Warehouse-To-Retailer Distribution Optimization Using a Transportation Linear Programming Model

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***Abstract*—This study develops and verifies a transportation linear programming model for allocation of homogeneous product from three Indian warehouses situated in Bhiwandi, Nagpur, and Gurugram to six retail hubs: Delhi, Mumbai, Bengaluru, Hyderabad, Kolkata, and Ahmedabad. The objective is the minimization of the monthly distribution cost while satisfying hub demand within warehouse capacity. Route costs include road distance, freight rate of INR 3 per tonne-kilometre, and handling charges of INR 150 per tonne. Initial feasible solutions were produced using the Northwest Corner Method and Vogel's Approximation Method, followed by optimality assessment through MODI and Excel Solver. Independent computational verification was done using PuLP/CBC and SciPy/HiGHS. All four exact approaches reproduced the same base optimum of INR 2,705,241, confirming the model's computational consistency. The optimal allocation includes eight of eighteen possible routes and fully utilizes the network's 1,600-tonne capacity. Four synthetic stress scenarios were then examined: a 10% increase in demand, a 20% increase in freight rates, a 30% reduction in Nagpur capacity, and the addition of a 200-tonne Chennai hub.**

Freight inflation increased the cost of transportation to INR 3,198,289 while maintaining full service. Conversely, demand surge, the Nagpur disruption, and Chennai expansion produced unmet demand of 160, 135, and 200 tonnes, which reduced service levels to 90.91%, 91.56%, and 88.89%, respectively. Shortages were concentrated in Bengaluru and Kolkata under a uniform penalty of INR 10,000 per unmet tonne. The findings demonstrate that the base solution is cost-efficient but structurally fragile due to the lack of reserve capacity. The study contributes a clear, reproducible workflow integrating classical transportation techniques, spreadsheet optimization, and two open-source solvers. However, its results should be interpreted as methodological evidence because the inputs and scenarios are synthetic. Future research should introduce real data, demand probability, vehicle constraints, fixed costs, multiple periods, and objectives affecting the environment.

I. INTRODUCTION

It costs Indian organizations significantly to undertake logistics operations. According to a report commissioned by the DPIIT and conducted by the NCAER, India's total logistics expenditure in 2023-24 was ₹24.01 lakh crore. This translates to 7.97% of the country's GDP and 9.09% of non-services output (NCAER, 2025). It is also important to highlight that India's dependency on road transportation remains high with fuel being one of the cost components. In the case of a firm which moves its goods from warehouses to retail markets, small savings on a tonne of transport on each journey translate into big savings in a year.

Choosing the amounts to ship from each warehouse to each market is an old problem in operations research known as the transportation problem. It was originally stated by Hitchcock (1941) and analyzed by Koopmans (1949), and was later solved using the simplex method by Dantzig (1951). It is a special case of linear programming where the objective is to transport commodities from sources having limited supplies to destinations with predetermined demands at minimum total costs. The basic starting solution is determined using Northwest Corner Rule or Vogel's Approximation Method (Reinfeld & Vogel, 1958). This is subsequently refined into an optimum solution using the stepping-stone method (Charnes & Cooper, 1954) or MODI method.

Despite its widespread application, there are continuous efforts to extend this model with multi-echelon networks and more than one transport mode under conditions of uncertainty in demand. In this paper, bibliometric analysis – the analysis and classification of existing literature (Donthu et al., 2020; Verma & Gustafsson, 2020) – is used to examine the progress in the development of research on transportation model in distribution.

The transportation model is applied to the distribution network in India. There are three warehouses that are located in Bhiwandi, Nagpur and Gurugram. They provide six retail hubs located in Delhi, Mumbai, Bengaluru, Hyderabad, Kolkata and Ahmedabad. Each warehouse has a constant capacity per month, and each hub has a constant demand per month. The cost of transportation of one ton of goods is equal to the road distance from warehouse to hub multiplied by the freight rate and added to the handling charge for one ton of goods. Road distances are real, obtained from OSRM, whereas supply, demand, the freight rate and the handling charge are hypothetical. The goal is to develop the optimal distribution plan of transportation and investigate sensitivity of this plan to changes in demand, costs and capacity.

The study addresses four research questions:

- RQ1. What is the minimum-cost distribution plan for the 3-warehouse, 6-hub network, and which routes does it use?
- RQ2. How close do the Northwest Corner and Vogel's Approximation initial solutions come to the optimal cost found by MODI and Excel Solver?
- RQ3. How sensitive is the optimal plan and total cost to changes in demand, freight cost, warehouse capacity and the addition of a new hub?
- RQ4. What does the bibliometric literature show about trends, dominant methods and gaps in transportation-model research for distribution?

The scope is that of one product only, in tonnes for one month and transported via road. Base Case is balanced, with total supply equal to total demand, i.e., 1,600 tonnes. In unbalanced cases, dummy row/column approach is used. Time windows, vehicle routing and holding cost are out of scope.

There are three contributions in the study. Comparison of manual heuristics (North West Corner Method and Vogel's Approximation Method) against

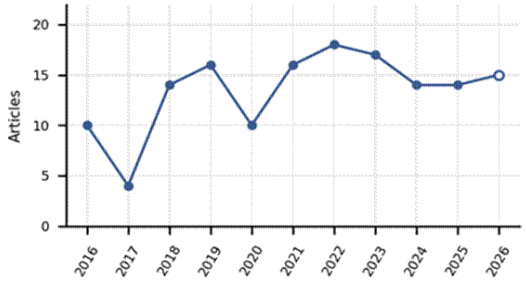
MODI and Excel Solver, and then verification in Python. Four different scenarios will be tested on a network based on actual road distance in India. Lastly, connection to the relevant literature via a bibliometric study.

The rest of the paper will be structured as follows. Literature and bibliometric studies will be presented in Section 2. Model and Methodology will be described in Section 3. Data, results and scenarios will be explained in Section 4. Managerial implications and limitations will be discussed in Section 5.

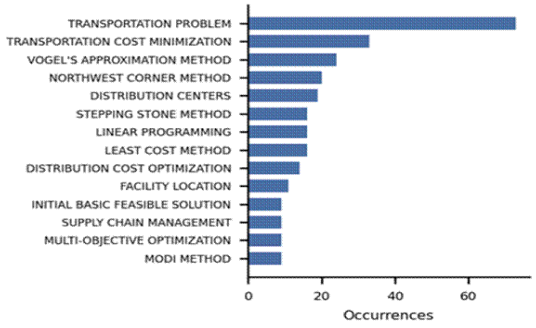
II. LITERATURE REVIEW

Bibliometric overview

The search shown in Section) showed 152 records on the transportation problem and warehouse to distribution cost minimization. The records are published between 2016 and 2026. Removing four duplicates left us with 148 different records and 107 contain an abstract. The number lf papers published are between 4 to 18 records per year with the maximum coming in 2022 with 18 different papers written (Figure 1)



The most common keywords are transportation problem (73 entries) and cost minimization (33 entries), followed by Vogel's approximation method (24 entries), northwest corner method (20 entries), and distribution centers (19). If we count topics in title, abstract, and keywords together, classical methods of finding the initial solution and optimal solutions (northwest corner, least cost, Vogel, MODI, stepping stone) have 45 entries, and linear or mixed-integer programming formulation has 51 entries. Location problems have 31 entries, while the two-stage or multi-echelon structure is found in 29 entries. Fuzzy or neutrosophic parameters (23), metaheuristics (24), and routing, drones or robot problems (22) are different side streams. Sensitivity analysis is mentioned in 7 entries and last mile delivery in 4.



Thematic synthesis

Table 1 shows the papers that are the core studies relating to using transportation problems in warehouses

Table 1: Core studies (coded from abstracts where available).

Study	Problem	Method	Data
Pop et al. (2017)	two-stage fixed-charge transportation, manufacturers to customers via distribution centers	hybrid genetic algorithm with local search	benchmark
Tanksale and Jha (2017)	multi-region multi-facility inventory allocation and transportation, Indian public distribution system	unclear	case study (tagged)
Cosma et al. (2019)	two-stage transportation with fixed charge for opening distribution centers	constructive heuristic	benchmark + synthetic
Mollanoori et al. (2019)	solid step fixed-charge transportation extended to two-stage networks and multi-item shipments	unclear (no abstract)	unclear
Akpan et al. (2020)	product distribution, two plants to three distribution centers (Nigeria)	Vogel's approximation LP model	real
Hosianna and Ningsih (2020)	detergent distribution from distribution centers to minimarkets (Serang, Indonesia)	Vogel's approximation method	case
Sahu (2021)	tutorial on single-product transportation from origins to warehouses	northwest corner, matrix minimum, Vogel, UV method	illustrative examples
Dalal (2022)	teaching exercise: warehouse siting and optimal distribution plan (India)	OpenSolver	spatial + synthetic
Maheswari (2022)	new initial-solution rule for transportation across plant, warehouse, distribution center and retail store	New Vogel's approximation method	unclear
Mallick et al. (2023)	drug shipments from factories to warehouses (India)	Vogel start, stepping stone	case
Stetsyuk et al. (2023)	two-stage transportation with unknown consumer demands within lower and upper bounds	mathematical model	unclear (theoretical)
Christofel et al. (2024)	shipping cost at a telecom warehouse (Cawang, Indonesia)	northwest corner start, stepping stone	case
Dong et al. (2025)	factories, intermediate warehouses and retail stores in the automobile industry (China)	integer linear programming with spanning-tree structure, CPLEX	case-based experiments

Classical methods and single case applications:

Most of the papers are focused on standard textbook methods and are tested on a single real-world scenario with no plans in case of unexpected events. Most of them prove that these classic techniques can cut transportation costs. For example, Akpan et al. used VAM for two factories with three depots in Nigeria, which saved around 9.76 million in the local currency. Christofel et al. (2024) used a mix of the Northwest Corner Rule and the Stepping Stone method, which led to a reduction in shipping costs of around 23%. After understanding India's pharmaceutical deliveries, Mallick et al. tested VAM and the Stepping Stone method. They found that the latter works better when

mathematical ties or degenerate solutions arise. The remaining papers studied are more technical in nature. Sahu has offered a step-by-step tutorial on these methods, while Dalal has made use of Open Solver to solve these problems and map out delivery routes.

Multi-stage, fixed charge, and network extensions:
Several researchers have focused on two stage networks in which each depot has a setup fee. Pop et al. (2017) and Cosma et al. (2019) have tackled the fixed charge by using specialised optimisation algorithms that are a hybrid genetic approach along with a step-by-step heuristic. These are tested on standard benchmark scenarios. Nollanoori et al. (2019) enhanced it more by using multiple products and tiered shipping rates. While Stetsyuk et al.2023 designs the two-stage setup in such a way where customers demand is fluctuating within the given bounds. Other papers focus on multi-tier supply chains. Dong et al. (2025) built an integers programming model which tracks the movement of goods across the three levels that are factories, regional depots, and retail shops. Across all these studies there are a balance on services and costs that are structural overheads like rents,

Research gaps
The research papers and articles, after analysis, showed three main research gaps. First, very few studies had evaluated the sensitivity or dealt with uncertainties if they had single-outcome plans without any scenario. Second, researchers have hardly explained where the cost of transportation comes from; they are not using real rates, but rather given numbers. Lastly, small regions are hardly studied, and they play a key role in the Indian economy. We aim to address these issues in a clear and focused way; hence, we model a network of three central warehouses in the rural parts of India supplying six major cities. We have also made use of elastic rates rather than a straight line.

III. BIBLIOMETRIC ANALYSIS: TRANSPORTATION PROBLEM & WAREHOUSE DISTRIBUTION

Bibliometric data collection
The bibliographic data were obtained from a single RIS export containing 152 records on the transportation problem in warehouse and distribution contexts, published from 2016 to 2026, retrieved from OpenAlex on 7 October 2026, restricted to the article document type, with no language filter applied. The Boolean search string, applied to titles, abstracts and keywords, was ("warehouse" OR "distribution center") AND "transportation problem". The export lists titles, authors, authors' affiliations, journal, year, keywords, DOIs and the abstracts of 107 of the 148 unique records. It does not include citation counts or reference lists, so citation-based indicators, co-citation and bibliographic coupling were not calculated. The file was parsed using a custom python script, and 4 duplicates (matched to normalized title) were eliminated, which resulted in 148 papers.
No screening was done beyond de-duplication, so 148 records were retained. All records returned by the search were treated as eligible, and no inclusion or exclusion criteria were applied. Variants were combined, countries extracted from the affiliation field (missing for 38 records), and themes tagged to keyword rules.

Performance analysis
Performance analysis covered annual production, publication outlets, authors and countries.

Annual scientific production

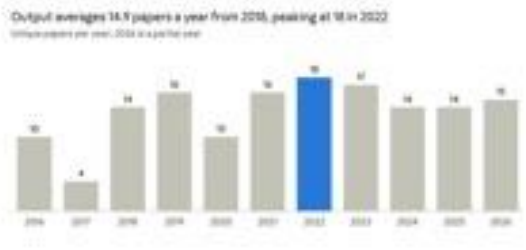


Figure 1: Annual scientific production, 2016–2026 (2026 is a partial year)

Production averaged 14.9 papers a year from 2018 to 2026 with no clear trend, peaking at 18 in 2022 (Figure 1). 2017 was the low point (4 papers), 2016 includes one re-dated older paper (Section 3.4), and 2026 is a partial year.

Sources and languages
There were 148 papers in 133 outlets – 122 (92%) of them had one contribution, 4 papers appeared in 4 journals [Expert Systems with Applications (4)]; others, 2 each: IEEE Access, Computers & Industrial

Engineering, the International Journal of Operational Research, RAIRO – Operations Research, Transportation Research Part E and Mathematics. Elsevier published 15 papers and Springer - 8. Records' language tags: 128 (86%) - English, 6 - Indonesian and 3 - Arabic.

Authors and collaboration

The papers had 397 distinct authors, 372 (94%) of whom wrote a single paper. Manoranjan Maiti was the most productive author (9 papers), followed by Dipak Jana (4). Papers averaged 2.95 authors, and 123 (83%) were co-authored. The largest cluster is Maiti's group on fuzzy, multi-item and solid transportation models, followed by Pop, Cosma and Sabo on two-stage fixed-charge problems and Tanksale and Jha on inventory-transportation models for Indian food distribution.

Country contributions

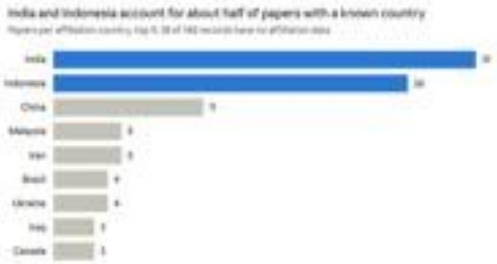


Figure 2: Papers by affiliation country (top 9; a paper count once per country)

Affiliation data identified a country for 110 of the 148 papers. India (31 papers) and Indonesia (26) accounted for 57 of them, followed by China (11), Malaysia (5), Iran (5) and Brazil (4) (Figure 2). Indonesian output was mostly applied case studies of local firms, whereas Indian output ranged from fuzzy and multi-item theory to public distribution models. These counts are minimums because 38 records lack affiliation data.

Science mapping

Science mapping used author keywords and rule-based themes, since the export holds no reference lists.

Keyword co-occurrence

The vocabulary is long (about 580 distinct terms after merging) but concentrated. "Transportation problem" appeared in 73 papers and was omitted from Figure 3 as the search anchor; the most frequent remaining

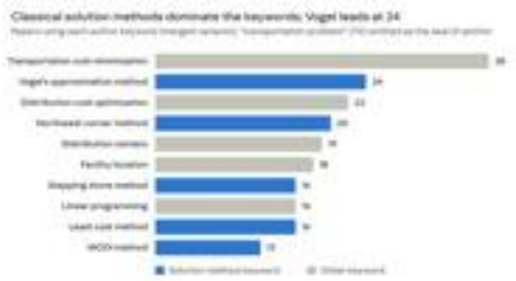


Figure 3: Most frequent author keywords (merged variants; "transportation problem" omitted)

keywords were transportation cost minimization (38), Vogel's approximation method (24), distribution cost optimization (22) and Northwest Corner method (20). The strongest pairs linked the problem to a method: Vogel (23 papers), Northwest Corner (20), Stepping Stone (15) and MODI (12). Stepping Stone and MODI ranked among the top keywords in 2023 to 2026 papers, whereas facility location, distribution centers and two-echelon supply chains led in 2016 to 2020, indicating a shift from model-driven to method-driven applied work.

Thematic structure

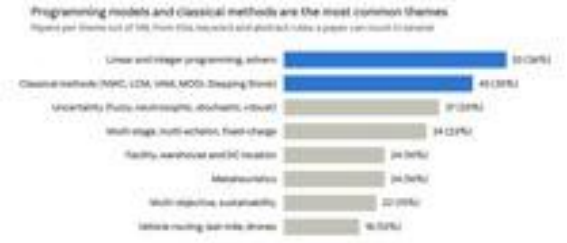


Figure 4: Thematic structure (rule-based tagging; a paper can count in several themes)

Eight rule-based themes were tagged (Figure 4). Linear and integer programming (53 papers, 36%) and classical transportation methods (45, 30%) were the most common, followed by uncertainty modelling (37), multi-stage and fixed-charge networks (34), facility and warehouse location (24) and metaheuristics (24). The literature splits into an applied stream of single-company case studies using classical methods and a theoretical stream on uncertainty, multi-stage networks and metaheuristics.

Implications and limitations

These findings suggest standard approaches to an operations research project: Construct the initial

feasible solution using VAM and verify optimality using MODI or Steppingstone, the most popular combination of techniques in the collection. In the papers that compared initial methods, VAM had the lowest initial cost.

Three gaps emerge. Distribution is sparsely covered (six-ish papers, mostly Indonesian and of location planning type). Classical approaches do not often combine with richer network types, since two-stage, fixed-charge and location type models are as likely to be seen in combination with metaheuristics. Uncertainty and sustainability are a theoretical stream, with little applied work on real companies.

This analysis has these caveats: no citation or reference data available for export; theme counts are based on keyword rules so it is possible for one paper to be in multiple themes; 2026 is incomplete; one record that lists 2016 as the publication date looks like an earlier paper that was simply re-indexed as from a later date. Future work should re-export the query with cited references, from Scopus, Web of Science or the referenced-works field in OpenAlex, and map it in Bibliometrix for citation and co-citation analysis.

IV. METHODOLOGY

The following subsection presents the way the distribution of goods from the warehouses to the retailers is mathematically structured and solved. The mathematical structuring used here involves a Balanced Transportation Problem that was first introduced by Hitchcock (1941), and then efficiently solved using the specialized simplex method created by Dantzig (1951). The procedure involved four major steps, namely; (i) formulate the Linear Programming, (ii) develop two Initial Basic Feasible Solutions using the Northwest Corner (NWC) and Vogel's Approximation Method (VAM), (iii) test and refine the more optimal Initial Basic Feasible Solution using the MODI method, and finally (iv) verify the solution using the Simplex LP engine of Excel Solver.

4.1. Problem setting and assumptions

One product is transported by road from three warehouses (Bhiwandi, Nagpur, Gurugram) to six retail centers (Delhi, Mumbai, Bengaluru, Hyderabad, Kolkata, Ahmedabad) within a one-month planning horizon. The capacity at each warehouse per month is 600, 450, and 550 tonnes while the demand at each retail center per month is 380, 320, 280, 220, 210, and 190 tonnes respectively. Both total supply and demand is equal to 1,600 tonnes so that the basic model will be balanced and there is no need for a dummy row or column. Supply, demand, cost per tonne and handling cost per tonne are artificial group data while distance is real data derived from OSRM on 5 October 2026.

4.2. Mathematical formulation

Let i = 1, 2, 3 index warehouses and j = 1,…,6 index retail hubs. The notation is as follows.

Decision variable: x_{ij} = tonnes shipped per month from warehouse i to hub j (18 variables).

Parameters: c_{ij} = unit transport cost (INR per tonne), s_i = supply of warehouse i (tonnes), d_j = demand of hub j (tonnes).

$c_{ij} = D_{ij} \times r + h$, where D_{ij} is road distance in km, r = 3 INR per tonne-km is the freight rate and h = 150 INR per tonne is the handling charge.

The linear programme is:

Minimise $Z = \Sigma_{i=1..3} \Sigma_{j=1..6} c_{ij} x_{ij}$

subject to $\Sigma_j x_{ij} = s_i$ for every warehouse i (supply constraints)

$\Sigma_i x_{ij} = d_j$ for every hub j (demand constraints)

$x_{ij} \geq 0$ for all i, j (non-negativity)

Because total supply equals total demand ($\Sigma s = \Sigma d$ = 1,600), all nine constraints are equalities and exactly one of them is redundant. A basic feasible solution therefore has $m + n - 1 = 3 + 6 - 1 = 8$ positive allocations. Because supply and demand are whole numbers, the transportation structure also guarantees an optimal solution with whole-tonne flows.

Table 1. Unit transport cost c(i,j), INR per tonne

Warehouse \ Hub	Delhi	Mumbai	Bengaluru	Hyderabad	Kolkata	Ahmedabad
Bhiwandi	4,042.8	267.9	3,132.6	2,282.4	5,713.5	1,619.4
Nagpur	3,266.4	2,487.3	3,415.5	1,651.8	3,523.8	2,799.9
Gurugram	240.0	4,047.9	6,148.2	4,661.7	4,620.0	3,111.3

Source: Costs tab of the data sheet; c = distance × 3 + 150.

4.3. Initial basic feasible solutions

North West Corner Method. Shipment begins from the top left cell (Bhiwandi, Delhi). At each step min (available supply, available demand) units are shipped, and one step to the right is made if the demand of a hub is satisfied or a step downwards if a warehouse is depleted. There are eight shipments made: Bhiwandi to Delhi 380 tons and to Mumbai 220 tons; Nagpur to Mumbai 100 tons, Bengaluru 280 tons and Hyderabad 70 tons; Gurugram to Hyderabad 150 tons, Kolkata 210 tons and Ahmedabad 190 tons. The method takes no account of costs; thus, it is fast but yields a poor initial solution. Its total cost is INR 5,176,500.

Vogel's Approximation Method. The penalty for each available row and column is defined as the difference between their two lowest unit costs. The row or column having the highest penalty value is chosen and the maximum amount that can be transported to the cheapest cell within the chosen row or column is determined. Penalties are updated after each shipment until all the supply/demand is allocated. Table 2 shows the eight steps of the algorithm. Total cost is INR 2,724,417.

Table 2. Vogel's Approximation: iteration summary

Step	Highest penalty line	Penalty	Allocation (route)	Qty (t)	Unit cost
1	Col Delhi	3,026.4	Gurugram to Delhi	380	240.0
2	Col Mumbai	2,219.4	Bhiwandi to Mumbai	320	267.9
3	Row Gurugram	1,508.7	Gurugram to Ahmedabad	170	3,111.3
4	Col Kolkata	2,189.7	Nagpur to Kolkata	210	3,523.8
5	Col Ahmedabad	1,180.5	Bhiwandi to Ahmedabad	20	1,619.4
6	Row Nagpur	1,763.7	Nagpur to Hyderabad	220	1,651.8
7	Row Nagpur	3,415.5	Nagpur to Bengaluru	20	3,415.5
8	Row Bhiwandi	3,132.6	Bhiwandi to Bengaluru	260	3,132.6

Full penalty tables for each iteration are in the Vogel tab of the workbook.

4.4. MODI optimality test

The MODI method assigns a row potential u(i) and column potential v(j) so that u(i) + v(j) = c(i,j) for every cell with a positive allocation, fixing u(Bhiwandi) = 0. For every empty cell the opportunity cost is d(i,j) = c(i,j) − u(i) − v(j), the change in total cost per tonne if that route were opened. The plan is optimal when all d(i,j) ≥ 0 for a minimisation problem.

MODI was started from the Vogel solution because it has a lower cost. The potentials are u = (0, 282.9, 1491.9) for Bhiwandi, Nagpur, Gurugram; v = (-1251.9, 267.9, 3132.6, 1368.9, 3240.9, 1619.4) for Delhi, Mumbai, Bengaluru, Hyderabad, Kolkata, Ahmedabad. Table 3 gives the opportunity costs. Only one empty cell is negative: Gurugram to Kolkata with d = −112.8 INR per tonne, so the plan is not yet optimal.

Table 3. MODI iteration 0: opportunity costs d(i,j) from the Vogel solution

d(i,j)	Delhi	Mumbai	Bengaluru	Hyderabad	Kolkata	Ahmedabad
Bhiwandi	5,294.7	basic	basic	913.5	2,472.6	basic
Nagpur	4,235.4	1,936.5	basic	basic	basic	897.6
Gurugram	basic	2,288.1	1,523.7	1,800.9	-112.8	basic

Negative value shown in red. “basic” marks cells with a positive allocation.

The route Gurugram to Kolkata comes into the picture. The closed route will be +Gurugram-Kolkata, −Gurugram-Ahmedabad, +Bhiwandi-Ahmedabad, −Bhiwandi-Bengaluru, +Nagpur-Bengaluru, −Nagpur-Kolkata. The minimum value in the negative cells will be θ = min (170, 260, 210) = 170 tonnes. Moving 170 tonnes through the loop decreases the cost by 170 × 112.8 = INR 19,176, making the total cost INR 2,705,241. Re-computing the values of potential (u = (0, 282.9, 1379.1); v = (-1139.1, 267.9, 3132.6, 1368.9, 3240.9, 1619.4).) reveals that every vacant cell will have d(i,j) > 0 (minimum d(i,j) being 112.8 in the case of Gurugram to Ahmedabad route), indicating that the current solution is optimal. Since none of the d(i,j) is equal to zero, the optimal solution obtained is unique. In case if we start with NW corner solution, MODI will require four iterations to arrive at the optimal solution.

4.5. Excel Solver base solution

Solver reported that it found a solution and that all constraints and optimality conditions were satisfied (Table 4). The minimum total cost is INR 2,705,241, identical to the MODI result, and the 18 variable values reproduce the optimal plan in Table 5 exactly. All nine supply and demand constraints are binding with zero slack, as expected for a balanced problem, and Solver returned whole-tonne flows without an integer restriction, consistent with the integrality property of the transportation problem.

Table 4. Excel Solver Answer Report summary

Item	Result
Solver engine	Simplex LP
Iterations / solution time	14 iterations, 0.032 seconds
Objective cell	$B19$ (Min); original value 0, final value INR 2,705,241
Variable cells	$B13{:}G15$ (18 cells, continuous, non-negative)
Constraints	9 equality constraints (3 supply, 6 demand), all binding, slack 0
Solver message	Solver found a solution. All constraints and optimality conditions are satisfied.

4.6. Comparison of the three solutions

Routes used in at least one route by each model have been presented in Table 5 and Table 6 presents cost. In the optimized solution, eight out of 18 routes are used and the cost per tonne stands at INR 1,690.8. The route from Gurugram goes to the full capacity of Delhi and 170 tonnes of Kolkata. The route from Bhiwandi caters to Mumbai, Ahmedabad, and partial Bengaluru, whereas the route from Nagpur caters to Hyderabad, remaining Bengaluru, and 40 tonnes of Kolkata. The cost of NWC is 91.4 percent higher than optimum, while VAM is just 0.71 percent higher than optimum (INR 19,176).

Table 5. Allocations (tonnes) by method; green cells carry flow

Route	Unit cost (INR/t)	NWC (t)	Vogel (t)	Optimal (t)
Bhiwandi to Delhi	4,042.8	380	-	-
Bhiwandi to Mumbai	267.9	220	320	320
Bhiwandi to Bengaluru	3,132.6	-	260	90
Bhiwandi to Ahmedabad	1,619.4	-	20	190
Nagpur to Mumbai	2,487.3	100	-	-
Nagpur to Bengaluru	3,415.5	280	20	190
Nagpur to Hyderabad	1,651.8	70	220	220
Nagpur to Kolkata	3,523.8	-	210	40
Gurugram to Delhi	240.0	-	380	380
Gurugram to Hyderabad	4,661.7	150	-	-
Gurugram to Kolkata	4,620.0	210	-	170
Gurugram to Ahmedabad	3,111.3	190	170	-

Routes not listed carry no flow under any method.

Table 6. Total cost by method

Method	Total cost (INR)	Gap to optimum (INR)	Gap (%)	MODI iterations
Northwest Corner	5,176,500	2,471,259	91.4 %	4
Vogel's Approximation	2,724,417	19,176	0.71 %	1
MODI / Excel Solver (optimal)	2,705,241	0	0.00 %	-

MODI iterations counted from each starting solution (entering cell = most negative d). Optimal cost verified independently with an LP solver in Python.

4.7. Limitations of the method

It is a model with one period, one product, and linear cost function. The truck capacity constraints, integer constraints for the number of vehicles used, and fixed route costs, time in-transit constraints, and stochastic demands are not included. These simplifying assumptions have been made to make sure that the comparison between the algorithms becomes the sole focus of attention.

V. SCENARIO ANALYSIS

Scenario Design

The base model establishes the distribution framework under fixed supply, demand and transportation cost parameters. To see how this plan changes when the operating environment shifts, we tested four scenarios. Each scenario modifies a single variable within the distribution network, allowing for an isolated assessment of changes in transportation costs, service levels and route allocations.

The selected scenarios are based on the factors outlined in the research question. They cover changes in demand, freight pricing, warehouse availability and the addition of a new retail hub.

This comprehensive approach evaluates not only fluctuations in total expenditure but also shifts in the best allocation strategy between warehouses and retail hubs.

Scenario Assumptions

In the 1st scenario the freight rate goes up by 10% from Rs. 3 To Rs. 3.30 per tonne kilometre. The handling charge remains fixed at Rs. 150 per tonne because this uniform increase applies strictly to the transport component across all routes, meaning the relative cost competitiveness of the existing pathways remains unchanged.

In the 2nd scenario demand at every retail hub rises by 10%. Total demand therefore scaling from 1,600 tonnes to 1,760 tonnes while total warehouse supply is kept at 1,600 tonnes. This creates a capacity stress test because the available supply is not enough to meet all demand.

The 3rd scenario looks at removing the Nagpur warehouse. The supply from the remaining warehouses drops to 1,150 tonnes while total demand stays at 1,600 tonnes. This scenario helps me see how losing one warehouse affects the networks' ability to serve the hubs.

The 4th scenario adds Pune as a retail hub. To isolate the effect of adding this hub while keeping total demand the same we also reduced Mumbai demand from 320 tonnes to 160 tonnes and gave Pune 160 tonnes of demand. The extra road distances for Pune are 148.5 km from Bhiwandi, 680.2 km from Nagpur and 1,266.9 km, from Gurugram.

Results

The results of all the four scenarios are compared with the base solution below. The base model serves all 1,600 tonnes at a total transportation cost of Rs. 2,705,241. The scenario results show how changes in different aspects of the network affect both transportation cost and the amount of demand that can be served.

Scenario	Demand	Tonnes served	Unmet demand	Transportation cost (INR)	Cost per tonne served (INR)	Routes used
Base case	1,600	1,600	0	2,705,241	1,690.78	8
Freight rate +10%	1,600	1,600	0	2,951,765.10	1,844.85	8
Demand +10%	1,760	1,600	160	2,375,246.40	1,484.53	7

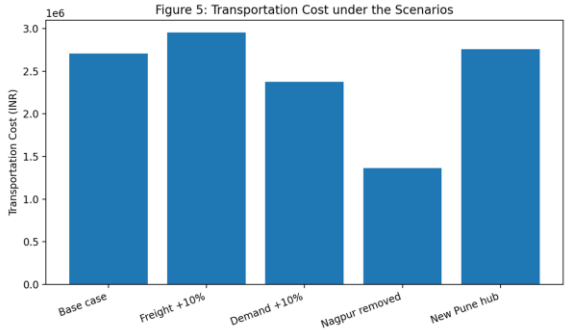
Nagpur warehouse removed	1,600	1,150	450	1,365,489	1,187.38	6
New Pune hub	1,600	1,600	0	2,757,657	1,723.54	9

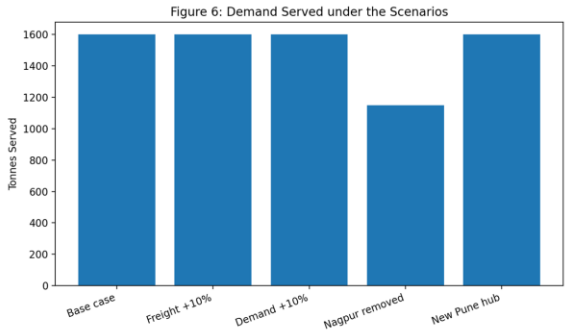
1st scenario: Increase in freight rate: The increase in freight rate does not change the allocation of the network. As the increase in freight rate is applied uniformly to every route the relative cost advantage of the existing routes stays the same. The overall transportation cost rises, showing that the network is still sensitive to changes in freight price even if the distribution pattern stays stable.

2nd scenario: Increase in Demand: The increase in demand creates a capacity constraint because the total requirement is now larger than the warehouse supply. The model therefore prioritises the supply at the retail hubs however some demand still remains unserved. The shortage is mainly seen at the Bengaluru and Kolkata hubs. This shows that the existing network has limited flexibility when demand rises beyond capacity. The lower transportation cost in this scenario should not be seen as an improvement because it comes from the network being unable to serve the demand rather than from being most efficient.

3rd scenario: removing a warehouse facility: the removal of the Nagpur warehouse has an impact on the network, losing this warehouse cuts the available supply and leaves a large part of demand unserved, especially at Bengaluru and Kolkata. This shows the importance of Nagpur in the existing distribution network. It shows that reducing transportation cost is not always beneficial if it comes at the expense of service coverage.

4th scenario: adding a new retail hub: the addition of Pune has an effect on the existing network. All demand continues to be served even though the overall transportation cost goes up a little. The existing warehouse allocation stays the same with some demand moving from Mumbai to the new Pune hub. This suggests that the network can fit the hub without a big disruption although the new route structure is a bit more costly, than the previous arrangement.





Changes in the Optimal Allocation

The changes in route allocation show more clearly how the network responds to each scenario. The freight rate increase does not change the allocation because every route experiences the same proportional increase in its freight component. The remaining scenarios result in changes because they alter either the available capacity or the demand structure of the network.

Scenario	Route affected	Base allocation	New allocation	What changed
Freight rate +10%	All routes	Same as base	Same as base	No allocation change
Demand +10%	Bhiwandi to Mumbai	320	352	Increase
	Bhiwandi → Bengaluru	90	39	Decrease
	Bhiwandi → Ahmedabad	190	209	Increase
	Nagpur → Bengaluru	190	208	Increase
	Nagpur → Hyderabad	220	242	Increase
	Gurugram → Delhi	380	418	Increase
	Gurugram → Kolkata	170	132	Decrease

Nagpur warehouse removed	Nagpur routes	450	0	Warehouse unavailable
	Bhiwandi → Hyderabad	0	220	New allocation
	Bhiwandi → Ahmedabad	190	60	Decrease
	Gurugram → Ahmedabad	0	130	New allocation
	Gurugram → Kolkata	170	40	Decrease
New Pune hub	Bhiwandi → Mumbai	320	160	Decrease
	Bhiwandi → Pune	0	160	New route

Interpretation

The scenario analysis shows that the distribution network remains relatively stable when the freight rate changes uniformly. The distribution network is considerably more sensitive to changes in demand and warehouse capacity. A ten percent increase in freight cost raises the transportation cost without changing the route allocation. This indicates that the base allocation is mainly determined by the distances and costs between the warehouses and retail hubs.

The increase in demand highlights the importance of supply capacity. Although the model continues to use all 1,600 tonnes of supply the model cannot meet the 1600 tonnes of demand. The unmet demand at Bengaluru and Kolkata shows that these spots become more difficult to serve when the distribution network faces a shortage of capacity.

The removal of Nagpur creates a larger service problem. The scenario leaves 450 tonnes of demand unmet, particularly affecting Bengaluru and Kolkata. This shows that Nagpur plays a role in the distribution network especially for Bengaluru and Kolkata. It also demonstrates why a lower transportation cost alone cannot be used to judge the quality of a solution when part of the demand remains unserved.

The addition of Pune has an effect on the overall distribution network. All demand continues to be served although the total transportation cost increases by 1.94%. The distribution network is therefore able to absorb the hub without a major disruption but the location of the additional demand still affects the overall transportation cost because the new route structure is less economical, than the original allocation.

Overall, the scenarios show that transportation cost and service level need to be considered. The cheapest solution is not necessarily the most effective solution if it leaves a significant amount of demand unmet. The analysis therefore gives a practical view of the base transportation model by showing how the optimal plan behaves under different operating conditions.

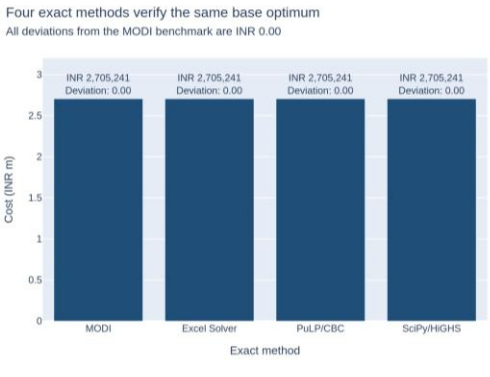
VI. RESULTS, DISCUSSION & LIMITATIONS

Aggregate Analysis of Five Graphs

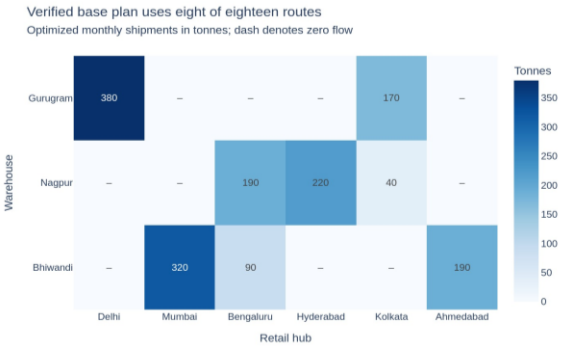
The five figures cumulatively establish the model's efficacy, efficiency, allocation logic, and vulnerability under stress. First, the verification chart shows that MODI, Excel Solver, PuLP/CBC, and SciPy/HiGHS independently generate the identical base objective of INR 2,705,241, with zero deviation. This result supports computational correctness and reduces the possibility of error. Second, the allocation heatmap demonstrates that only eight of eighteen possible routes carry positive shipments. Gurugram primarily serves Delhi and Kolkata, Bhiwandi serves Mumbai, Ahmedabad, and part of Bengaluru, while Nagpur serves Hyderabad and shares Bengaluru and Kolkata.

The network therefore minimizes cost through selective route concentration. Third, the method comparison showcases that the Northwest Corner solution is 91.35% above the optimum, whereas Vogel's Approximation Method is only 0.71% higher. VAM consequently provides a stronger initial feasible solution, although exact optimization remains essential. Fourth, scenario cost decomposition shows that a 20% freight-rate increase raises transport expenditure to INR 3.198 million without disrupting service. In contrast, demand growth, the Nagpur capacity reduction, and Chennai expansion generate penalty costs because supply becomes insufficient. Their lower transportation expenditure cannot be understood as savings because fewer tonnes are served.

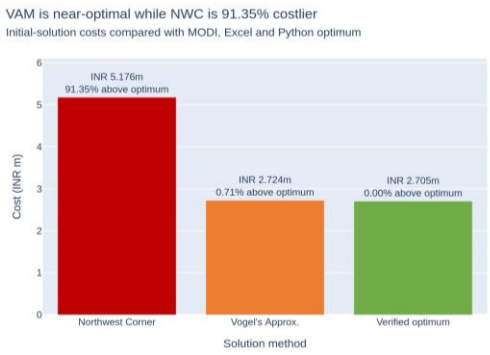
Finally, the resilience chart confirms 100% service in the base and freight-inflation cases, but service declines to 90.9%, 91.6%, and 88.9% under demand growth, disruption, and expansion. Shortages fall on Bengaluru and Kolkata, demonstrating that full capacity utilization creates efficiency at the cost of resilience and that expansion requires additional supply or service prioritization.



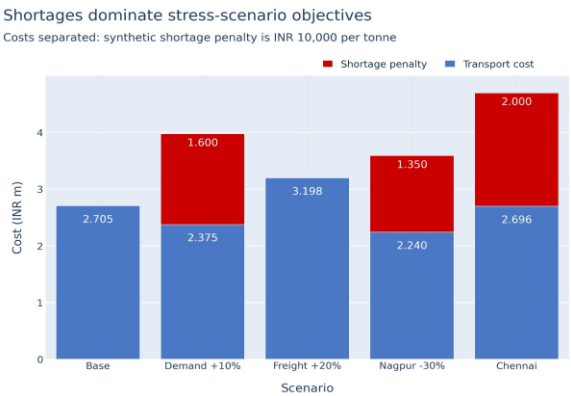
Graph 1



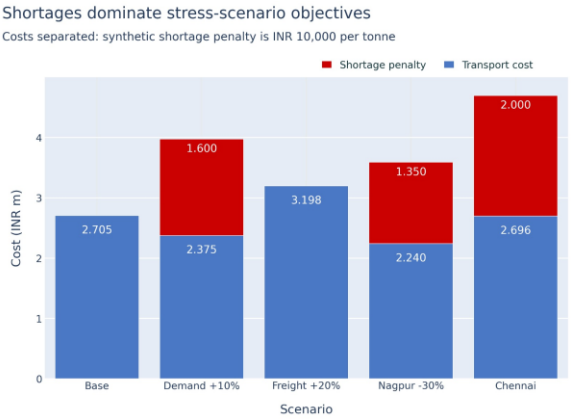
Graph 2



Graph 3



Graph 4



Graph 5

Discussion

The results portray that computational verification is a substantial part of the analysis rather than a procedural afterthought. MODI, Excel Solver, PuLP/CBC, and SciPy/HiGHS all provide the same minimum cost of INR 2,705,241, while the Python models satisfy supply, demand-balance, and non-negativity conditions. This convergence shows strong evidence of internal accuracy. The route heatmap also reveals a highly concentrated solution: only eight of eighteen arcs are active, and every warehouse is fully utilized. Such concentration is economically efficient because the model gives priority to comparatively inexpensive routes, but it eliminates capacity that is spare and increases exposure to disruption.

The scenarios clarify this efficiency-resilience trade-off. Freight inflation preserves the physical allocation and 100% service but leads to an increase in transport cost by 18.23%, slightly below the 20% freight shock because handling charges remain unchanged. The other scenarios create structural shortages and not optimization failures. Demand growth is greater than the capacity by 160 tonnes, the Nagpur disruption removes 135 tonnes, and Chennai adds 200 tonnes without any increment in supply. Consequently, service plummets below 92% in each case. Under a common shortage penalty, the optimizer concentrates unmet demand in Bengaluru and Kolkata because these hubs are comparatively expensive to serve. Although cost-minimizing, this allocation may lead to a conflict with contractual priorities or strategic customer relationships. Managers should therefore supplement the base model with provision for capacity buffers, hub-specific shortage penalties, constraints

for minimum service, and contingency sourcing before using the solution operationally.

Limitations

The study has a few limitations that impact managerial generalization. First, the model examines the distribution of a single product during one monthly planning period. It therefore fails to capture seasonal fluctuations in demand, product-specific handling requirements, inventory carryovers, or changes in warehouse capacity over time. Second, the formulation is deterministic. It treats demand, capacity, and route cost as known constants, thereby excluding forecast error, fuel-price volatility, travel-time uncertainty, and disruption duration. Third, the model assumes linear variable costs and divisible shipments. It omits fixed dispatch costs, truck capacities, minimum-load requirements, integer vehicle counts, route-specific tolls, quantity discounts, return logistics, and rounding to practical truckloads.

The objective also excludes holding of inventory, operating costs of warehouse, delivery lead times, time windows, product differentiation, carbon emissions, and service-quality considerations. In shortage scenarios, demand that is unmet is valued uniformly at INR 10,000 per tonne. Because this penalty is a value that is assumed managerially rather than an economic loss, the generalized objective and shortage allocation are penalty-sensitive. Equal penalties also permit the solver to prioritize cheaper hubs even when customer importance differs. Additionally, Chennai distances are routing estimates and may vary with geocoding, road restrictions, commercial vehicle rules, or route-provider updates.

VII. CONCLUSION

This study demonstrates that a classical transportation linear program can provide a plan this is reproducible for a three-warehouse, six-hub Indian distribution network. The base solution minimizes monthly transportation cost at INR 2,705,241 and uses eight of eighteen available routes. Identical results from MODI, Excel Solver, PuLP/CBC, and SciPy/HiGHS, establish strong computational consistency and demonstrate the value of combining traditional operations-research methods with independent open-source verification.

The scenario analysis provides the key managerial insight. The base solution is cost-optimal, however full utilization of the 1,600-tonne capacity leaves the network exposed to increases in demand and disruptions in supply. Freight inflation leads to an increase in cost while preserving service, whereas demand growth, a reduction at the Nagpur capacity, and Chennai expansion create shortages which plummets service below 92%. The concentration of unmet demand in Bengaluru and Kolkata further depicts that a uniform cost-minimization objective may not reflect strategic customer priorities. the verified optimum should be treated as a benchmark rather than a complete operating policy. Implementation requires reserve capacity, contingency supply, and customer-level constraints.

Overall, the study demonstrates that optimization is most valuable when its results are interpreted alongside operational realities. The combined use of manual techniques, Excel Solver, and Python verification provides decision-makers with a transparent basis for evaluating distribution choices and responding more systematically to changes in cost, demand, and capacity.

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