properties, crop distribution, water allocation) are held constant, and that crops are grown in a mixed pattern in which no fixed area can be assigned to each crop. The authors note that the model will be extended to a stochastic version.

5.3 Solution methodology and reported results

The model is solved in LINGO 18.0 and a standard sensitivity (ranging) report is produced with reduced costs, shadow prices and allowable increases and decreases. The reported solution is $x_2 = 6$, $x_5 = 1{,}320.33$, $x_6 = 6$ and $x_8 = 6$, with all other crops at zero, and an optimal objective value of 100,375. The shadow price of field preparation is 12.5. The authors interpret the zero-valued crops as contributing little to optimality and argue that the solution lies in the convex hull of previous planting decisions. Validation consists of the sensitivity report only; no comparison with current farm practice or actual farm income is given.

5.4 Independent re-computation and audit

Re-solving the printed model reveals a decisive inconsistency. The reported solution has $x_5 = 1{,}320.33$, far above the printed bound $x_5 \leq 6$. The reported optimum is reproduced exactly (Z = 100,375.00 and a field-preparation shadow price of 12.5) when the cap on x_5 is dropped and the other seven caps are retained, which is consistent with the sensitivity report listing

15 constraint rows when the formulation has 16. With all eight caps imposed as stated, the optimum is Z = 3,102 with every crop at 6 units (48 units in total); no resource limit exceeds 3.8% utilisation. With no caps at all, the LP is driven by field preparation and gives Z = 160,000 with x_2 = 2,000. Table 8 summarises. Two further points arise: the "profit per hectare" in the paper's Table 5 are in fact each crop's contribution to the objective (for example 80 × 6 = 480), and the formulation's water constraint (Equation 8) and interaction effects (Table 1) do not appear in the solved model, although water is the paper's stated motivation.

Table 8 *Reported and Re-Computed Solutions for Paper 2*

Case	Objective Z	Solution	Binding resource
Reported (paper's Table 6)	100,375.0	$x_2 = 6$, $x_5 = 1{,}320.33$, $x_6 = 6$, $x_8 = 6$	Field preparation (shadow price 12.5)
Re-computed: cap on x_5 omitted	100,375.0	Identical to reported	Field preparation (12.5)
Re-computed: all eight caps ($x_j \le 6$)	3,102.0	All $x_j = 6$	None (all caps; resources ≤ 3.8% used)
Re-computed: no land caps	160,000.0	$x_2 = 2{,}000$; others 0	Field preparation (shadow price 20)

Note. Computed by the present authors in Python with SciPy/HiGHS from the coefficients in Table 7. Units follow the paper.

This result matters for interpretation. As printed, the stated resource limits are so large relative to the per-hectare requirements that they never restrict the plan; the only active restrictions are the arbitrary caps of 6 units per crop.

A model in which land caps alone determine the solution cannot support the inferences drawn about water or cost constraints, and the "convex hull" discussion does not follow from the LP itself. The model nevertheless usefully illustrates how a mixed-cropping system could be encoded if the interaction terms and water balance were retained.

VI. EXTENDED ANALYSIS USING PAPER 1'S DATA

Because Paper 1's data are complete and its model reproduces, we use it to address the gaps identified in Section 2.4. All analyses use the Table 3 coefficients and HiGHS.

6.1 Net-income versus gross-income objective

Replacing the gross-income coefficients by gross margins (Table 3) and re-solving gives the same plan and a net income of R612,112. The reason is that the resource limits, not the cost differences between crops, drive the allocation, and the highest-margin crops (tomatoes, cabbages) are also the highest-income crops. The result validates the authors' choice of objective in this instance but not in general, since a crop with high income and very high costs could be favoured by the gross objective and rejected by the net one.

6.2 Diversification requirements

The optimum places 37.7% of land under cabbage, which exposes the farm to a single-crop price shock. Adding a cap on any crop's share of the farm costs little (Table 9): a 40% cap has no effect, a 30% cap lowers net income by 0.53% (R3,235), and a 25% cap lowers it by 1.23% (R7,517). Under a 30% cap, land shifts from cabbage toward tomatoes and mustard (cabbage 2.10 ha, tomato 1.27 ha, mustard 1.43 ha, maize 1.00 ha). A risk-averse farmer could therefore buy considerable diversification at a price of about half a percent of income.

6.3 What drives the gain: land use or crop mix?

The farmer's plan used 5 ha. Restricting the LP to 5 ha gives a net income of R413,020 against R366,000 for the farmer's plan under the model's own cost data (R375,000 using the paper's Table 1 costs). Re-mixing crops on the same land therefore adds about R47,000. Allowing the full 7 ha raises net income by a further R199,092 to R612,112. Of the total improvement of R246,112, about 19% comes from crop mix and about 81% from using idle land. The value of optimisation is real, but its largest part could have been captured by planting the unused hectares with any reasonable mix, which is a more modest message than "the LP raised profit by 38%". Figure 2 shows that net income rises steeply with land until about 7 to 8 ha, after which land

stops binding and other resources, especially seed and chemicals, become the limit (net income is R626,764 for 8 ha or more).

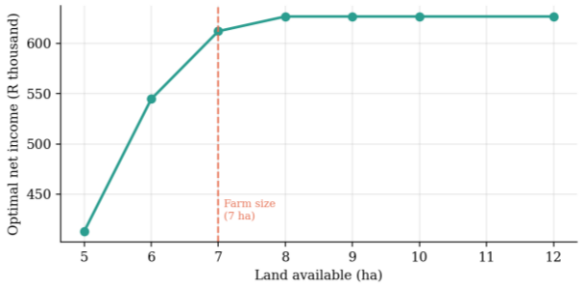

Figure 2 Optimal Net Income as a Function of Available Land (Paper 1 Data)

Note. Net-income objective. Dashed red line marks the actual farm size. Land stops being the binding resource between 7 and 8 ha. Computed by the present authors.

Table 9 *Scenario Analysis on Net Income (Paper 1 Data)*

Scenario	Net income (R)	Change vs base	Notes
Base (7 ha, net objective)	612,112	–	Maize 1.16, cabbage 2.64, tomato 0.80, mustard 1.20
Cap any crop at 40% of land	612,112	0.0%	Not binding
Cap any crop at 30% of land	608,877	−0.5%	Cabbage 2.10, tomato 1.27, mustard 1.43
Cap any crop at 25% of land	604,595	−1.2%	Cabbage 1.75, tomato 1.50, mustard 1.55
Land 5 ha (farmer's used area)	413,020	−32.5%	Crop mix only, no idle land
Land 6 ha	544,678	−11.0%	
Land 8 ha or more	626,764	+2.4%	Seed and chemicals bind
Chemicals limit −10% / +10% / +25%	580,990 / 637,514 / 651,214	−5.1% / +4.2% / +6.4%	Most valuable input
Seed limit −10% / +10%	595,717 / 617,440	−2.7% / +0.9%	Saturates above +10%
Fertiliser −10%; Capital −10%	604,369; 604,964	−1.3%; −1.2%	Become binding when reduced

Note. Computed by the present authors. "Change vs base" is relative to R612,112.

6.4 Price sensitivity and uncertainty

We next test the plan's robustness to market prices, a gap in the original sensitivity analysis. Varying cabbage income per hectare shows that the base plan is optimal only in a narrow band: it changes below about 98% and above about 103% of the base cabbage price (Figure 3). Outside that band, re-optimisation helps: at 70% of the base price, re-optimising raises net income by about R38,000 relative to holding the base plan fixed; around the base price the difference is negligible. The farmer's plan earns R366,000 at base prices and, with only 0.5 ha of cabbage, is hardly exposed to the cabbage price; it remains well below the re-optimised curve throughout.

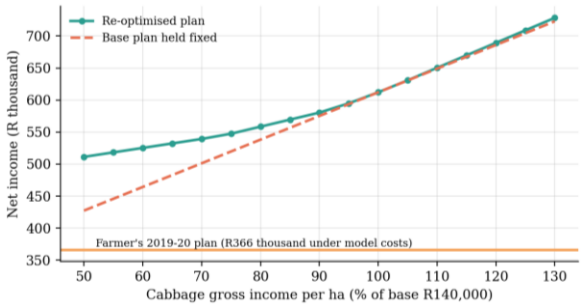

Figure 3 Effect of Cabbage Price on Net Income: Re-Optimised versus Fixed Plan

Note. Cabbage gross income per hectare varied from 50% to 130% of R140,000, all else constant. Computed by the present authors.

To assess joint price risk, we ran a Monte Carlo experiment of 5,000 draws in which each crop's gross income per hectare was multiplied independently by a uniform factor between 0.8 and 1.2, mimicking ±20% price or yield variation. Holding the base plan fixed gave a mean net income of R613,449 (standard deviation R47,982; 5th to 95th percentile R535,831 to R690,007).

Re-optimising after prices were revealed gave a mean of R620,340 (standard deviation R45,880; 5th to 95th percentile R542,734 to R692,768). The average regret of the fixed plan was R6,892 (1.13%), with a maximum of R47,191, and in no draw did the fixed LP plan earn less than the farmer's plan evaluated at the same prices (the LP plan earned on average 1.68 times as much, minimum 1.43). The optimal cabbage area varied from 1.00 to 2.79 ha (mean 2.31, standard deviation 0.56). Figure 4 shows the distributions.

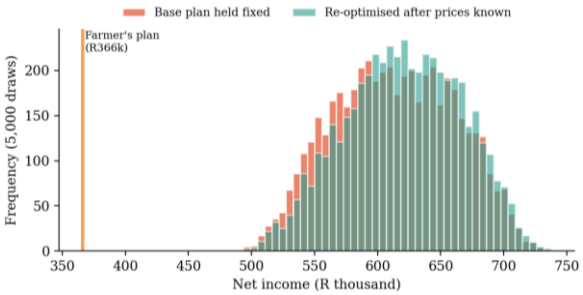

Figure 4 Simulated Net Income Under Independent ±20% Price Variation (5,000 Draws)

Note. Orange-red: base plan held fixed. Teal: plan re-optimised after prices known. Amber line: farmer's plan at base prices (R366,000). The ±20% uniform variation is illustrative, not estimated from market data.

Two limits apply. The simulation assumes independent price shocks, whereas prices of vegetables often move together, and it treats the plan as a once-off decision. It does show that, for this farm, the LP recommendation is robust: the superiority over the traditional plan does not depend on exact prices, and the expected loss from not re-optimising is small. That finding would be valuable to report in any LP crop paper but is almost never reported.

VII. DISCUSSION

7.1 Key findings and implications

Both papers confirm the long-standing view that LP is an accessible and informative tool for farm planning. Paper 1 shows how a small farmer could add roughly R237,000 to R246,000 of net income per season by planting all available land and favouring cabbages and mustard, and the dual solution tells the farmer that land, seed and especially the chemicals budget are the limiting resources.

Paper 2 shows how a mixed-cropping model for an arid region can be encoded, although its published solution does not follow from its published constraints. Beyond these specific results, the study suggests three implications for practice: report the base-plan land use alongside any "improvement" percentage; interpret shadow prices and reduced costs as managerial guidance, not only as software output; and test robustness to price and diversification, which cost seconds to run once the model exists.

7.2 Strengths of the models

Paper 1's strengths are its clear problem, complete data tables, a transparent single-objective LP, sensible lower and upper bounds that capture market commitments, a standard solution method and a sensitivity analysis that, once the labels are corrected, is correct. Its model is also easy for a farmer or extension officer to understand and rerun. Paper 2's strengths are its relevance to water-scarce smallholder farming, its recognition of mixed-cropping and livestock integration, and a rich formulation (multiple objectives and crop interaction effects) that points toward a more realistic model. Both papers use field data from real farms rather than synthetic data, and both make the full problem definition available to the reader.

Table 10 *Strengths, Limitations and Improvements for the Two Models*

Aspect	Paper 1 (Muzhinji & Ndou, 2022)	Paper 2 (Bhatia & Rana, 2020)
Strengths	Complete data; clear objective; realistic bounds; reproducible optimum; simple to communicate	Relevant to arid smallholders; mixed-crop and livestock integration; multi-objective ambition
Limitations	Deterministic, single season; gross objective; misreported gain; swapped sensitivity rows; idle-land baseline; coefficient typos	Solved model drops water balance and interactions; reported solution violates stated bound; caps drive solution; no validation against practice
Assumptions at risk	Fixed prices and yields; equal land suitability; no rotation	Constant climate and market; linear scaling of mixed-crop effects
Improvements	Report base land use; corrected sensitivity; price scenarios or stochastic LP; diversification and rotation constraints	Solve the stated model (water, costs, interaction); use goal programming or ε-constraint; calibrate resource limits to survey data; report validation

Note. Assessment by the present authors based on Sections 4 to 6.

7.3 Limitations of this study

This is a literature-based study of two papers, not a field study, and the audit is limited to what the papers print. Where we infer a cause (for example, that the x_5 cap was omitted in the solved model), the inference is consistent with the evidence but not proven; the authors may have used data or files not in the publication. The extended analysis relies on the same single-farm data and on illustrative ±20% price ranges that are not estimated from market series. Results are deterministic or simulated on a model with LP's usual assumptions, and the findings should not be generalised to other farms without new data.

7.4 Possibilities for extension

The review and audit point to a clear roadmap (Table 11). The most valuable extensions are stochastic or robust LP for price and yield risk (building on the risk-oriented work of Rădulescu & Rădulescu, 2012), goal or multi-objective programming for profit, water and risk (Sarker & Quaddus, 2002; Adekanmbi & Olugbara, 2015), mixed-integer formulations for indivisible plots and rotation sequences (Filippi et al., 2017; Castellazzi et al., 2008; Dury et al., 2012), multi-period models linking seasons, and explicit irrigation-water balances (Rao et al., 1990; Li & Guo, 2014). For Indian conditions, it would be natural to add minimum-support-price or contract-price scenarios and water-use limits at district level.

Table 11 *Research Gaps and Proposed Extensions*

Gap identified	Evidence in this study	Proposed extension
Misleading baselines	About 81% of Paper 1's gain is idle land	Report gains on equal land; decompose by source
Reporting and labelling errors	Swapped rows; infeasible rounded plan; inconsistent coefficients	Publish model files; automated feasibility check; supplementary code
Price and yield risk	Plan optimal in a narrow cabbage-price band	Scenario or stochastic LP; robust optimisation
Concentration risk	37.7% of land in one crop	Diversification constraints; mean-variance or portfolio LP
Water and interactions omitted	Paper 2 solved model lacks water balance	Include irrigation and interaction terms; test linearisation
Single season	Both models are one-season	Multi-period model with rotations and cash flow

Note. Compiled by the present authors.

VIII. CONCLUSION

This study reviewed LP-based crop planning, analysed two published applications against the structure of an OR modelling study, and independently re-solved both models.

In terms of the research questions: (RQ1) both papers formulate deterministic, single-objective LPs with land, input and cost constraints, solved by simplex-based software, and validated mainly by sensitivity analysis; (RQ2) Paper 1's optimum is reproduced exactly but its headline gain is misreported (63.2% net rather than 37.8% or 38.7%), two sensitivity rows are swapped, and most of the gain stems from planting previously idle land, while Paper 2's reported optimum is reproduced only when one land cap is dropped and the fully constrained model gives a very different result; (RQ3) extending Paper 1 shows that land, seed and especially the chemicals budget are limiting, that diversification up to a 30% crop share costs about 0.5% of income, and that the plan is robust to ±20% price variation with an average regret of about 1%.

The broader lesson for operations research students and practitioners is that LP's value lies in the model's transparency, but that transparency must be earned through full data reporting, feasibility checking and honest baselines.

Future work should extend the audited models toward stochastic, multi-objective and rotation-based formulations and should publish code and data so that each result can be checked in seconds.

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APPENDIX A

Re-computation Procedure for Paper 1

The following procedure reproduces all Paper 1 results in this article. Variables are ordered as maize, cabbage, tomato, mustard, spinach, butternut, sweet potato.

```
from scipy.optimize import linprog
gross = [100000,140000,160000,60000,44000,44000,50000]
A = [[1]*7,                          # land (ha)
[900,9000,8000,4000,4500,1000,4500], # seed
[4000,4000,4000,3000,5000,4500,5000],# fertiliser
[200,200,300,0,0,0,0],               # chemicals
[500,200,3000,300,400,1000,700],     # fungicides
[4000]*7,                            # labour
[5000,4000,4000,3000,3000,2000,5000]]# other costs
b = [7,40000,30000,1000,7000,30000,30000]
bounds = [(1, None), (1, None), (.8, None), (.8, None), (.4,.8), (.4,.8), (.4,.8)]
res = linprog([-g for g in gross], A_ub=A, b_ub=b, bounds=bounds, method="highs")
# res.x -> [1.1605, 2.6395, 0.8, 1.2, 0.4, 0.4, 0.4]; -res.fun -> 740,780.25
# res.ineqlin.marginals -> shadow prices (land 40,247; seed 4.94; chemicals 276.54)
```

The capital row (direct cost per hectare) is added as a further constraint with limit 140,000 using the direct-cost values in Table 3 and does not bind. The Paper 2 model is solved analogously with the coefficients in Table 7 and variable bounds of (0, 6), with the bound-on x_5 removed to reproduce the published solution.