Applications of Operations Research in LPG Cylinder Distribution: A Literature-Based Analytical Study of Transportation Models

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I. INTRODUCTION

Operations Research (OR) applies mathematical models and analytical methods to help decision-makers choose the best course of action when resources are limited and constraints must be met (Taha, 2017). A typical OR study describes the real problem, defines the decision, builds a model with an objective function and constraints, solves it, and tests the result before it is used. The value of OR lies in this translation: a loose managerial question such as "how many cylinders should each plant send to each agent?" becomes a problem with a defined best answer.

Distribution is one of the most common OR applications because transport cost is large, measurable and controllable. LPG (liquefied petroleum gas) cylinder distribution suits the transportation problem, a linear programme introduced by Hitchcock (1941), for four reasons. The product is uniform (a cylinder of one size). The sources are few (filling plants). The destinations are known (agents or districts). The cost of moving a cylinder along each route can be estimated. In the two cases studied in depth here, distribution cost was set by driver experience or by serving each demand point from its nearest facility, with no formal allocation plan (Adianta & Mussafi, 2024; Syahruddin et al., 2023).

This report therefore studies LPG distribution as a transportation problem. Routing, vehicle scheduling and safety, which appear in other LPG studies, are treated as extensions (Sections 4 and 13), not as the core model.

Research question. How can transportation models and classical optimisation techniques be used to minimise LPG cylinder distribution costs while satisfying source capacities and destination demands?

Objectives.

- To examine the application of transportation models to LPG cylinder distribution.
- To analyse the mathematical formulation of selected LPG distribution studies.
- To understand the use of the North-West Corner (NWC), Least Cost and Vogel-type methods and the Modified Distribution (MODI) method.
- To compare optimised transportation costs with existing distribution practices.
- To evaluate critically the assumptions, strengths and limitations of the models.

II. RESEARCH METHODOLOGY

This is a literature-based analytical study. It collects no new field data. It reads published studies, extracts their models and data, recomputes their results and evaluates them critically.

Selection of papers. The group selected five papers on LPG cylinder distribution for this study. The selection did not follow a documented search protocol (no stated database, search terms or screening log), so this report does not present itself as a systematic literature review.

Why LPG distribution. The topic appears under Transportation Problems in the project guidelines. LPG is distributed from a few filling points to many demand points in a single standard product, which matches the structure of the classical model.

Selection criteria. Each paper was screened against five criteria (Table 1).

Table 1. Screening of the five papers

Criterion	P1	P2	P3	P4	P5
1. LPG or gas-cylinder distribution	Yes	Yes	Yes	Yes	Yes
2. Operations Research application	Yes	Yes	Yes	Yes	Yes
3. Transportation optimisation (source-to-destination allocation)	Yes	Yes	No (routing)	No (flow distribution and dispatch)	No (delivery sequencing)
4. Sufficient model information (full cost matrix, supplies, demands)	Yes	Yes	Partly (benchmark data)	Partly (illustrative network)	Partly (sample distance matrix)
5. Solution method and results reported	Yes	Yes	Yes	Yes	Yes (preprint)

P1 = Syahruddin et al. (2023); P2 = Adianta & Mussafi (2024); P3 = Singamsetty & Thenepalle (2021); P4 = Jia et al. (2018); P5 = Prithvirajan & Mathirajan (2025).

Only P1 and P2 meet all five criteria. They are analysed in full in Sections 5 to 10. P3 to P5 are reviewed in Section 4 to show what lies beyond the classical model.

Verification and labelling. Every number in Sections 5 to 10 was recomputed from the published tables, using Python and a linear-programming solver (SciPy). Recomputed results are labelled "Recalculated from the published data". Statements about the papers are introduced by "According to the original paper"; our own findings by "Our analysis indicates". Bibliographic details of P2, Ratnasari et al. (2019) and Wardrop (1952) were checked against online journal records. Those of P1, P3, P4 and P5 were checked against the first pages of the papers themselves. Where a source does not give enough information, the report says so.

III. REAL-LIFE CONTEXT: LPG CYLINDER DISTRIBUTION

3.1 The LPG supply chain



Figure 1. The LPG supply chain and the part modelled as a transportation problem

In the classical model the filling plants are the sources, each with a limited filling or supply capacity (a_i). The agents or districts are the destinations, each with a demand (b_j). The cost of moving one cylinder along each plant-to-destination route is c_{ij}. In P1, five facilities supply six retail agents, and 6,000 cylinders are delivered twice a week. In P2, two bulk filling stations (SPPBE) supply 15 districts of Bantul Regency, with 54,880 cylinders in September 2023.

Distribution cost matters for three reasons. First, the 3 kg cylinder in these studies is a subsidised product with a regulated retail ceiling. P2 reports a highest retail price of Rp 15,500 set under a regional regulation. Second, P1 reports a monthly distribution cost of IDR 45 million with no distribution strategy, and P2 reports a manually calculated monthly cost of IDR 9,307,000. Third, the cost depends on which source serves which destination, which is exactly what an allocation model decides.

LPG also creates operational challenges that most products do not. Cylinders are pressurised and flammable, so safe handling and transport matter (Jia et al., 2018). Empty cylinders are collected on the return journey (reverse logistics) (Singamsetty & Thenepalle, 2021). Both lie outside the classical transportation model and are not analysed here.

3.2 Classical transportation problem versus real-world LPG logistics

Table 2. What the classical model covers

Aspect	Classical transportation problem (P1, P2)	Real-world LPG logistics
Decision	How many cylinders to ship from each source to each destination	Also, which vehicle, which route, and at what time
Cost	Fixed unit cost c_{ij}, linear in quantity	Trips, vehicle capacity, fuel, traffic
Vehicles	Not modelled	Limited cylinder capacity; several restocking trips (5 or 12 cylinders in P5)
Demand and supply	Known and fixed	Vary by period; stock-outs possible
Safety	Not modelled	Hazardous product; risk varies by time of day (P4)
Empty cylinders	Not modelled	Collected on return trips (P3)

The classical model answers one question: how many cylinders should be shipped from each source to each destination? Routing, vehicle scheduling and safety are extensions of that allocation decision, not part of it.

IV. LITERATURE REVIEW: THE FIVE SELECTED PAPERS

Table 3. The five papers

ID	Authors (year)	Title and source	Model
P1	Syahruddin, Gaffar, Burhanudin & Amarilies (2023)	Comparison of LPG bottle distribution cost using North West Corner and Least Cost methods (case study at PT ABC in Samarinda). ICO-SEID 2022, EAI.	Transportation problem; NWC, LCM, MODI
P2	Adianta & Mussafi (2024)	Optimization of transportation distribution costs using Improved Vogel's Approximation Method (IVAM) (case study: PT. Sinar Putra Pertam). IJIEM 5(2), 417-428.	Transportation problem; IVAM, MODI
P3	Singamsetty & Thenepalle (2021)	Designing optimal route for the distribution chain of a rural LPG delivery system. Int. J. Ind. Eng. Comput. 12, 221-234.	Capacitated truncated VRP with pickup and delivery; Lexi-search
P4	Jia, He, Zhang & Chai (2018)	A bi-level programming model of liquefied petroleum gas transportation operation for urban road network by period-security. Sustainability 10(12), 4714.	Bi-level model; genetic algorithm with Frank-Wolfe
P5	Prithvirajan & Mathirajan (2025, preprint)	Development of an optimal and efficient solution methodologies for delivery sequencing in last mile delivery of LPG cylinders in India. SSRN 5242495 (not peer reviewed)	Multiple travelling salesman problem; four metaheuristics

P1 (transportation problem). According to the original paper, PT ABC distributes 3 kg LPG cylinders from five facilities to six retail agents in Samarinda, Indonesia, at an existing cost of IDR 45 million per month. NWC followed by MODI, and the Least Cost Method, both reach IDR 3,003,888 per delivery, or IDR 24,031,104 per month over eight deliveries, a 46.6% reduction. The data are secondary, taken from Ratnasari et al. (2019), whose case company is named PT Tri Pribumi Sejati. That study reports the same monthly optimum (IDR 24,031,104) and the same 46.6% saving against the same IDR 45,000,000 baseline, using VAM and the stepping-stone method. The source does not state whether the two companies are the same.

P2 (transportation problem). According to the original paper, PT Sinar Putra Pertam supplies 15 districts of Bantul Regency from two SPPBE filling stations. IVAM gives IDR 6,816,230 in three iterations, and MODI confirms it is optimal. The company's manual cost is IDR 9,307,000, so the reported saving is IDR 2,490,770 (26.76%).

P3 (routing extension). Singamsetty and Thenepalle (2021) model rural LPG delivery as a capacitated truncated vehicle routing problem. Vehicles deliver filled cylinders and collect the same number of empties, and the agency can serve only a fixed number of centres because filled stock is scarce. Their exact Lexi-search algorithm matched the best-known solutions on 16 benchmark instances, with run times

of 10 to 1,652 seconds. The data are benchmark instances, not company data.

P4 (safety extension). Jia et al. (2018) ask which time period and line an LPG supply station should use in a congested city. A lower model spreads flow across lines and periods by safety cost; an upper model chooses the number of vehicles at least cost. The paper reports 48, 57 and 66 vehicles for demands of 60,000, 72,000 and 90,000 kg, with the two peak periods unused. A discrepancy identified during our analysis: the paper gives the station's daily capacity as 104,000 kg, but its own inputs (12 × 15 × 100 × 6) give 108,000 kg (recalculated from the published data).

P5 (delivery-sequencing extension). According to the original preprint, delivery agents in Chennai make several restocking trips a day because their vehicles carry only 5 or 12 cylinders. The authors model the order of deliveries as a multiple travelling salesman problem. An exact model solved a 20-order, two-trip case in about 54 seconds but did not finish a four-trip case in six hours. Tabu Search gave the shortest routes among four metaheuristics on 216 real test cases.

4.1 Two terms used in P4

Wardrop's principle (user equilibrium). Wardrop (1952) described how drivers choose routes on a congested network: each picks the route that looks cheapest, but a route gets costlier as flow rises, until no driver can gain by switching. P4 applies the idea to safety cost, so the cost of a line in a period rises with the LPG flow it carries:

$$c_p^k(x_p^k) = v_p^k\left(1 + \alpha \frac{x_p^k}{n_p^k}\right)$$

Here v_p^k is the safety cost at zero flow, n_p^k the maximum flow on the line and $\alpha = 1.5$ a calibration parameter. The effect is to spread the load across the safe periods instead of piling it into one.

Frank-Wolfe algorithm. Frank and Wolfe (1956) proposed an iterative method for problems whose cost rises with flow: it repeatedly assigns all demand to the currently cheapest line and moves part of the way toward that assignment (P4 reports a tolerance of 10^{-4} after 15 iterations). In P4 it runs inside a genetic algorithm to score each candidate vehicle plan.

4.2 Synthesis

P1 and P2 address the classical allocation question with full data. P3 to P5 add routing, pickups, safety and sequencing. Allocation decides how many cylinders move between each pair of points; routing decides how a vehicle carries them (Section 13).

V. MATHEMATICAL MODEL

P1 and P2 use the same classical transportation formulation. The notation below follows both papers.

Decision variable. x_{ij} = number of LPG cylinders transported from source i to destination j.

Parameters. c_{ij} = transportation cost per cylinder from source i to destination j; a_i = supply available at source i; b_j = demand at destination j; m = number of sources; n = number of destinations.

Objective: minimise total transportation cost.

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, \ldots, m \qquad \text{(supply constraints)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, \ldots, n \qquad \text{(demand constraints)}$$

$$x_{ij} \geq 0 \quad \text{for all } i, j \qquad \text{(non–negativity)}$$

Why these equations describe the LPG problem. The objective adds up the cost of every cylinder moved. The supply constraint says a filling plant cannot send out more cylinders than it has available. The demand constraint says every agent or district receives exactly the cylinders it needs. Non-negativity rules out shipping a negative number of cylinders. All three are directly observable in the LPG operations described by P1 and P2.

Balanced or unbalanced. A problem is balanced when total supply equals total demand. If it is not, a dummy source or destination at zero cost is added. Both selected problems are balanced (Table 4), so no dummy is needed. Both are therefore written with equality constraints.

Table 4. Structure of the two problems (recalculated from the published data)

Feature	P1: Samarinda	P2: Bantul
Sources (m)	5 facilities	2 SPPBE filling stations
Destinations (n)	6 retail agents	15 districts

Decision variables (m × n)	30	30
Total supply	6,000 per delivery	54,880
Total demand	6,000 per delivery	54,880
Status	Balanced	Balanced
Basic cells needed (m + n − 1)	10	16

Assumptions stated in the papers.

- P1: distribution is made twice a week (8 deliveries a month); the company never has a stock-out of subsidised LPG; cost data are secondary data from Ratnasari et al. (2019). The paper also states a truck capacity of 3,000 cylinders per delivery (see discrepancy D1-2 in Section 12.3).
- P2: costs depend on vehicle fuel consumption and the mileage between each source and destination, obtained from interviews with the company director; total supply equals total demand for September 2023.
- Both: the unit cost of a route is fixed and does not depend on the quantity shipped, and only the allocation is decided (not the vehicle route).

A notation discrepancy identified during our analysis: P1 prints its demand constraint with the summation limit n and the symbol b_i; the intended meaning (sum over the m sources, demand b_j) is clear from the variable definitions and does not affect any result.

VI. ACTUAL TRANSPORTATION TABLES

The tables below reproduce the published data. Nothing has been altered except where marked.

Table 5. P1: transportation cost matrix, PT ABC, Samarinda (IDR per cylinder)

Facility	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5	Agent 6	Supply a_i
Facility 1	152	934	934	1,738	913	956	1,123
Facility 2	1,756	319	878	1,317	1,038	319	1,256
Facility 3	1,330	1,296	102	2,251	443	205	1,203
Facility 4	525	1,002	715	1,955	692	739	1,124
Facility 5	1,438	863	246	2,506	329	246	1,294
Demand b_j	920	985	1,255	950	890	1,000	6,000

Source: Syahruddin et al. (2023), Table 2. The paper defines c_{ij} as the cost per unit of product; it does not state the unit explicitly, so we read it as IDR per cylinder. Supply and demand are the average per-delivery figures given in the paper.

Table 6. P2: transportation cost matrix, PT Sinar Putra Pertam, Bantul (IDR per cylinder)

No.	District	c1j: from S1	c2j: from S2	Demand b_j
1	Bambanglipuro	141	134	2,410
2	Banguntapan	148	83 / 85†	2,860
3	Bantul	119	100	10,710
4	Dlingo	194	134	1,530
5	Imogiri	163	119	2,150
6	Jetis	146	100	8,820
7	Kasihan‡	107	85	5,430
8	Kretek	172	158	1,490
9	Pajangan‡	121	124	1,340
10	Pandak	138	121	2,150
11	Pleret	172	97	2,050
12	Pundong	160	136	3,100
13	Sanden	153	160	3,430
14	Sewon	131	87	7,060
15	Srandakan	155	180	350
	Supply a_i	40,320	14,560	54,880

Source: Adianta & Mussafi (2024), Tables 2 to 4. S1 = SPPBE Kosan, PT Lestari Pelita Graha (Kulon Progo); S2 = SPPBE PT Alam Narrizqi Airmas (Piyungan, Bantul). The paper lists 15 destinations across the columns; the table is shown transposed here for readability. The data are unchanged.

† A discrepancy identified during our analysis: the cost table (Table 3 in the paper) gives the S2–Banguntapan cost as 85, but the transportation table and the objective function use 83. Both values are shown; Section 9 tests both.

‡ The key beneath the paper's cost table labels destinations D7 and D9 as "Pity" and "Display",

whereas its demand table lists Kasihan and Pajangan. We use the demand-table names; the source does not provide sufficient information to establish which labels are correct. The numerical data are unaffected.

VII. SOLUTION METHODOLOGY

A transportation problem is solved in two stages: find an initial feasible solution, then test it for optimality and improve it if needed (Taha, 2017). The initial methods differ in effort and quality. MODI is the optimality test used in both papers.

7.1 North-West Corner (NWC) method

Purpose. To obtain a feasible allocation quickly, with no use of cost data.

Steps.

- Start at the top-left (north-west) cell of the table.
- Allocate the smaller of that row's remaining supply and that column's remaining demand.
- Cross out the exhausted row or column, and move one cell right (if demand is met) or down (if supply is exhausted).
- Repeat until all supply and demand are allocated.

The result satisfies every constraint, so it is feasible. It is not necessarily optimal, because the allocation depends only on the position of cells in the table, not on their costs. In P1 the NWC solution costs IDR 3,812,054 per delivery against an optimum of IDR 3,003,888, about 26.9% higher (recalculated from the published data).

7.2 Least Cost Method (LCM)

Steps.

- Select the cell with the lowest unit cost in the whole table.
- Allocate as much as possible to it (the smaller of supply and demand).
- Cross out the row or column that is exhausted.
- Select the lowest-cost cell among those remaining, and repeat until all supply and demand are met.

LCM uses cost information, so it usually starts closer to the optimum than NWC. It can still be improved by MODI, because choosing the cheapest cell first can force expensive allocations later. In P1 it happens to reach the optimum at once.

7.3 Vogel's Approximation Method (VAM) and Improved VAM (IVAM)

VAM.

For each row and column, compute a penalty: the difference between its two lowest costs.

Select the row or column with the highest penalty.

Allocate as much as possible to the lowest-cost cell in that row or column.

Cross out the exhausted row or column, recompute penalties, and repeat.

The penalty measures what is lost if the cheapest route is not used, so VAM usually gives a better start than NWC or LCM.

IVAM as used in P2 (after Korukoğlu & Ballı, 2011). IVAM replaces raw costs by a Total Opportunity Cost (TOC) matrix. For each cell, TOC is the sum of its row opportunity cost (cost minus the row minimum) and its column opportunity cost (cost minus the column minimum). Penalties are then computed on the TOC values. In the published application, the three highest penalties are served in each iteration, each at the source with the lowest TOC for that destination, and the process repeats until supply and demand are exhausted. This report describes IVAM only to the extent that P2 uses it.

7.4 Modified Distribution (MODI) method

Why MODI is used. An initial solution does not prove optimality. MODI tests it by asking, for every unused route, whether shipping along that route would reduce the total cost.

Steps.

Check that the solution has $m + n - 1$ occupied (basic) cells.

Set $u_1 = 0$ and solve $u_i + v_j = c_{ij}$ for every occupied cell to obtain all u_i and v_j.

For every unoccupied cell compute the improvement (opportunity cost) index $\Delta_{ij} = c_{ij} - u_i - v_j$.

Optimality condition (minimisation): the solution is optimal if every $\Delta_{ij} \geq 0$.

A negative Δ_{ij} means each cylinder shifted onto that route lowers cost by $|\Delta_{ij}|$, so the allocation is not optimal.

Choose the unoccupied cell with the most negative Δ_{ij} as the entering cell. Trace a closed loop from it through occupied cells, turning only at occupied cells and alternating + and − signs.

Let θ be the smallest quantity on a minus cell. Add θ to plus cells and subtract it from minus cells. The cost falls by $\theta \times |\Delta_{ij}|$.
Recompute u, v and Δ for the new solution, and repeat until all $\Delta_{ij} \geq 0$.

VIII. WORKED ANALYSIS OF PAPER 1: SYAHRUDDIN ET AL. (2023), PT ABC, SAMARINDA

According to the original paper, PT ABC solves a balanced 5 × 6 problem by NWC with MODI and by the Least Cost Method. Every step below was recomputed from Table 5.

Step 1: Original transportation table. See Table 5. Total supply and demand are both 6,000 cylinders per delivery (balanced; 30 variables).
Step 2: Initial solutions.
North-West Corner. Starting at Facility 1 / Agent 1, the allocation is as in Table 7. It has 10 occupied cells (= m + n − 1).

Table 7. P1: NWC initial allocation x_{ij} (cylinders)

Facility	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5	Agent 6	Supply
Facility 1	920	203					1,123
Facility 2		782	474				1,256
Facility 3			781	422			1,203
Facility 4				528	596		1,124
Facility 5					294	1,000	1,294
Demand	920	985	1,255	950	890	1,000	6,000

Least Cost Method. Allocations are made in order of cell cost 102, 152, 246, 246, 319, 329, 692, 1,317, 1,738 and 1,955. Our recalculation reproduces the allocation reported in the paper, which is identical to the optimal allocation in Table 10.

Step 3: Transportation cost of the initial solution.

$$Z_{NWC} = 920(152) + 203(934) + 782(319) + 474(878) + 781(102)$$
$$+ 422(2,251) + 528(1,955) + 596(692) + 294(329) + 1000(246)$$
$$= 3,812,054$$

This matches the paper's IDR 3,812,054 per delivery. The Least Cost allocation costs IDR 3,003,888 per delivery (Step 7).
Step 4: MODI optimality test on the NWC solution. With $u_1 = 0$, the occupied cells give u = (0, −615, −1,391, −1,687, −2,050) and v = (152, 934, 1,493, 3,642, 2,379, 2,296). The improvement indices $\Delta_{ij} = c_{ij} - u_i - v_j$ is in Table 8 (B = occupied cell).

Table 8. P1: MODI indices for the NWC solution

Facility	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5	Agent 6
Facility 1	B	B	−559	−1,904	−1,466	−1,340
Facility 2	2,219	B	B	−1,710	−726	−1,362
Facility 3	2,569	1,753	B	B	−545	−700
Facility 4	2,060	1,755	909	B	B	130
Facility 5	3,336	1,979	803	914	B	B

Nine indices are negative, so the NWC solution is not optimal. The most negative is Facility 1 / Agent 4 (−1,904). These values agree with the paper's calculation.

Step 5: Improvement loops. Table 9 gives the improvement steps recalculated using the most-negative-index rule. For the first pivot, the entering cell is Facility 1 / Agent 4; the loop runs F1A4 (+),

F1A2 (−), F2A2 (+), F2A3 (−), F3A3 (+), F3A4 (−); the smallest minus quantity is θ = 203, so cost falls by 203 × 1,904 = 386,512.

Table 9. P1: MODI pivots from the NWC solution (recalculated from the published data)

Pivot	Entering cell (index)	Loop (+ / −)	θ	Cost after pivot (IDR per delivery)
Start		NWC solution		3,812,054
1	F1–A4 (−1,904)	+F1A4, −F1A2, +F2A2, −F2A3, +F3A3, −F3A4	203	3,425,542
2	F2–A4 (−1,710)	+F2A4, −F2A3, +F3A3, −F3A4	219	3,051,052
3	F5–A3 (−907)	+F5A3, −F5A5, +F4A5, −F4A4, +F2A4, −F2A3	52	3,003,888

A discrepancy identified during our analysis: the paper states that MODI needed two iterations, but it shows only the starting (NWC) tableau and the final tableau. Our recalculation, using the most-negative-index rule, needed three improving pivots before the optimality check. The count depends on the entering-cell rule and the order of pivots, and the source does not provide sufficient information to establish how its intermediate steps were handled. The final solution and cost are the same.

Step 6: Final optimal transportation table.

Table 10. P1: optimal allocation x_{ij} (cylinders)

Facility	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5	Agent 6	Supply
Facility 1	920			203			1,123
Facility 2		985		271			1,256
Facility 3			1,203				1,203
Facility 4				476	648		1,124
Facility 5			52		242	1,000	1,294
Demand	920	985	1,255	950	890	1,000	6,000

Both supply constraints and demand constraints hold with equality (checked).

Step 7: Final cost and optimality check.

$$Z^* = 920(152) + 203(1{,}738) + 985(319) + 271(1{,}317) + 1203(102) + 476(1{,}955) + 648(692) + 52(246) + 242(329) + 1000(246) = 3{,}003{,}888$$

For the MODI test of this solution, u = (0, −421, −290, 217, −146) and v = (152, 740, 392, 1,738, 475, 392). The indices are in Table 11. All are positive (the smallest is 45), so the solution is optimal.

Table 11. P1: MODI indices for the final solution

Facility	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5	Agent 6
Facility 1	B	194	542	B	438	564
Facility 2	2,025	B	907	B	984	348
Facility 3	1,468	846	B	803	258	103
Facility 4	156	45	106	B	B	130
Facility 5	1,432	269	B	914	B	B

The paper reports the same indices (it uses $u_5 = 0$, which shifts u and v by a constant and leaves every Δ_{ij} unchanged). Recalculated from the published data, a linear-programming solver also returns a minimum of IDR 3,003,888.

Step 8: Comparison with the existing cost. According to the original paper, PT ABC's existing distribution cost is IDR 45,000,000 per month, based on 8 deliveries a month.

Table 12. P1: monthly cost and saving (recalculated from the published data)

Solution	Cost per delivery (IDR)	Monthly cost, 8 deliveries (IDR)	Saving against IDR 45,000,000
Existing practice	not given	45,000,000	
NWC (initial solution)	3,812,054	30,496,432	14,503,568 (32.2%)
NWC + MODI = Least Cost (optimal)	3,003,888	24,031,104	20,968,896 (46.6%)

Step 9: Percentage saving. (45,000,000 − 24,031,104) / 45,000,000 = 46.6%. The source does not provide the allocation or calculation behind the IDR 45,000,000 baseline, so the baseline cannot be independently verified.

IX. WORKED ANALYSIS OF PAPER 2: ADIANTA & MUSSAFI (2024), PT SINAR PUTRA PERTAM, BANTUL

According to the original paper, the company supplies 15 districts from two SPPBE stations. The paper applies IVAM and then MODI. Every step below was recomputed from Table 6.

Step 1: Original transportation table. See Table 6. Total supply and demand are both 54,880 cylinders (balanced; 2 × 15 = 30 variables).

Step 2: IVAM allocation. The TOC matrix adds each cell's row and column opportunity costs. Because there are only two sources, the penalty of each destination is the absolute difference between its two TOC values.

Table 13. P2: TOC values and penalties (recalculated from the published data)

District	TOC at S1	TOC at S2	Penalty
1 Bambanglipuro	41	51	10
2 Banguntapan	106	0	106
3 Bantul	31	17	14
4 Dlingo	147	51	96
5 Imogiri	100	36	64
6 Jetis	85	17	68
7 Kasihan	22	2	20
8 Kretek	79	75	4
9 Pajangan	14	44	30
10 Pandak	48	38	10
11 Pleret	140	14	126
12 Pundong	77	53	24
13 Sanden	46	84	38
14 Sewon	68*	4	64*
15 Srandakan	48	122	74

A discrepancy identified during our analysis: the paper's Table 5 gives Sewon's opportunity costs at S1 as 24 (row) and 44 (column), which sum to 68, but its TOC matrix prints 66 (and a penalty of 62). With 68, Sewon ties with Imogiri (penalty 64) for third place in iteration 2, but S2 is already exhausted by then, so either is served from S1 and the allocation is unaffected.

The allocation proceeds as follows (recalculated; it reproduces Tables 7 to 9 of the paper).

- Iteration 1. The three highest penalties are Pleret (126), Banguntapan (106) and Dlingo (96). Each is served from S2, which has the lowest TOC: 2,050, 2,860 and 1,530 cylinders. S2 has 8,120 left.
- Iteration 2. The three highest penalties are Srandakan (74), Jetis (68) and Imogiri (64). Srandakan takes 350 from S1. Jetis takes S2's remaining 8,120, and its last 700 come from S1. S2 is now exhausted, so Imogiri takes 2,150 from S1.
- Iteration 3. Only S1 has supply left, so all remaining demand is met from S1.

This gives the IVAM solution in three iterations, as the paper reports. Recalculated, standard VAM gives the same allocation and cost, which supports the paper's statement that VAM and IVAM gave the same initial solution; the paper itself does not show the VAM calculation.

Step 3: Transportation cost.

Table 14. P2: final allocation and cost (IDR; cost data as in the objective function, S2–Banguntapan = 83)

District	c1j	x1j (S1)	Cost S1	c2j	x2j (S2)	Cost S2
1 Bambanglipuro	141	2,410	339,810	134	0	0
2 Banguntapan	148	0	0	83	2,860	237,380
3 Bantul	119	10,710	1,274,490	100	0	0
4 Dlingo	194	0	0	134	1,530	205,020
5 Imogiri	163	2,150	350,450	119	0	0
6 Jetis	146	700	102,200	100	8,120	812,000
7 Kasihan	107	5,430	581,010	85	0	0
8 Kretek	172	1,490	256,280	158	0	0
9 Pajangan	121	1,340	162,140	124	0	0
10 Pandak	138	2,150	296,700	121	0	0
11 Pleret	172	0	0	97	2,050	198,850
12 Pundong	160	3,100	496,000	136	0	0
13 Sanden	153	3,430	524,790	160	0	0
14 Sewon	131	7,060	924,860	87	0	0
15 Srandakan	155	350	54,250	180	0	0
Total		40,320	5,362,980		14,560	1,453,250

Total cost Z = 5,362,980 + 1,453,250 = IDR 6,816,230, which matches the paper. There are 16 occupied cells (= m + n − 1), so the solution is non-degenerate. Supply and demand constraints hold with equality (checked).

Step 4: MODI optimality test. With $u_1 = 0$, the occupied cells give $u_2 = -46$ and v = (141, 129, 119, 180, 163, 146, 107, 172, 121, 138, 143, 160, 153, 131, 155). The 14 improvement indices $k_{ij} = c_{ij} - u_i - v_j$ is in Table 15.

Table 15. P2: MODI improvement indices for the IVAM solution

Cell	Index	Cell	Index
S1–Banguntapan (k12)	19	S2–Kasihan (k27)	24
S1–Dlingo (k14)	14	S2–Kretek (k28)	32
S1–Pleret (k1,11)	29	S2–Pajangan (k29)	49
S2–Bambanglipuro (k21)	39	S2–Pandak (k2,10)	29
S2–Bantul (k23)	27	S2–Pundong (k2,12)	22
S2–Imogiri (k25)	2	S2–Sanden (k2,13)	53
S2–Sewon (k2,14)	2	S2–Srandakan (k2,15)	71

These values agree with the paper's Table 13. All indices are positive (smallest 2), so the IVAM solution is optimal. Recalculated from the published data, a linear-programming solver returns the same allocation and the same minimum of IDR 6,816,230.

Step 5: Improvement loop. None is required: no index is negative, so MODI makes no change. The paper describes this as one MODI iteration; in our terms it is one optimality check and zero improving pivots.

Step 6: Final optimal allocation. Identical to Table 14.

Step 7: Optimal cost. IDR 6,816,230. Divided by 54,880 cylinders, this is about IDR 124 per cylinder (recalculated). The paper does not state the unit of its cost data, so we read the costs as IDR per cylinder; this is consistent with the total.

Step 8: Existing company cost. According to the original paper, the manual cost, calculated from driver experience, is IDR 9,307,000 for a month. The source does not give the allocation or calculation behind this figure.

Step 9: Absolute and percentage saving.

$$9{,}307{,}000 - 6{,}816{,}230 = 2{,}490{,}770 \qquad \frac{2{,}490{,}770}{9{,}307{,}000} = 26.76\%$$

The figures match the paper. Sensitivity to the data discrepancy (Table 6, †): if S2–Banguntapan costs 85 instead of 83, the optimal allocation is unchanged and still optimal (k12 becomes 17), the cost becomes IDR 6,821,950, the saving IDR 2,485,050 and the percentage 26.70% (recalculated). The conclusion does not change.

Step 10: Interpretation. Allocating cylinders by TOC and MODI instead of by driver experience reduces monthly cost by about a quarter. The optimal plan serves 12 districts from S1 (40,320 cylinders, 700 of them for Jetis) and uses S2 (14,560 cylinders) fully for the four districts where it is much cheaper: Banguntapan, Dlingo, Jetis and Pleret. The source does not show the company's own allocation, so the exact origin of the saving cannot be traced.

X. RESULTS AND COMPARISON

Table 16. Comparison of P1 and P2 (figures recalculated from the published data and found to match the papers)

Measure	P1: Samarinda	P2: Bantul
Initial / current cost	IDR 45,000,000 per month (existing practice; cannot be independently verified)	IDR 9,307,000 per month (manual, driver experience)
Optimised cost	IDR 24,031,104 per month (IDR 3,003,888 per delivery × 8)	IDR 6,816,230
Absolute saving	IDR 20,968,896	IDR 2,490,770
Percentage saving	46.6%	26.76%
Number of sources	5 facilities	2 SPPBE stations
Number of destinations	6 agents	15 districts
Volume	6,000 cylinders per delivery	54,880 cylinders (September 2023)
Method	NWC + MODI; Least Cost Method	IVAM + MODI
Initial solution optimal?	NWC no; LCM yes	IVAM yes
Optimality confirmed?	Yes: all MODI indices at least 45	Yes: all MODI indices at least 2
Source of cost data	Secondary (Ratnasari et al., 2019)	Interview with company director; fuel and mileage

Which study achieved the greater percentage improvement? P1 (46.6% against 26.76%). This does not show that its method is superior, for four reasons.

- Every method reaches the same minimum. NWC with MODI, Least Cost and IVAM with MODI each end at the minimum cost of their own problem, so the saving percentage measures how far the existing practice was from that minimum, not which method is better.
- The baselines differ. P1's IDR 45 million comes from an earlier study and P2's IDR 9.3 million is a manual estimate by the company; neither calculation is shown. A poorer baseline produces a larger percentage.
- The problems differ. One is a 5 × 6 problem with 6,000 cylinders per delivery, the other a 2 × 15 problem with 54,880 cylinders, and the cost data are built differently (the unit of P1's costs is not stated).
- The time bases differ. P1 scales a per-delivery cost by eight deliveries; P2 uses one month's data.
- Our analysis indicates that the two studies are not directly comparable. They support the same conclusion: a formal allocation model lowers cost against unplanned practice.

XI. SOFTWARE AND VALIDATION

Table 17. Software and validation for P1 and P2

Question	P1	P2
Software or tool used	The source does not name any software.	The source does not name any software.
Calculated manually?	The calculations are presented as hand-worked tables.	The calculations are presented as hand-worked tables.
How optimality was verified	MODI indices: negative for NWC, all positive for the final solution	MODI indices: all positive for the IVAM solution
MODI confirmed optimality?	Yes	Yes

Compared with company practice?	Yes (IDR 45,000,000 per month)	Yes (IDR 9,307,000 per month)
Our independent check	Recomputed in Python; a linear-programming solver returns the same minimum	Recomputed in Python; a linear-programming solver returns the same minimum and allocation

Neither paper claims to have used software for the transportation calculations, and we do not claim otherwise. Validation in both is internal (a MODI test) plus a comparison with the company's own figure; neither tests its allocation in operations. For the extension papers, the sources state that P3 used MATLAB 2017a with 16 benchmark instances, P4 used MATLAB 2014a with a numerical example, and P5 used Python (PuLP and a map API) with 216 real test cases.

XII. CRITICAL ANALYSIS

12.1 Strengths

Simplicity and transparency. The model needs only unit costs, supplies and demands, and every step, from the NWC allocation to the MODI index, can be reproduced by hand and checked by a manager with no OR training. Our recomputation matched all 20 MODI indices printed for P1 and all 14 printed for P2.

Explicit constraints and a proof of optimality. Supply and demand balance guarantee a plan that can be executed, and both final allocations satisfy every constraint exactly. Because every improvement index for an unused route is strictly positive (at least 45 in P1 and at least 2 in P2), no other allocation under the stated data costs the same or less, and an independent linear-programming solver confirmed both minima. A manager can therefore state that the plan is the cheapest possible for the given data, not merely better than before.

Easy managerial interpretation. The output is a table of cylinders per route that converts directly into dispatch instructions, and the benefit is stated in money (IDR 20,968,896 a month in P1 and IDR 2,490,770 in P2).

Flexible methods and low requirements. The initial method affects effort, not the final answer: LCM reached the optimum with no further work in P1, NWC needed several improving steps, and IVAM's start was already optimal in P2. Both papers solved their problems by hand, so a small distributor can repeat the calculation in a spreadsheet whenever inputs change.

12.2 Limitations

- Linear, fixed unit costs. The cost of a route is a fixed amount per cylinder, whatever the quantity. Real transport costs depend on trips, fuel and vehicle loading.
- Fixed supply and demand, static view. Quantities are known and constant, and one planning period is solved at a time. Demand for cooking fuel varies by period, and P1 assumes there is never a stock-out.
- No route sequencing or vehicle limits. The model decides how many cylinders move between each pair of points, not the order in which a vehicle visits customers (see P3 and P5). P1 mentions a truck capacity but does not use it as a constraint, and neither model counts trips.
- Traffic, time windows and safety risk are ignored. Delivery times, road conditions, customer availability and the risk of a route or time of day do not enter the cost matrix (see P4).
- Dependence on data quality. The result is only as good as the cost matrix. P2 derives costs from fuel and mileage and P1 takes them from an earlier study, and neither paper reports validating them against actual delivery records.

12.3 Data and methodological issues in the selected papers

The items below are inconsistencies that the sources themselves support. None changes the main conclusion, but each should be clarified before the reported results are treated as fully reproducible. We do not suggest that the authors acted improperly.

Table 18. Discrepancies identified during our analysis

ID	Paper	Discrepancy	Effect on the results
D1-1	P1	The paper reports two MODI iterations but shows only the NWC tableau and the final tableau. Our recalculation needed three improving pivots.	None on the final cost; the intermediate steps cannot be reproduced from the paper.

D1-2	P1	Assumption 2 states a truck capacity of 3,000 cylinders per delivery, but the dataset and model have 6,000 cylinders per delivery.	None on the model, which has no capacity constraint; the assumption conflicts with the data.
D1-3	P1	The Introduction refers to delivery to five retail agents, but the dataset and model have six.	The source does not provide sufficient information to establish whether the IDR 45 million baseline covers all six agents.
D1-4	P1	The optimum (IDR 24,031,104) and the saving (46.6%) against IDR 45,000,000 are identical to those reported by Ratnasari et al. (2019), the stated source of the data. That study names its company PT Tri Pribumi Sejati; P1 names PT ABC.	P1 acts as a method comparison on an existing dataset; the baseline is inherited, not independently measured.
D2-1	P2	Cost S2–Banguntapan is 85 in the cost table but 83 in the transportation table and objective function.	With 85, the cost is IDR 6,821,950 and the saving 26.70% (26.76% with 83). The allocation stays optimal.
D2-2	P2	The TOC value for S1–Sewon is printed as 66; recomputed from the paper's own Table 5 it is 68.	None; the allocation is unaffected.
D2-3	P2	Destinations D7 and D9 are labelled "Pity" and "Display" in the cost-table key but are Kasihan and Pajangan in the demand table.	None on the numbers; a labelling issue.
D2-4	P2	The paper says VAM and IVAM gave the same initial solution but does not show the VAM calculation.	None; our recalculation confirms the statement.
D2-5	P2	The unit of the cost data is not stated, and the calculation of the IDR 9,307,000 baseline is not shown.	The saving cannot be independently verified.
D-R	P1, P2	P1 cites Ratnasari et al. as 2019 and P2 as 2020. The journal records show Eksponensial vol. 10 no. 2, November 2019.	A citation error in P2. We cite 2019.

12.4 Applicability to India

Although both selected cases are Indonesian, the transportation formulation transfers to Indian LPG distribution, because the structure of sources, destinations, supplies, demands and unit costs is the same. P5 describes an Indian network in which appointed distributors deliver cylinders to households using small vehicles. Applying the model there would need local data and would be subject to differences in distribution structure, vehicle capacity, cost levels, regulation, demand patterns and safety requirements. No Indian numerical data are used or assumed.

12.5 Practical benefits and implications

In the two cases the models reduced the reported monthly cost by IDR 20,968,896 (P1) and IDR 2,490,770 (P2). For a subsidised product sold under a regulated maximum price, our analysis indicates that a lower distribution cost gives a distributor more room to operate within that price. Both companies relied on driver experience or on serving each demand point from its nearest facility; the models replace this with a documented allocation that can be repeated every month, audited and explained to others. Because P2 builds its costs from fuel consumption and mileage, a lower modelled cost is likely to come with lower fuel use, although neither paper measures fuel or emissions directly. Both models meet all demand exactly, which supports continuous availability of cooking fuel if the data are accurate. Finally, the allocation can feed a routing model (Section 13), so a distributor can start with a simple tool and add detail later.

XIII. POSSIBILITIES FOR IMPROVEMENT

The improvements below are extensions. None is part of the classical transportation model analysed in P1 and P2.

Two-stage optimisation. The most practical extension combines the models in this report with those of the extension papers:

- Stage 1: transportation model. Decide how many cylinders each filling plant sends to each agent or district (the allocation analysed in Sections 8 and 9).
- Stage 2: vehicle routing model. Given those quantities, decide which vehicle visits which customers and in what order (Dantzig & Ramser, 1959). P3 and P5 show how this can be done for LPG, with exact and metaheuristic methods.

Other improvements. Vehicle capacity and the number of trips could be included in the cost, since P1 mentions capacity but does not use it. A routing stage could add time windows and travel-time variation. Dynamic demand would allow re-planning during the period, multiple depots would allow several filling points to serve one area, and reverse logistics would cover the collection and refilling of empty cylinders (P3). Safety risk by route and time of day (P4) and fuel use or emissions could be added as further measures, giving a multi-objective model that shows the trade-off between cost, risk and emissions. Finally, unit costs should be measured from actual delivery records and road distances, and the optimised plan tested in operation.

XIV. CONCLUSION

This report asked how transportation models and classical optimisation techniques can minimise LPG cylinder distribution costs while satisfying source capacities and destination demands. The two studies analysed in depth give the following answer.

- Transportation models suit source-to-destination LPG allocation. Both problems are balanced allocation problems with a uniform product, known supplies and demands, and unit costs (5×6 and 2×15).
- Classical OR methods reduce distribution cost. NWC with MODI and the Least Cost Method reduced P1's monthly cost from IDR 45,000,000 to IDR 24,031,104 (46.6%). IVAM with MODI reduced P2's cost from IDR 9,307,000 to IDR 6,816,230 (26.76%).
- MODI provides an optimality check. Every improvement index was non-negative at the final solution (at least 45 in P1 and 2 in P2), and an independent solver confirmed both minima.
- Results depend on data quality and assumptions. The baselines cannot be independently verified, each paper has inconsistencies to be clarified (Table 18), and the two percentages are not directly comparable.
- The classical model does not capture routing and safety. It decides quantities, not routes, and ignores vehicle capacity, time and risk. A two-stage model, a transportation stage followed by a routing stage, would combine the transparency of P1 and P2 with the route detail of P3 to P5.

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