Study of Longitudinal Wave Propagation in Magnetoelastic Circular Rod of Solid Cross Section under the Influence of Hydrostatic Stress.

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***Abstract*—In this paper we studied the method of simultaneous differential equations, the frequency equation of longitudinal wave propagation in magnetoelastic circular rod of solid cross section under the influence of hydrostatic stress. The radius of the cylinder is very small and retaining only first order Bessel functions and derivatives. The longitudinal wave velocity is obtained in terms of Lames constant. The obtained results were presented graphically.**

***Index Terms*—Longitudinal waves, Magnetoelasticity, Circular rod, Wave propagation, Frequency, Wave number, Hydrostatic stress.**

I. INTRODUCTION

The study of wave propagation in cylinders is important in various applications such as ultrasonic nondestructive evaluation techniques. With the advancement of a deep insight in the behavior of materials, especially of the elastic ones that are so frequently used in missiles and other allied systems. Most cylindrical components, such as rods, wires, tubes, pipes and fibers are manufactured by processes that induce elastic properties in them. Without taking into the consideration the effect of the magnetic field, the analysis of longitudinal wave propagation in an elastic and circular cylinder, according to the theory of elasticity, has been done by many authors. Longitudinal wave propagation in a cylinder was studied with in the frame work of classical theory of elasticity by pochammer [1], Chree [2] and Oliver [3] have pulse techniques to study the propagation of waves in cylindrical rods. The problem of the velocity of longitudinal waves in cylindrical bars was explained by Bancroft [4]. Wave propagation in elastic solids was discussed by Achenbach [5].The problem of a mathematical model for longitudinal wave propagation in a magnetoelastic hollow circular cylinder of anisotropic material under the influence of initial hydrostatic stress investigated by N Abd-alla and Alshaikh[6],Sree Lakshmi, Somaiah and Rama [7,8] studied the longitudinal wave propagation in a circular rod of solid cross section and Effect of rotation on longitudinal wave propagation in elastic solid with a cylindrical hole. Some of the analogous results on magnetoelastic wave propagation problems, but in an anisotropic medium, were obtained by Abd-alla [9,10] and Datta [11].

In this paper, the longitudinal wave propagation in magnetoelastic circular rod of solid cross section under the influence of hydrostatic stress is investigated. The displacement components are obtained by founding the analytical solutions of the motion's equations. After applying suitable boundary conditions, the frequency equation is presented as a determinant with elements containing Bessel functions [12, 13], General details and many references on these subjects may be found in mono graphs by Eringen and Suhubi [14]. Finally, the results are given in the form of graphics.

II. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Longitudinal wave propagation in a circular cylinder of elastic material of solid radius a, subjected to an axial magnetic field is considered. Longitudinal waves are axially symmetric waves characterized by the presence of displacements components in the radial and axial directions only. We use the cylindrical coordinate system and (r,θ,z) represent the

coordinates of a point. Therefore, the displacement vector $\vec{u}$ can be taken as

$$\vec{u} = (u_r, 0, u_z) \tag{1}$$

Where

$$u_r = u_r(r,z,t), \quad u_\theta = 0, \quad u_z = u_z(r,z,t)$$

u_r, u_θ, u_z are the displacement components in the radial, circumferential and axial directions, respectively, and all other quantities involved are functions of r, z and t only, where t denotes the time. The equations of motion for a perfectly conducting homogeneous elastic solid under uniform magnetic field and under hydrostatic stress.

$$\nabla.T - P_0\ \nabla.\nabla u + f = \rho\, \ddot{u} \tag{2}$$

Max well's equations may be written in Gaussian units as

$$\nabla \times H = \frac{4\pi}{c} J, \qquad \nabla \times E = \frac{-1}{c}\frac{\partial B}{\partial t},$$

$$\nabla.B = 0, \qquad \nabla.D = 4\pi\rho_e \tag{3}$$

Where ρ_e is the charge density in Gaussian units (which vanishes in our case), while the magnetic field B and generalized Ohm's law are given respectively by

$$B = \mu_e H, \; J = \sigma\left(E + \frac{1}{c}\frac{\partial u}{\partial t} \times B\right) \tag{4}$$

Where $-P_0$ is the hydrostatic tension or compression (tension when $p_0 < 0$ and compression when $p_0 > 0$)

Lorentz force f is given by

$$f = J \times B \tag{5}$$

By substituting the equation (3), and equation (4), into equation (5), we have

$$f = \frac{\mu_0}{4\pi}\left[\nabla \times H\right] \times H_0 \tag{6}$$

Where $\mu_0 \equiv c\mu_e \approx 1$ is the magnetic permeability in the medium and where we have assumed that the magnetic field $H = H_0 + h$ is given by the superposition of a constant magnetic field $H_0 = (0,0,H_0)$, where H_0 is its intensity parallel to the axis of the cylinder, and a small perturbation h.

As is known, in perfectly conducting medium the conductivity $\sigma \to \infty$, and this yields

$$E = \frac{-1}{c}\left(\frac{\partial u}{\partial t} \times B\right), h = \nabla \times (u \times H_0) \tag{7}$$

Where the operator ∇ is defined in terms of unit vectors of the cylindrical coordinates (e_r, e_θ, e_z) as $\nabla = \frac{\partial}{\partial r} e_r + \frac{1}{r}\frac{\partial}{\partial \theta} e_\theta + \frac{\partial}{\partial z} e_z$.

Now, equation (7) may be written as

$$h = H_0 \frac{\partial u_r}{\partial z} e_r - H_0\left[\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right] e_z \tag{8}$$

Using (6) and (7) in (3), Lorentz force becomes

$$f = \rho\alpha^2\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2}\right] e_r \tag{9}$$

where $\alpha^2 = \frac{\mu_0 {H_0}^2}{4\pi\rho}$ and α is the so- called Alfven wave velocity, and the electric field intensity is given as the form

$$E_0 = \frac{\mu_0 H_0}{c}\left[\frac{\partial u_r}{\partial t}\right] \tag{10}$$

Where we have also assumed the form (1) of the components of the displacement field.

The electro -magnetic field equations in vacuum are given by [14].

$$\nabla^2(E^*, h^*) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(E^*, h^*)\ \nabla \times (E^*, h^*) = \frac{1}{c}\frac{\partial}{\partial t}(-h^*, E^*) \tag{11}$$

Where the Laplace operator is defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The strain components are given in terms of the displacements by

$$e_{rr} = \frac{\partial u_r}{\partial r}, \tag{12}$$

$$e_{rz} = \frac{1}{2}\left[\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right]$$

For a tetragonal elastic body, piola stress tensor components are given in terms of the independent elastic constants as follows

$$t_{rr} = (\lambda + 2\mu)\frac{\partial u_r}{\partial r} + \lambda\frac{u_r}{r} + \lambda\frac{\partial u_z}{\partial z} = 0 \quad (13)$$

$$t_{rz} = \mu\left(\frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial r}\right) = 0 \quad (14)$$

For more details on constitutive laws of elastic material, the reader is referred, Substituting (12) in (2), using (9), (10) and (11), we may get the equations of motion in terms of the displacement components as

Now, the equations of motion given by equation (2) reduce to

$$(\lambda + 2\mu)\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{1}{r^2}u_r\right] + \lambda\left[\frac{\partial^2 u_z}{\partial r \partial z}\right] + \rho\alpha^2\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2}\right] - p_0 \quad (15)$$

$$\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2}\right] = \rho\frac{\partial^2 u_r}{\partial t^2}$$

$$(\lambda + \mu)\left[\frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z}\right] + (\lambda + 2\mu)\left[\frac{\partial^2 u_z}{\partial z^2}\right] + \mu\left[\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right]$$

$$- p_0\left[\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2}\right] = \rho\frac{\partial^2 u_z}{\partial t^2} \quad (16)$$

III. SOLUTION OF THE PROBLEM

We use trigonometric functions in order to represent $\vec{u}$ attenuated waves propagating in the z - direction. Hence, we seek the solution of simultaneous differential equations (15) and (16) as

$$u_r = A\,J_1(hr)\cos(qz - wt) \quad (17)$$

$$u_z = B\,J_0(hr)\sin(qz - wt) \quad (18)$$

Where $J_0(hr)$ and $J_1(hr)$ denote the Bessel functions of order zero and one respectively and q is the wave number. The phase velocity of wave propagation in the z – direction is $\frac{w}{q} = c$ Substituting equations (17) and (18) in equations (15) and (16) we get,

$$\left[-(\lambda + 2\mu)h^2 - q^2\mu + \rho\omega^2 + (p_0 - \rho\alpha^2)(h^2 + q^2)\right]A - \left[(\lambda + \mu)hq\right]B = 0 \quad (19)$$

$$\left[-(\lambda + \mu)hq\right]A + \left[-(\lambda + 2\mu)q^2 - \mu h^2 + p_0(h^2 + q^2) + \rho\omega^2\right]B = 0 \quad (20)$$

For the existence of non -trivial solution of (19) and (20), the determinant of the coefficient matrix must be zero

$$i.e., \left|a_{ij}\right| = 0 \quad (21)$$

were

$$a_{11} = \left[-(\lambda + 2\mu)h^2 - q^2\mu + \rho\omega^2 + (p_0 - \rho\alpha^2)(h^2 + q^2)\right]$$

$$a_{12} = -(\lambda + \mu)hq$$

$$a_{21} = -(\lambda + \mu)hq$$

$$a_{22} = \left[-(\lambda + 2\mu)q^2 - \mu h^2 + p_0(h^2 + q^2) + \rho\omega^2\right]$$

For solving the determinant (21), we obtain the following quadratic equation in h^2 so assuming two roots say h_1^2 and h_2^2. As h_1 and h_2 roots of an equation corresponding to the simultaneous differential equation (15) and (16), the solution of these equations is given by the form

$$\left(h^2\right)^2 + Ph^2 + Q = 0 \quad (23)$$

Were

$$P = \left[-(\lambda + 3\mu)(\rho\omega^2 + 2q^2p_0 - \rho\alpha^2q^2) + 2\mu(\lambda + 2\mu)q^2 + \rho\omega^2(2p_0 - \rho\alpha^2)\right](\mu - p_0)^{-1}$$
$$\left[(\lambda + 2\mu) - (p_0 - \rho\alpha^2)\right]^{-1} \quad (24)$$

$$Q = \rho^2\omega^4 + q^2\left[-(\lambda + 3\mu)\rho\omega^2 + \rho\omega^2(2p_0 - \rho\alpha^2)\right]$$
$$+ q^4\left[\mu(\lambda + 2\mu) - p_0(\lambda + 3\mu) + \rho\alpha^2(\lambda + 2\mu) + (p_0 - \rho\alpha^2)p_0\right] \quad (25)$$
$$(\mu - p_0)^{-1}\left[(\lambda + 2\mu) - (p_0 - \rho\alpha^2)\right]^{-1}$$

so, assuming two roots say h_1^2 and h_2^2. As h_1 and h_2 roots of an equation corresponding to the simultaneous differential equations (15) and (16), the solution of these equations is given by the form

$$u_r = \left[J_1(h_1r)A + J_1(h_2r)B\right]\cos(qz - \omega t) \quad (26)$$

$$u_z = \left[J_0(h_1r)A - \frac{h_2}{q}J_0(h_2r)B\right]\sin(qz - wt) \quad (27)$$

where $h_1^2, h_2^2 = \frac{1}{2}\left[-p \pm (p^2 - 4Q)^{\frac{1}{2}}\right]$

IV. FREQUENCY EQUATION

Substituting equations (26) and (27) in equations (15) and (16). We have the following simultaneous homogeneous linear equations in A and B and after elimination of unknowns A, B We get the frequency equation

$$\det(b_{ij}) = 0 \quad (28)$$

Were

$$b_{11} = \frac{-2\mu}{a}J_1(h_1a) + (\lambda + 2\mu)h_1J_0(h_1a) + \frac{\lambda q^2}{h_1}J_0(h_1a)$$

$$b_{12} = \frac{-2\mu}{a}J_1(h_2a) + (\lambda + 2\mu)h_2J_0(h_2a) - \lambda h_2\,J_0(h_2a)$$

$$b_{21} = -2\mu q\,J_1(h_1a) \quad (29)$$

$$b_{22} = \frac{\mu}{q}h_2^2\,J_1(h_2a) - \mu\,q\,J_1(h_2a)$$

V. NUMERICAL RESULTS AND DISCUSSION

For investigation of the dispersion relation (29), in detail, we shall find the variation of frequency and wave number. For numerical computations in detail, we taken material values from [15],

Magnesium

$\lambda = 9.4 \times 10^{10}\, Nm^{-2}, \mu = 40 \times 10^{10}\, Nm^{-2},$

$\rho = 1.74 \times 10^{3}\, kgm^{-3}$,

$p_0 = -5 \times 10^{11}, 0, 5 \times 10^{11}, \alpha^2 = 1,2,3$; $a = 1$,

$H_0 = 10^5\, oersted.$

And Zinc material values

$\lambda = 0.385 \times 10^{11}\, Nm^{-2}, \mu = 0.508 \times 10^{11}\, Nm^{-2},$

$\rho = 7.14 \times 10^{3}\, kgm^{-3},$

$p_0 = -5 \times 10^{11}, 0, 5 \times 10^{11}, \alpha^2 = 1,2,3\,; H_0 = 10^5\, oersted.$

The frequency equations (28), (29) are solved numerically, Figures 1-2 represent the first, second and third modes, respectively of the frequency versus different values of wave number for three different values of the initial stress $p_o = (-5,0,5) \times 10^{11}\, N/m^2$ and m=1 for the value of the primary magnetic field $H_o = 10^5$ oersted.

Fig1, 2 shows that the frequency versus wave number are plotted. From Fig 1,2, it is observed that the frequency decreases as the wave number increases for both magnesium and Zinc materials, indicating an inverse relationship between them. Under different hydrostatic stress conditions (=0, +5, -5), all curves follow the same decreasing trend, indicating that hydrostatic stress changes the magnitude but not the overall behavior.

Magnesium shows higher frequency values than Zinc across the entire range, while Zinc exhibits a comparatively steeper decrease, indicating greater sensitivity to changes in wave number. If we neglect magnetic field and hydrostatic stress reduces to classical case [8]. Thus, hydrostatic stress shifts the frequency values without altering the general inverse relationship between frequency and wave number.

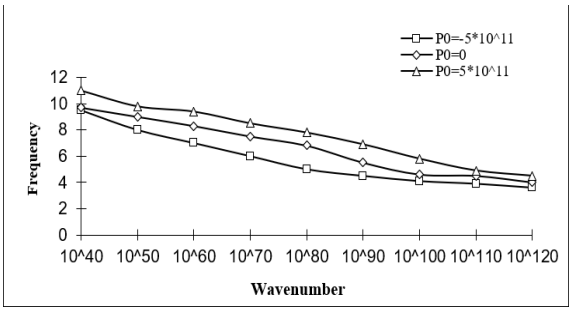

Fig: 1 Variation of frequency with wave number (magnesium material)

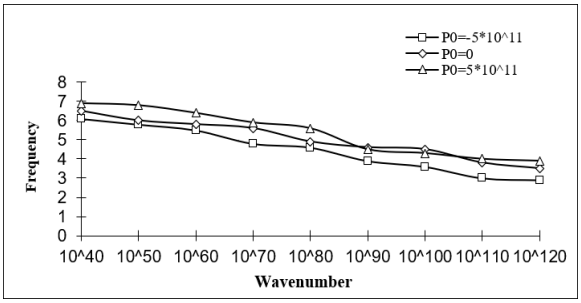

Fig: 2 Variation of frequency with wave number (zinc material)

VI. CONCLUSION

longitudinal wave propagation in magnetoelastic circular rod of solid cross section under the influence of hydrostatic stress has been investigated after deriving the secular equation and we observed that the natural frequent of longitudinal wave propagation depends on the wave number and showing that the frequency equation is dispersive curves.

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