Mathematical Analysis of Bianchi Type-V Space-Time in f(R, T)Gravity with Variable Deceleration Parameter

Anjani Kumar Kushwaha[1], Dr. R.K. Dubey[2]
[1]*Research Scholar Department of Mathematical Sciences Awadhesh Pratap Singh University Rewa 486003 (M.P.), India*
[2]*Professor Department of Mathematics Govt. T.R.S. College, Rewa (M.P.)*

***Abstract*—In this paper, we investigate a spatially homogeneous and anisotropic Bianchi Type-V cosmological model in the framework of f(R, T)gravity, where Rdenotes the Ricci scalar and Trepresents the trace of the energy-momentum tensor. To obtain exact solutions of the modified field equations, the functional form f(R, T) = R + 2λTis considered and a variable deceleration parameter is assumed. The adopted condition enables the determination of the metric potentials and the average scale factor in explicit form. Various physical and geometrical parameters of the model, including the Hubble parameter, expansion scalar, shear scalar, anisotropy parameter, energy density, and isotropic pressure, are derived and analyzed. The obtained solution describes an expanding universe and exhibits the essential features of anisotropic cosmological evolution. The behavior of the deceleration parameter indicates the possibility of accelerated cosmic expansion within the modified gravity framework. It is shown that the anisotropic effects decrease with the evolution of the universe, leading to an approach toward isotropy at late times. The singular and asymptotic properties of the model are also discussed through a mathematical examination of the cosmological parameters.**

The study demonstrates that Bianchi Type-V space-time in f(R, T)gravity provides a viable mathematical cosmological model for describing the large-scale evolution of the universe without introducing exotic dark energy components explicitly. The exact solutions obtained herein may be useful for further investigations of anisotropic cosmological scenarios in modified theories of gravitation.

***Index Terms*—Bianchi Type-V Space-Time, f(R, T)Gravity, Exact Solutions, Variable Deceleration Parameter, Anisotropic Cosmology, Accelerating Universe.**

I. INTRODUCTION

The accelerated expansion of the universe is one of the most important discoveries in modern cosmology. Observations based on Type Ia Supernovae, Cosmic Microwave Background (CMB) radiation, and large-scale structure surveys indicate that the present universe is undergoing an accelerated phase of expansion. Although Einstein's General Theory of Relativity successfully describes many gravitational phenomena, it faces difficulties in explaining this late-time acceleration without introducing additional exotic matter components. Consequently, modified theories of gravity have been proposed as possible alternatives to General Relativity.

Among these theories, f(R, T)gravity introduced by Harko et al. has received considerable attention. In this theory, the gravitational Lagrangian is expressed as a function of the Ricci scalar Rand the trace Tof the energy-momentum tensor. The coupling between matter and geometry provides a useful framework for studying different phases of cosmic evolution, including accelerated expansion and anisotropic cosmological behaviour.

Anisotropic cosmological models play an important role in understanding the early universe. Although present observations suggest that the universe is nearly isotropic on large scales, anisotropic effects may have been significant during the initial stages of cosmic evolution. In this context, Bianchi space-times provide a natural generalization of the standard isotropic cosmological models.

Among the Bianchi models, the Bianchi Type-V space-time is of particular interest due to its homogeneous but anisotropic nature and negative spatial curvature. It represents a generalized open

universe model and has been widely used in the study of anisotropic cosmology within different modified theories of gravity. Several investigations have demonstrated that Bianchi Type-V models can successfully describe the evolution of the universe from an anisotropic state toward isotropy at late times. Motivated by these developments, we study a Bianchi Type-V cosmological model in the framework of $f(R,T)$gravity by considering the linear functional form

$$f(R,T) = R + 2\lambda T,$$

where $\lambda$is a coupling constant. To obtain exact solutions of the modified field equations, a constant deceleration parameter is assumed. The resulting solutions are used to analyze important cosmological quantities such as the Hubble parameter, expansion scalar, shear scalar, anisotropy parameter, energy density, and pressure.

The paper is organized as follows. Section 2 presents the basic field equations and the Bianchi Type-V metric. Exact solutions of the model are obtained in Section 3. The physical and geometrical properties of the cosmological model are discussed in Section 4. Finally, the main conclusions are summarized in Section 5.

II. FIELD EQUATIONS OF $F(R,T)$GRAVITY AND BIANCHI TYPE-V SPACE-TIME

To investigate the anisotropic evolution of the universe, we consider the gravitational action of $f(R,T)$gravity proposed by Harko et al., which is given by

$$S = \int \left[\frac{f(R,T)}{16\pi} + L_m\right]\sqrt{-g}\, d^4x,$$

where $f(R,T)$is an arbitrary function of the Ricci scalar $R$and the trace $T$of the energy-momentum tensor T_{ij}, $L_m$is the matter Lagrangian density, and $g$denotes the determinant of the metric tensor.

Variation of the above action with respect to the metric tensor g_{ij}yields the gravitational field equations

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij},$$

were

$$f_R = \frac{\partial f}{\partial R},\ f_T = \frac{\partial f}{\partial T},$$

and

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m.$$

For a perfect fluid distribution, the energy-momentum tensor is

$$T_{ij} = (\rho + p)u_iu_j + p\, g_{ij},$$

where $\rho$and $p$represent the energy density and isotropic pressure respectively, and $u_i$is the four-velocity vector satisfying

$$u^iu_i = -1.$$

Following the commonly adopted linear functional form

$$f(R,T) = R + 2\lambda T,$$

where $\lambda$is a constant coupling parameter, the modified field equations reduce to

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\lambda)T_{ij} + \lambda(2p + T)g_{ij}.$$

Since

$$T = g^{ij}T_{ij} = -\rho + 3p,$$

the field equations can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\lambda)T_{ij} + \lambda(\rho - p)g_{ij}.$$

The above equations represent the gravitational field equations governing the cosmological model under consideration.

Bianchi Type-V Space-Time

We consider the spatially homogeneous and anisotropic Bianchi Type-V metric in the form

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + e^{2mx}[B^2(t)\,dy^2 + C^2(t)\,dz^2],$$

where $A(t)$, $B(t)$, and $C(t)$are directional scale factors and $m$is a non-zero constant.

The average scale factor $a(t)$is defined by

$$a = (ABC)^{1/3}.$$

Consequently, the spatial volume is

$$V = a^3 = ABC.$$

The generalized mean Hubble parameter is given by

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right),$$

or equivalently

$$H = \frac{\dot{a}}{a}.$$

The directional Hubble parameters along the three spatial directions are defined as

$$H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{B}, H_z = \frac{\dot{C}}{C}.$$

Using the metric coefficients of the Bianchi Type-V space-time, the modified field equations assume the form

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}$$
$$=-(8\pi+3\lambda)p+\lambda\rho,\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}$$
$$-\frac{m^2}{A^2}$$
$$=-(8\pi+3\lambda)p+\lambda\rho,\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}$$
$$-\frac{m^2}{A^2}=-(8\pi+3\lambda)p+\lambda\rho,$$

and

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3m^2}{A^2}=(8\pi+3\lambda)\rho-\lambda p.$$

The off-diagonal component yields the important relation

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0.$$

Integrating, we obtain

$$A^2=k\,BC,$$

where kis an integration constant. Without loss of generality, taking $k=1$, we get

$$A^2=BC.$$

This relation significantly simplifies the system of field equations and facilitates the determination of exact solutions.

Thus, the modified Einstein field equations corresponding to the Bianchi Type-V space-time have been established. In the next section, these equations will be solved by assuming a suitable law for the deceleration parameter, leading to exact cosmological solutions.

III. EXACT SOLUTION OF THE MODEL

The system of field equations obtained in the previous section contains more unknowns than the available independent equations. Therefore, an additional physically reasonable condition is required to obtain a determinate solution. For this purpose, we assume a constant deceleration parameter, which has been extensively used in cosmological investigations to describe different phases of cosmic evolution.

The deceleration parameter is defined by

$$q=-\frac{a\ddot{a}}{\dot{a}^2},$$

where adenotes the average scale factor of the universe.

Assuming the deceleration parameter to be constant, say

$$q=\beta-1,$$

where βis a positive constant, the above differential equation yields

$$\frac{a\ddot{a}}{\dot{a}^2}=1-\beta.$$

Integrating, we obtain the average scale factor in the form

$$a(t)=a_0t^{\frac{1}{\beta}},\ \beta\neq0,$$

where $a_0$is an integration constant.

Without loss of generality, we choose

$$a_0=1,$$

so that

$$a(t)=t^{\frac{1}{\beta}}.$$

Hence, the spatial volume becomes

$$V=a^3=t^{\frac{3}{\beta}}.$$

Since

$$V=ABC,$$

we have

$$ABC=t^{\frac{3}{\beta}}.$$

From the off-diagonal field equation

$$A^2=BC,$$

it follows that

$$A=a=t^{\frac{1}{\beta}}.$$

To determine the remaining metric potentials Band C, we introduce the relation

$$B=A\,e^{F(t)},C=A\,e^{-F(t)},$$

which automatically satisfies

$$A^2=BC.$$

Subtracting the second and third field equations, we obtain

$$\frac{d}{dt}\left(\ln\frac{B}{C}\right)=\frac{k}{ABC},$$

where kis an integration constant.

Using

$$ABC=t^{\frac{3}{\beta}},$$

we get

$$\frac{d}{dt}\left(\ln\frac{B}{C}\right)=kt^{-\frac{3}{\beta}}.$$

Integrating,

$$\ln\frac{B}{C}=\frac{k\beta}{\beta-3}t^{\frac{\beta-3}{\beta}}+c_1,\ (\beta\neq3),$$

where $c_1$is a constant of integration.

Choosing $c_1=0$for simplicity, we obtain

$$\frac{B}{C}=\exp\left[\frac{k\beta}{\beta-3}t^{\frac{\beta-3}{\beta}}\right].$$

Consequently,

$$B = t^{\frac{1}{\beta}} \exp\left[\frac{k\beta}{2(\beta-3)} t^{\frac{\beta-3}{\beta}}\right],$$

and

$$C = t^{\frac{1}{\beta}} \exp\left[-\frac{k\beta}{2(\beta-3)} t^{\frac{\beta-3}{\beta}}\right].$$

Thus, the metric potentials are completely determined as

$$A = t^{\frac{1}{\beta}}, B = t^{\frac{1}{\beta}} \exp\left[\frac{k\beta}{2(\beta-3)} t^{\frac{\beta-3}{\beta}}\right], C = t^{\frac{1}{\beta}} \exp\left[-\frac{k\beta}{2(\beta-3)} t^{\frac{\beta-3}{\beta}}\right].$$

Therefore, the exact Bianchi Type-V space-time assumes the form

$$ds^2 = -dt^2 + t^{\frac{2}{\beta}} dx^2 + e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2].$$

The directional Hubble parameters are given by

$$H_x = \frac{1}{\beta t}, H_y = \frac{1}{\beta t} + \frac{k}{2} t^{-\frac{3}{\beta}}, H_z = \frac{1}{\beta t} - \frac{k}{2} t^{-\frac{3}{\beta}}.$$

The mean Hubble parameter becomes

$$H = \frac{1}{\beta t}.$$

It is evident that the expansion rate decreases as cosmic time increases. Furthermore,

$$\lim_{t\to\infty}(H_y - H_z) = 0,$$

which indicates that the anisotropic expansion gradually diminishes during the evolution of the universe.

Hence, an exact anisotropic cosmological solution of the modified field equations in f(R, T)gravity has been obtained. In the next section, the physical and geometrical properties of the model will be investigated through important cosmological parameters such as the expansion scalar, shear scalar, anisotropy parameter, energy density, and pressure.

IV. PHYSICAL AND GEOMETRICAL BEHAVIOUR OF THE MODEL

In this section, we investigate the physical behaviour of the obtained Bianchi Type-V cosmological model through various kinematical and dynamical parameters. These parameters play a fundamental role in describing the expansion history and anisotropic evolution of the universe.

4.1 Mean Hubble Parameter

The mean Hubble parameter is defined as

$$H = \frac{1}{3}(H_x + H_y + H_z).$$

Using the directional Hubble parameters obtained in the previous section, we have

$$H = \frac{1}{\beta t}.$$

Since $H > 0$ for $t > 0$, the universe remains in an expanding phase throughout its evolution.

Furthermore,

$$\lim_{t\to\infty} H = 0.$$

Thus, although the universe continues to expand, the rate of expansion gradually decreases with cosmic time.

4.2 Expansion Scalar

The scalar expansion is defined by

$$\theta = u^i_{;i}.$$

For the present model,

$$\theta = 3H.$$

Hence,

$$\theta = \frac{3}{\beta t}.$$

Clearly,

$$\lim_{t\to 0} \theta = \infty,$$

and

$$\lim_{t\to\infty} \theta = 0.$$

Therefore, the model starts with an infinitely large rate of expansion and expands continuously as the universe evolves.

4.3 Shear Scalar

The shear scalar measures the deviation from isotropic expansion and is given by

$$\sigma^2 = \frac{1}{2}\left(H_x^2 + H_y^2 + H_z^2 - \frac{1}{3}\theta^2\right).$$

Substituting the directional Hubble parameters

$$H_x = \frac{1}{\beta t}, H_y = \frac{1}{\beta t} + \frac{k}{2} t^{-\frac{3}{\beta}}, H_z = \frac{1}{\beta t} - \frac{k}{2} t^{-\frac{3}{\beta}},$$

we obtain

$$\sigma^2 = \frac{k^2}{4} t^{-\frac{6}{\beta}}.$$

or

$$\sigma = \frac{k}{2} t^{-\frac{3}{\beta}}.$$

Since

$$\lim_{t\to\infty} \sigma = 0,$$

the anisotropy decreases with time.

The anisotropy parameter is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2.$$

After simplification, we obtain

$$A_m = \frac{k^2\beta^2}{6}t^{2-\frac{6}{\beta}}.$$

For suitable values of β,

$$\lim_{t\to\infty} A_m = 0.$$

Hence, the universe evolves from an anisotropic state toward an isotropic state at late times.

4.5 Deceleration Parameter

The deceleration parameter of the model is

$$q = \beta - 1.$$

The nature of cosmic expansion depends on the value of β.

Case I

If

$$\beta > 1,$$

then

$$q > 0,$$

which corresponds to a decelerating universe.

Case II

If

$$0 < \beta < 1,$$

then

$$q < 0,$$

which represents an accelerating universe.

Case III

If

$$\beta = 1,$$

then

$$q = 0,$$

and the model expands with constant velocity.

Thus, the obtained solution is capable of describing both accelerating and decelerating cosmological phases.

4.6 Energy Density

Using the modified field equation

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda p,$$

and substituting the exact solutions, the energy density can be written in the form

$$\rho = \frac{1}{8\pi + 4\lambda}\left[\frac{3}{\beta^2 t^2} - \frac{k^2}{4}t^{-\frac{6}{\beta}} - \frac{3m^2}{t^{\frac{2}{\beta}}}\right].$$

The density diverges near the initial singularity and decreases as time progresses.

Therefore,

$$\lim_{t\to 0}\rho = \infty,$$

while

$$\lim_{t\to\infty}\rho = 0.$$

This behaviour agrees with the standard cosmological picture.

4.7 Isotropic Pressure

The isotropic pressure is obtained from the remaining field equations and assumes the form

$$p = \frac{1}{8\pi + 4\lambda}\left[\frac{2\beta - 3}{\beta^2 t^2} + \frac{k^2}{4}t^{-\frac{6}{\beta}} + \frac{m^2}{t^{\frac{2}{\beta}}}\right].$$

The pressure remains a decreasing function of cosmic time and approaches zero at sufficiently large times.

4.8 Singular and Asymptotic Behaviour

From the above analysis, the following observations can be made:

1. The spatial volume
$$V = t^{\frac{3}{\beta}}$$
vanishes at
$$t = 0,$$
indicating a point-type singularity.
2. The Hubble parameter and expansion scalar diverge at the initial epoch.
3. The energy density and pressure become infinitely large near the singularity.
4. The shear scalar decreases continuously during cosmic evolution.
5. At late times,
$$\frac{\sigma}{\theta} \to 0,$$
showing that anisotropy disappears asymptotically.
6. The universe evolves toward an isotropic expanding phase.

Hence, the obtained Bianchi Type-V model in $f(R,T)$gravity represents a physically acceptable anisotropic cosmological solution that begins with a Big-Bang-type singularity and gradually approaches isotropy during its evolution.

V. RESULTS AND DISCUSSION

The exact solution obtained in the framework of $f(R,T)$gravity describes a spatially homogeneous and anisotropic Bianchi Type-V universe. The model is characterized by a power-law expansion governed by the average scale factor

$$a(t) = t^{\frac{1}{\beta}},$$

which leads to an increasing spatial volume

$$V = t^{\frac{3}{\beta}}.$$

Hence, the universe expands continuously with the progress of cosmic time.

The directional scale factors $A(t)$, $B(t)$, and $C(t)$remain finite for $t > 0$and satisfy the relation

$$A^2 = BC,$$

which is a characteristic property of the present Bianchi Type-V model. The exact solutions show that the directional expansion rates are different during the early stages of evolution, indicating the existence of anisotropy in the primordial universe.

The mean Hubble parameter

$$H = \frac{1}{\beta t}$$

is a positive decreasing function of time. This behaviour implies that the model represents an expanding universe whose expansion rate decreases gradually as the universe evolves.

The expansion scalar

$$\theta = \frac{3}{\beta t}$$

becomes infinite at

$$t = 0,$$

which indicates the presence of an initial Big-Bang-type singularity. As time increases, the expansion scalar decreases and approaches zero asymptotically.

The anisotropic nature of the model is measured by the shear scalar

$$\sigma^2 = \frac{k^2}{4} t^{-\frac{6}{\beta}}.$$

Since

$$\lim_{t\to\infty} \sigma = 0,$$

the anisotropic effects gradually disappear during the cosmic evolution. This result is further confirmed by the behaviour of the mean anisotropy parameter A_m, which tends to zero at late times. Therefore, the universe evolves from an anisotropic phase to an isotropic phase, consistent with the observed large-scale isotropy of the present universe.

The deceleration parameter

$$q = \beta - 1$$

plays an important role in determining the expansion dynamics. For

$$0 < \beta < 1,$$

the deceleration parameter becomes negative, indicating accelerated expansion. Thus, the present model successfully explains the possibility of a late-time accelerating universe within the framework of modified gravity without introducing any additional dark energy component.

The physical quantities such as energy density and pressure are positive in the early epoch and decrease with time. Their behaviour remains compatible with the standard cosmological picture in which matter domination gradually weakens as the universe expands.

Overall, the obtained model provides a mathematically consistent description of cosmic evolution and demonstrates that the matter-geometry coupling present in $f(R,T)$gravity has significant effects on the dynamics of anisotropic universes.

VI. CONCLUSION

In the present paper, an anisotropic Bianchi Type-V cosmological model has been investigated in the framework of $f(R,T)$gravity. By selecting the functional form

$$f(R,T) = R + 2\lambda T,$$

and assuming a constant deceleration parameter, exact solutions of the modified gravitational field equations have been obtained.

The resulting model describes a spatially homogeneous universe undergoing power-law expansion. Exact expressions for the metric potentials have been determined, and various physical parameters including the Hubble parameter, expansion scalar, shear scalar, anisotropy parameter, energy density, and isotropic pressure have been evaluated.

The analysis reveals that the model originates from a Big-Bang-type singularity where the cosmological parameters diverge. The spatial volume increases continuously with cosmic time, while the expansion and shear scalars decrease during evolution. The shear

scalar tends to zero at late times, indicating that the universe gradually approaches isotropy.

The behaviour of the deceleration parameter shows that both decelerating and accelerating phases can be described depending upon the choice of model parameters. In particular, the model admits accelerated expansion for suitable values of the parameter β, which is consistent with current cosmological observations.

Therefore, the present investigation demonstrates that Bianchi Type-V space-time in f(R, T)gravity provides a viable mathematical cosmological model capable of explaining the evolution of an anisotropic universe and its transition toward isotropy without invoking exotic dark energy components explicitly. The model may be further extended by considering variable deceleration parameters, bulk viscous fluids, or other generalized modified theories of gravity.

REFERENCES

[1] K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, "Dark energy cosmology: The equivalent description via different theoretical models and cosmography tests," *Astrophysics and Space Science*, vol. 342, no. 1, pp. 155–228, 2012, doi: 10.1007/s10509-012-1181-8.

[2] L. Bianchi, "Sugli spazi a tre dimensioni che ammettono un gruppo continuo di movimenti," *Memorie di Matematica e di Fisica della Società Italiana delle Scienze*, vol. 11, pp. 267–352, 1898.

[3] C. Brans and R. H. Dicke, "Mach's principle and a relativistic theory of gravitation," *Physical Review*, vol. 124, no. 3, pp. 925–935, 1961, doi: 10.1103/PhysRev.124.925.

[4] S. Capozziello and M. De Laurentis, "Extended theories of gravity," *Physics Reports*, vol. 509, nos. 4–5, pp. 167–321, 2011, doi: 10.1016/j.physrep.2011.09.003.

[5] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, "Modified gravity and cosmology," *Physics Reports*, vol. 513, nos. 1–3, pp. 1–189, 2012, doi: 10.1016/j.physrep.2012.01.001.

[6] J. Daimary, K. K. Das, and H. K. Barman, "Anisotropic LRS Bianchi type-V cosmological models in Saez-Ballester scalar-tensor theory," *Journal of Scientific Research*, vol. 16, no. 1, pp. 115–125, 2024.

[7] G. F. R. Ellis and M. A. H. MacCallum, "A class of homogeneous cosmological models," *Communications in Mathematical Physics*, vol. 12, no. 2, pp. 108–141, 1969, doi: 10.1007/BF01645908.

[8] T. Harko, F. S. N. Lobo, S. Nojiri, and S. D. Odintsov, "f(R, T) gravity," *Physical Review D*, vol. 84, no. 2, Art. no. 024020, 2011, doi: 10.1103/PhysRevD.84.024020.

[9] S. W. Hawking and G. F. R. Ellis, *The Large-Scale Structure of Space-Time*. Cambridge, U.K.: Cambridge University Press, 1973.

[10] R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure*. Cambridge, U.K.: Cambridge University Press, 2000.

[11] W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*. San Francisco, CA, USA: W. H. Freeman and Company, 1973.

[12] R. Myrzakulov, K. Myrzakulov, and L. Sebastiani, "Cosmological models in teleparallel gravity with Bianchi type-V metric," *Astrophysics and Space Science*, vol. 368, Art. no. 45, 2023.

[13] S. Nojiri and S. D. Odintsov, "Introduction to modified gravity and gravitational alternative for dark energy," *International Journal of Geometric Methods in Modern Physics*, vol. 4, no. 1, pp. 115–146, 2007, doi: 10.1142/S0219887807001928.

[14] S. Perlmutter *et al*., "Measurements of Ω and Λ from 42 high-redshift supernovae," *The Astrophysical Journal*, vol. 517, no. 2, pp. 565–586, 1999, doi: 10.1086/307221.

[15] Planck Collaboration, "Planck 2015 results. XIII. Cosmological parameters," *Astronomy & Astrophysics*, vol. 594, Art. no. A13, 2016, doi: 10.1051/0004-6361/201525830.

[16] G. Riess *et al*., "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *The Astronomical Journal*, vol. 116, no. 3, pp. 1009–1038, 1998, doi: 10.1086/300499.

[17] A. Starobinsky, "A new type of isotropic cosmological models without singularity," *Physics Letters B*, vol. 91, no. 1, pp. 99–102, 1980, doi: 10.1016/0370-2693(80)90670-X.

[18] S. Tsujikawa, "Modified gravity models of dark energy," *Lecture Notes in Physics*, vol. 800, pp.

99–145, 2010, doi: 10.1007/978-3-642-10598-2_3.

[19] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. New York, NY, USA: John Wiley & Sons, 1972.

[20] S. Nojiri and S. D. Odintsov, "Unified cosmic history in modified gravity: From \(F(R)\) theory to Lorentz non-invariant models," *Physics Reports*, vol. 505, nos. 2–4, pp. 59–144, 2011, doi: 10.1016/j.physrep.2011.04.001.